Nucleon Structure at Jefferson Lab
I) the 6 GeV findings

Rolf Ent, 12th HANUC Lecture Week, Mar. 26, 2009

- Deep Exclusive Reactions:
  - Constraints on Angular Momentum
  - Proton Tomography

- Semi-Inclusive Deep Inelastic Scattering (SIDIS)
  - Flavor Decomposition
  - Transverse Momentum Dependence

II) the 12 GeV program

§ Intro to "12 GeV Upgrade" and Status
§ Form Factors – Constraints on the GPDs
§ Valence Quark Structure and Parton Distributions
§ Deep Exclusive Scattering and GPDs

The Spin Structure of the Proton

Proton helicity sum rule:

$$\frac{1}{2} = \frac{1}{2} \qquad + \quad G + L_q + L_g$$

~ 0.3

Small?

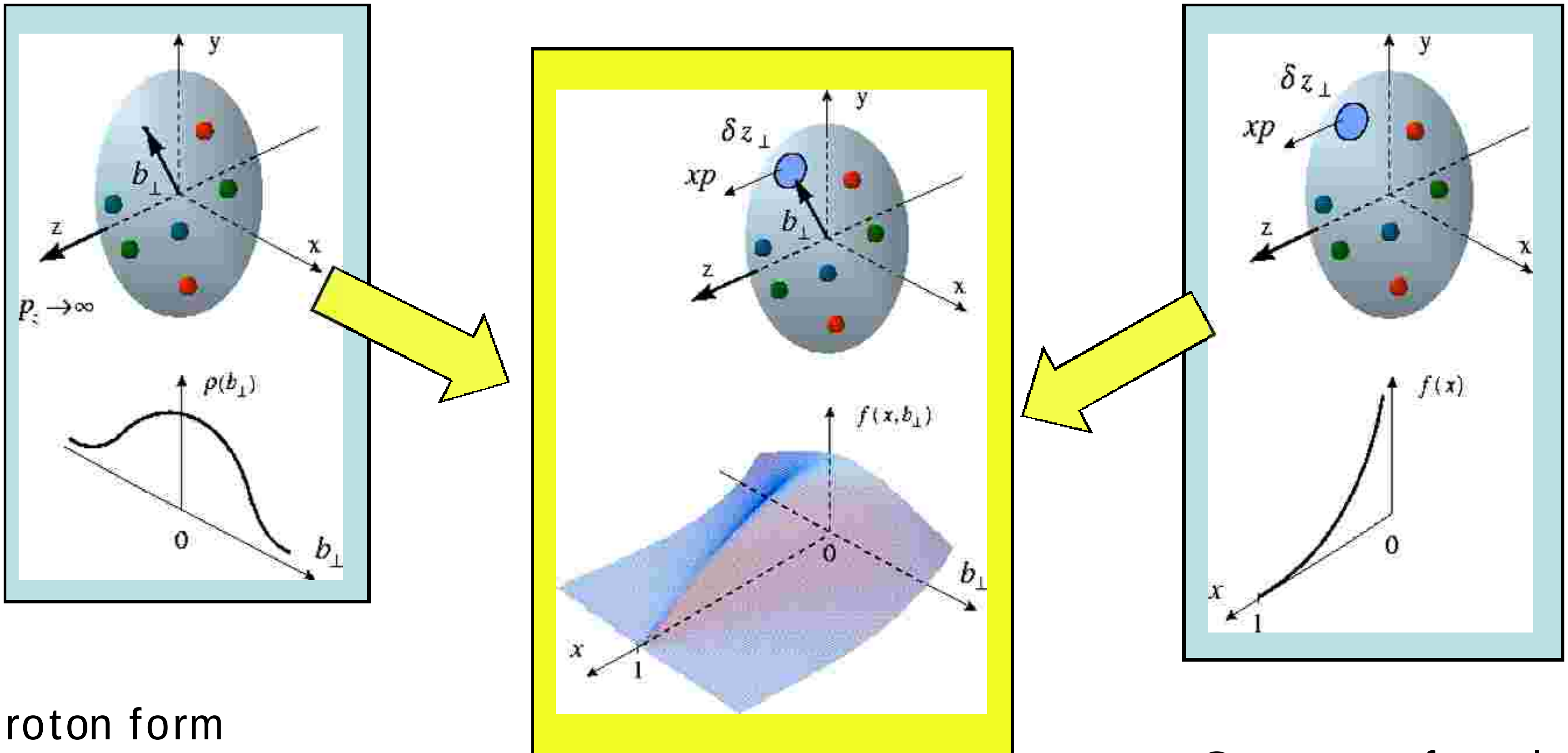
? Large ?

The Impact of Quark and Gluon Motion on the Nucleon Spin

“TMDs and GPDs”

Beyond form factors and quark distributions – Generalized Parton Distributions (GPDs)

X. Ji, D. Mueller, A. Radyushkin (1994-1997)



Proton form factors, transverse charge & current densities

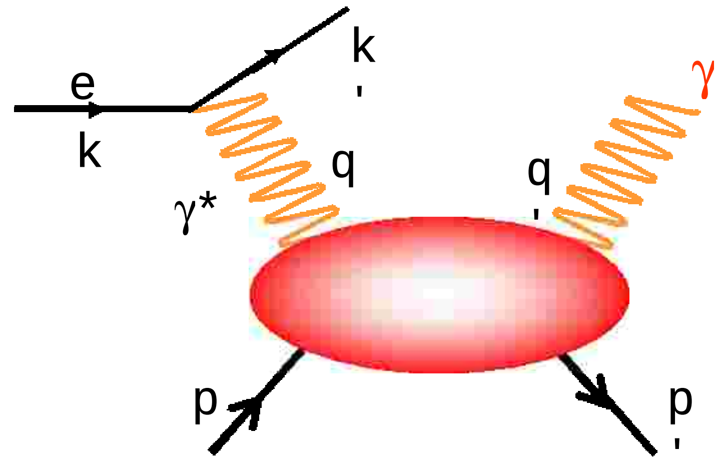
Correlated quark momentum and helicity distributions in transverse space - GPDs

Structure functions, quark longitudinal momentum & helicity distributions

Generalized Parton Distributions and Nucleon Tomography

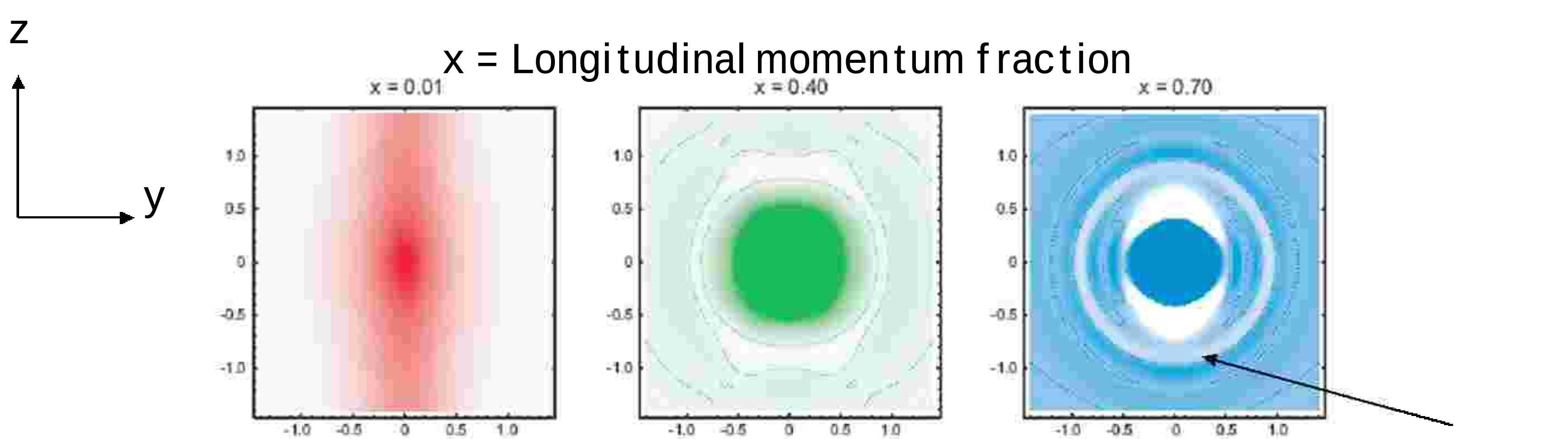
Accessible through (Deep) Exclusive Reactions

A Major new direction in Hadron Physics aimed at the 3-D mapping of the quark structure of the nucleon.



Simplest process:
Deep-Virtual Compton Scattering

x = Longitudinal momentum fraction



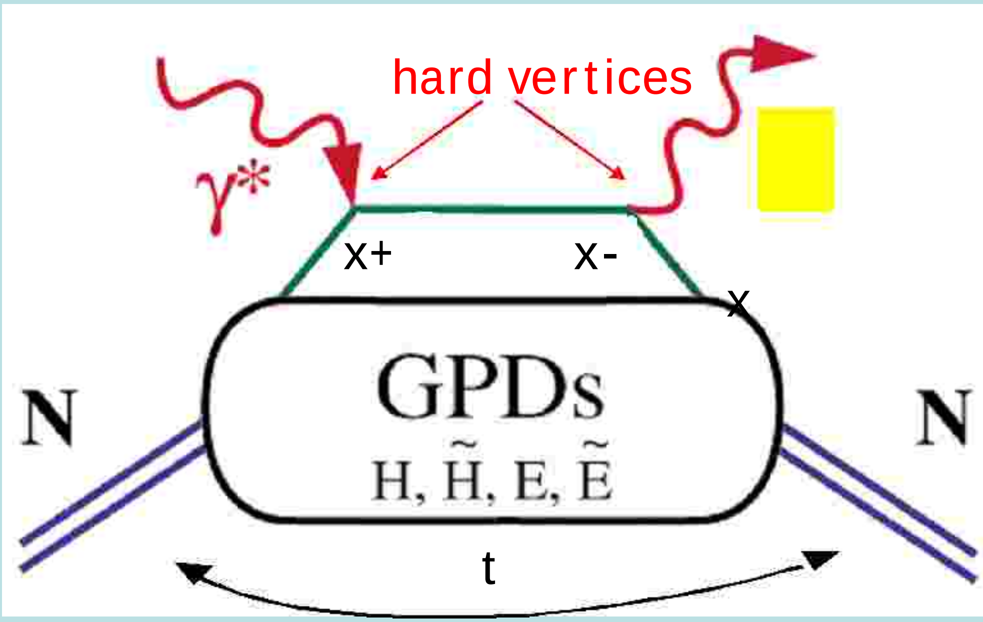
Charge density distributions for u-quarks

3D image is obtained by rotation around the z-axis

GPDs & Deeply Virtual Exclusive Processes

"handbag" mechanism

Deeply Virtual Compton Scattering (DVCS)



x – quark momentum fraction

ξ – longitudinal momentum transfer

$\sqrt{-t}$ – Fourier conjugate to transverse impact parameter

At large Q^2 : QCD factorization theorem à hard exclusive process can be described by 4 transitions (Generalized Parton Distributions) :

$H(x,\xi,t), E(x,\xi,t), \ldots$ "Generalized Parton Distributions"

Vector : $H(x,\xi,t)$ Axial-Vector : $\tilde{H}(x,\xi,t)$

Tensor : $E(x,\xi,t)$ Pseudoscalar : $\tilde{E}(x,\xi,t)$

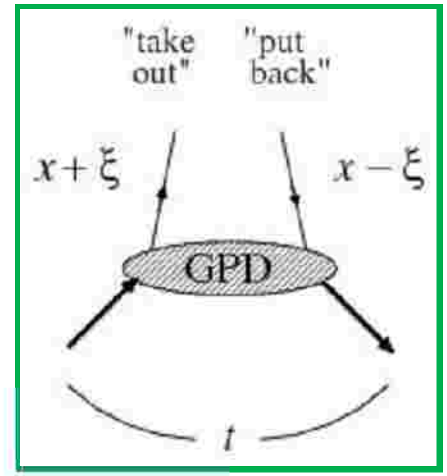
Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx\,\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

What's the use of GPDs?

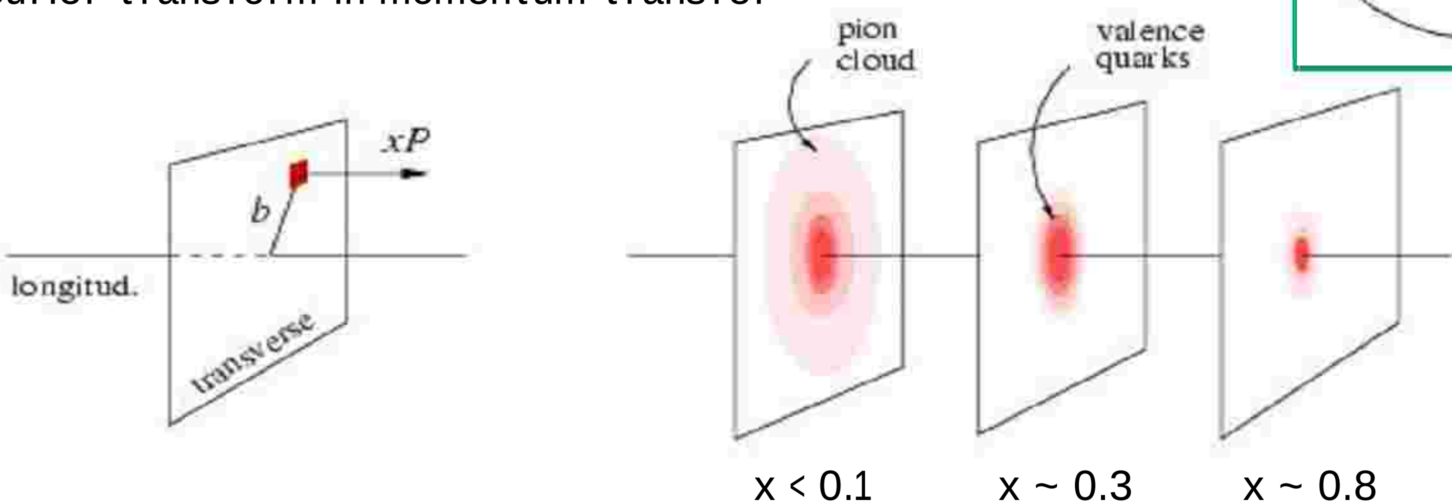
1. Allows for a unified description of form factors and parton distributions



2. Describe correlations of quarks/gluons

3. Allows for Transverse Imaging

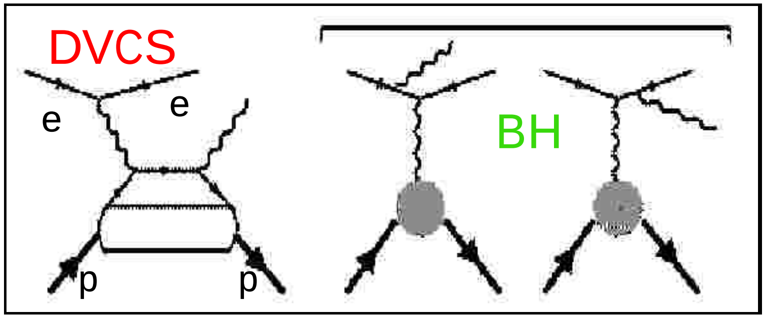
Fourier transform in momentum transfer



gives transverse spatial distribution of quark (parton) with momentum fraction x

4. Allows access to quark angular momentum (in model-dependent way)

Accessing GPDs through DVCS

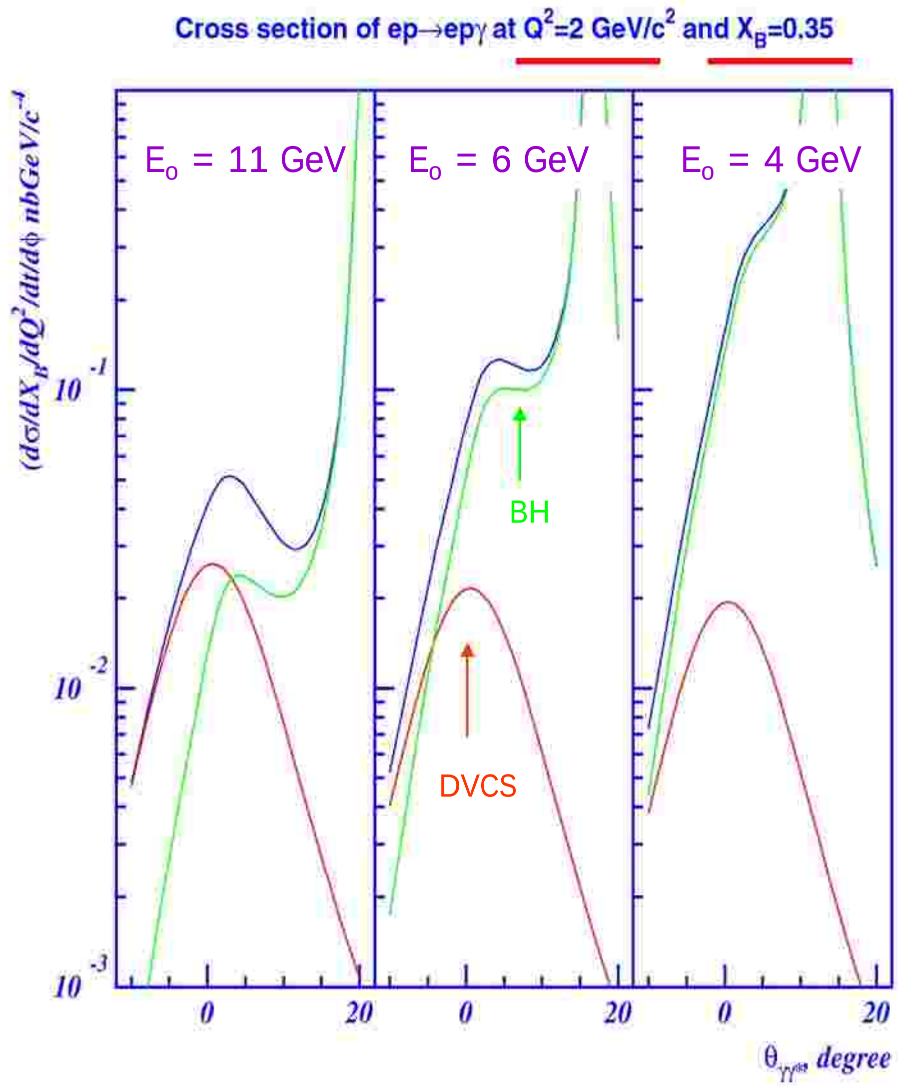


$$\frac{d^4\sigma}{dQ^2 dx_B dt d\phi} \sim |T^{DVCS} + T^{BH}|^2$$

T^{BH}: given by elastic form factors F_1, F_2
T^{DVCS}: determined by GPDs

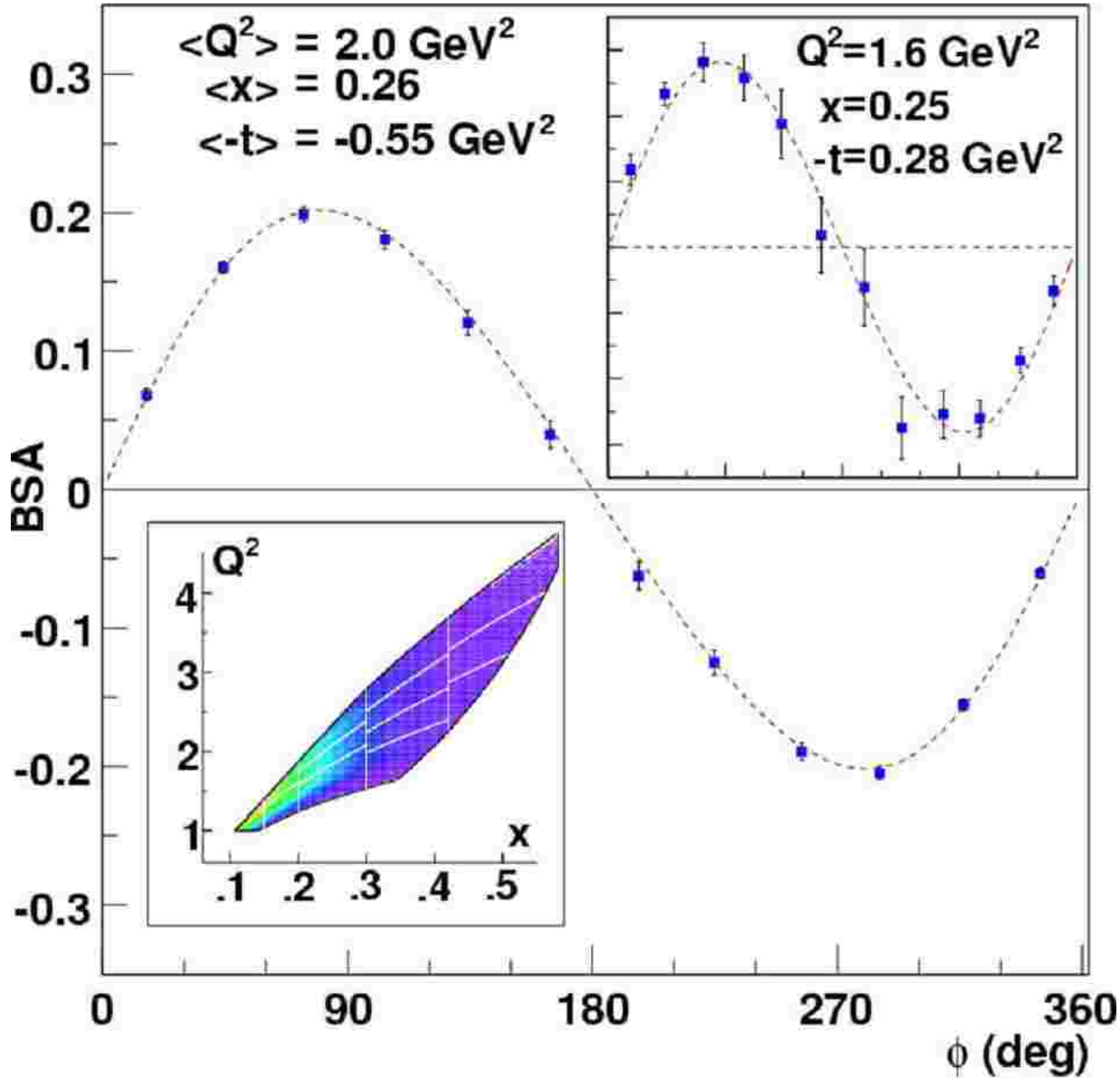
$$I \sim (T^{BH}) Im(T^{DVCS})$$

BH-DVCS interference generates *beam and target polarization asymmetries* that carry the proton structure information.



Transverse size of quark (parton) with longitudinal momentum fraction x

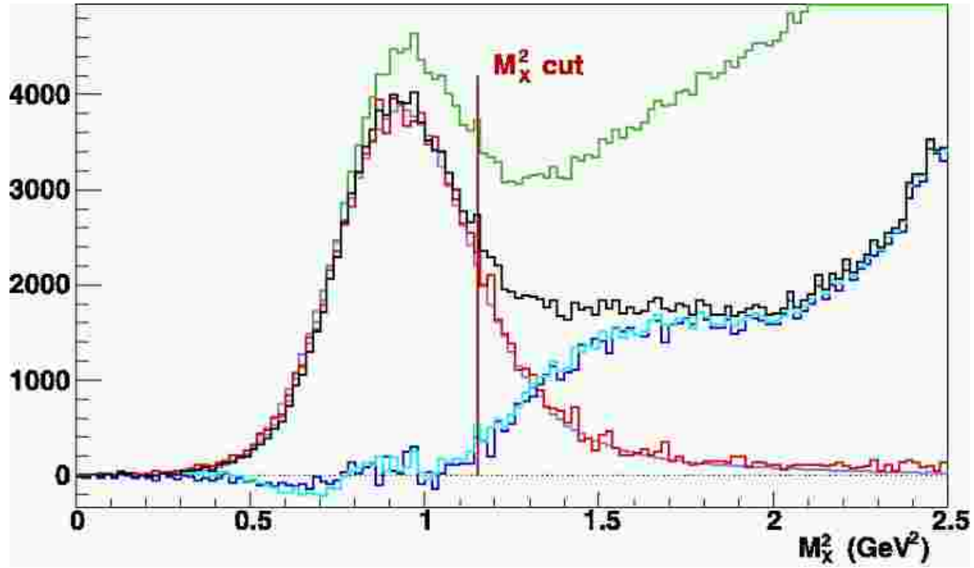
Next Round of DVCS Experiments: Hall B

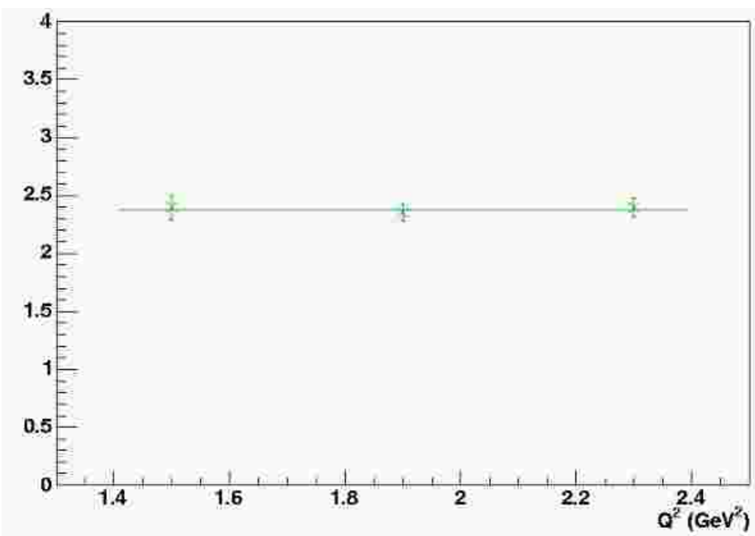


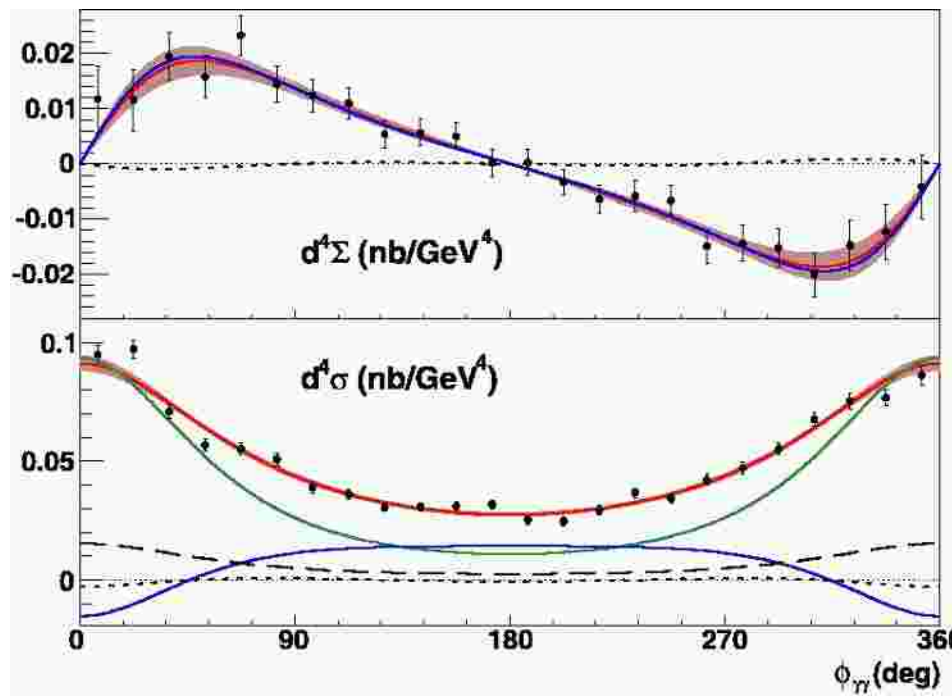
Range in (x, Q^2, t)

Next Round of DVCS Experiments: Hall A

- First explicit demonstration of exclusivity in the DVCS reaction, due to excellent missing-mass resolution.
- For a selected bin in Q^2 and t, the helicity-dependent (top) and helicity-independent (bottom) cross sections, compared with the twist-2 (blue, long-dashed) and twist-3 (dotted) contributions.
- Q^2-variation of the twist-2 term
  à indication of scaling







Next Round of DVCS Experiments: Hall B

Asymmetries, 0.4 < -t < 0.6 GeV^2

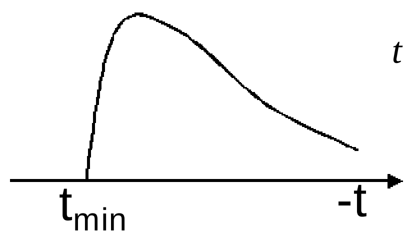
a = A_{LU}(90) as a function of t

Target Unpolarized
Beam Longitudinally Polarized

- F distributions compatible with (leading-twist) parameterization

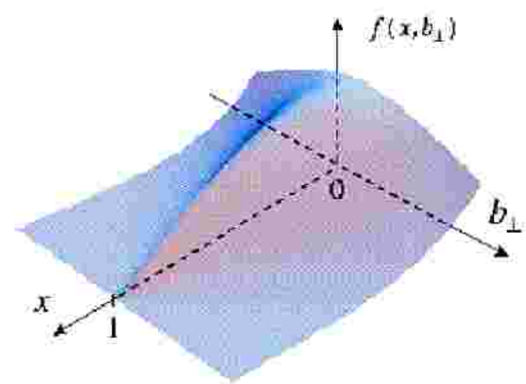
$$A(f) = \frac{a \sin f}{1 + b \cos f}$$

- a = A_{LU}(90)



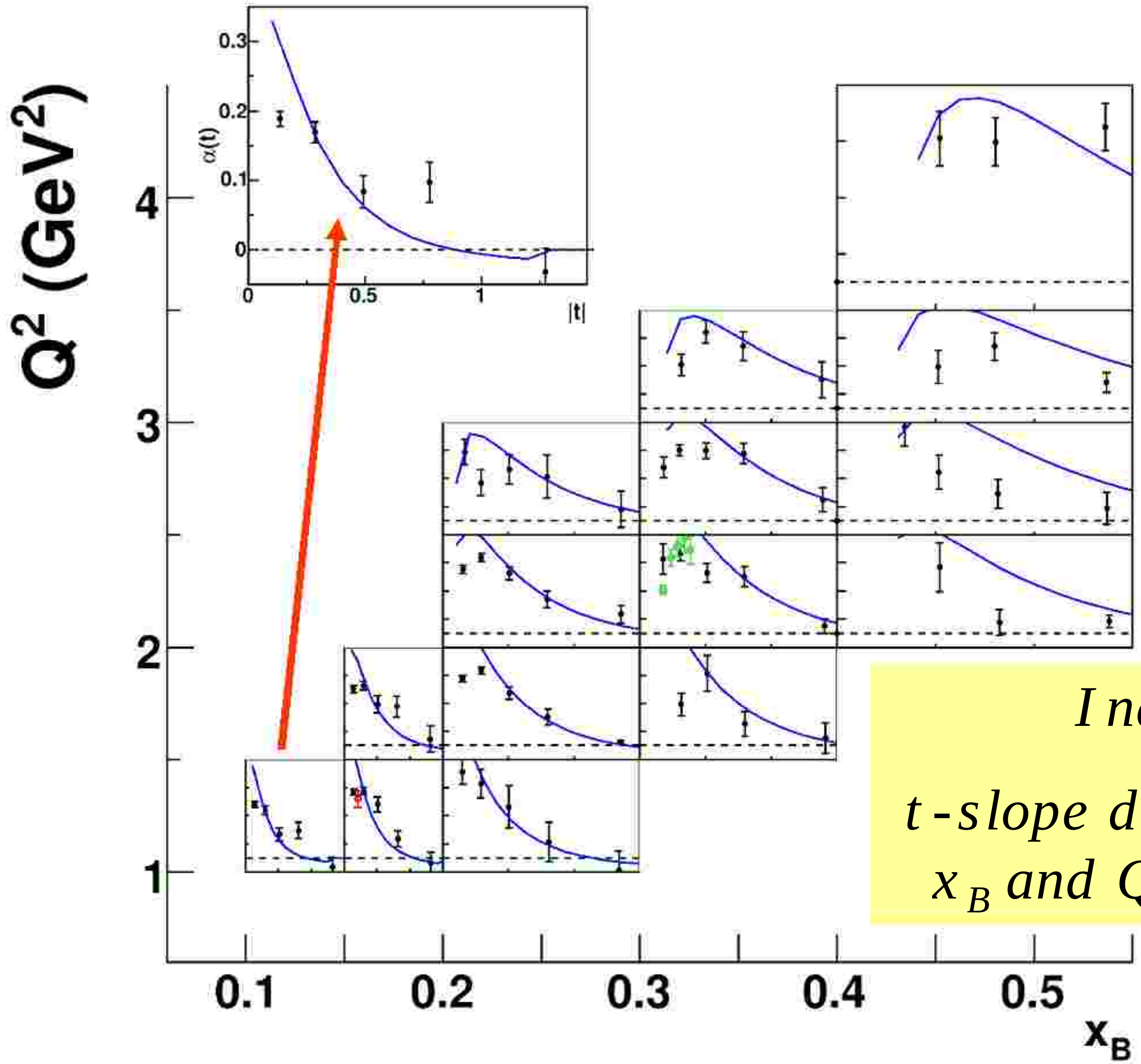
t-slope decreases as x_B and Q^2 increase

- Comparison to Vanderhaeghen, Guichon, Guidal GPD Model Calculation à



Next Round of DVCS Experiments: Hall B

a = $A_{LU}(90)$ as a function of t



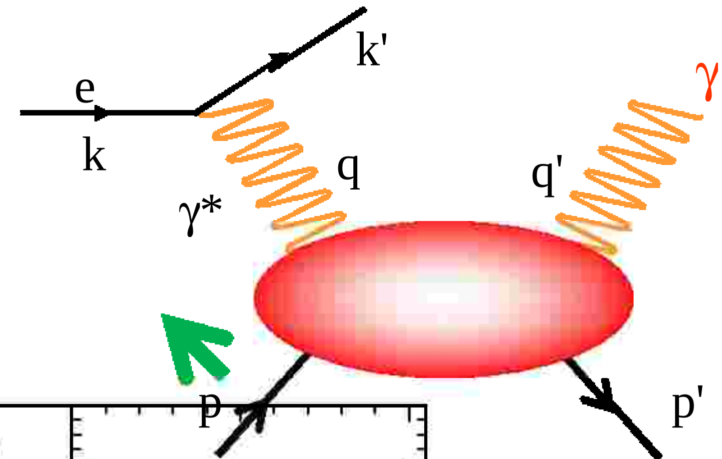
Indeed:

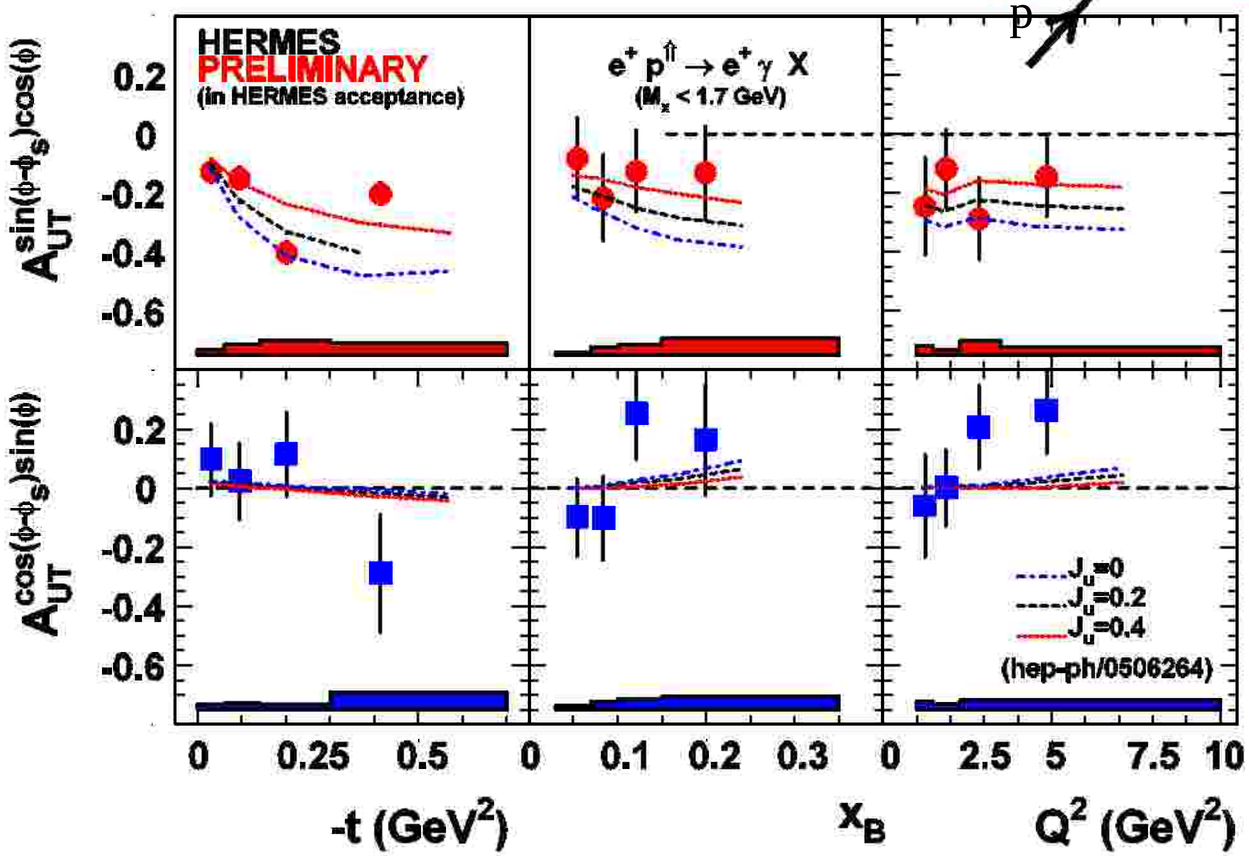
t-slope decreases as x_B and Q^2 increase

Next Round of DVCS Experiments: HERMES

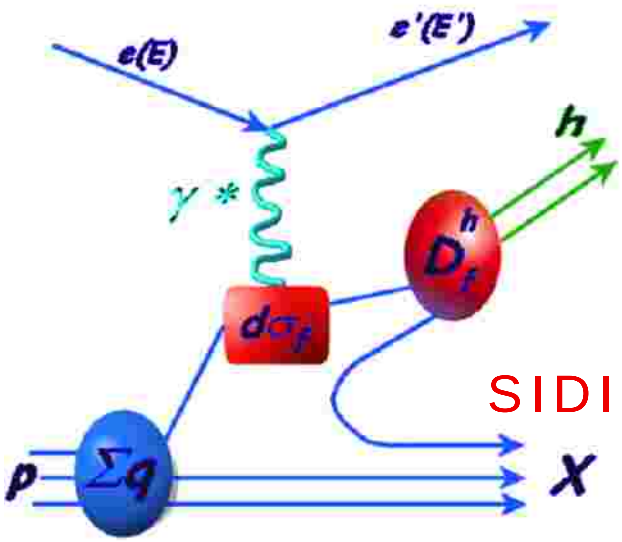
DVCS: "TTSA"

First model-dependent extraction of J_u possible (J_d assumed to be zero)





SIDIS – Flavor Decomposition



SIDIS

(M,0)

DIS probes only the sum of quarks and anti-quarks à requires assumptions on the role of sea quarks $\sum e_q^2(q+\bar{q})$

Solution: Detect a final state hadron in addition to scattered electron

à Can 'tag' the flavor of the struck quark by measuring the hadrons produced: 'flavor tagging'

(e,e')

$$M_x^2 = W^2 = M^2 + Q^2 (1/x - 1)$$

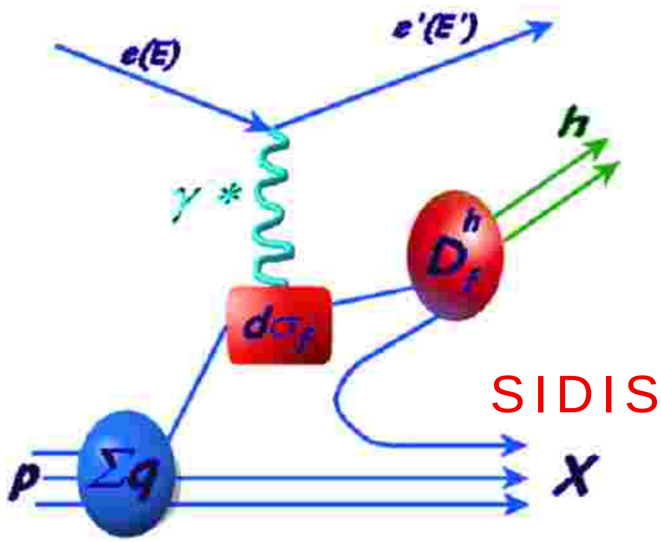
(For M_m small, $\vec{p}_m$ collinear with $\vec{\ }$, and $Q^2/\ ^2 \ll 1$)

(e,e'm)

$$M_x^2 = W'^2 = M^2 + Q^2 (1/x - 1)(1 - z)$$

$z = E_m/$

SIDIS – Flavor Decomposition



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$$\frac{1}{\sigma_{(e,e')}} \frac{d\sigma}{dz}(ep \to hX) = \frac{\sum_q e_q^2 f_q(x) D_q^h(z)}{\sum_q e_q^2(x) f_q(x)}$$

$f_q(x)$: parton distribution function

$D_q^h(z)$: fragmentation function

Measure inclusive (e,e') at same time as (e,e'h)

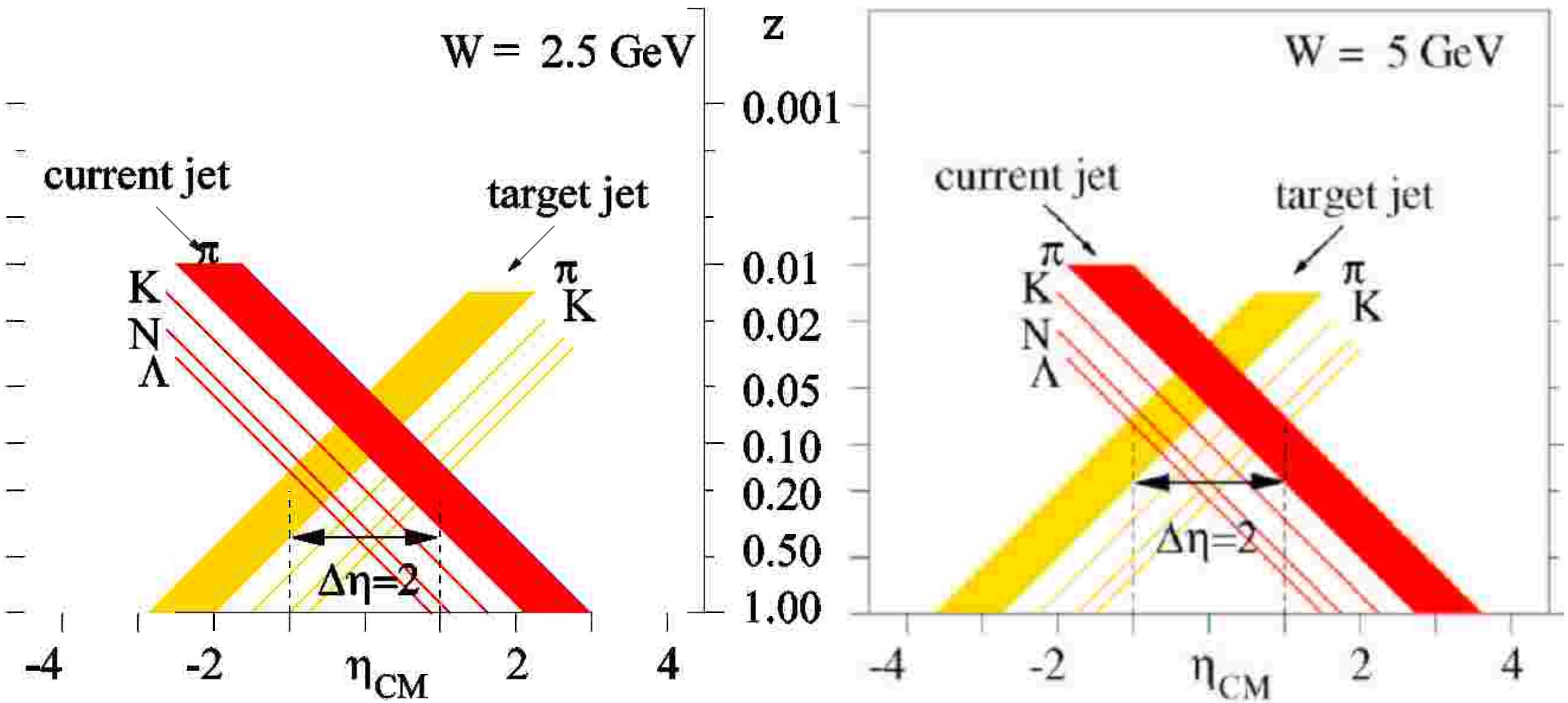
- Leading-Order (LO) QCD
- after integration over p_T and
- NLO: gluon radiation mixes x and z dependences

Low-Energy x-z Factorization

$$\sum_q e_q^2\, q(x)\, D_{q\to M}(z)$$

(after integration over p_T and)

P.J. Mulders, hep-ph/0010199 (EPIC Workshop, MIT, 2000)



At large z-values easier to separate current and target fragmentation region

à for fast hadrons factorization (Berger Criterion) “works” at lower energies

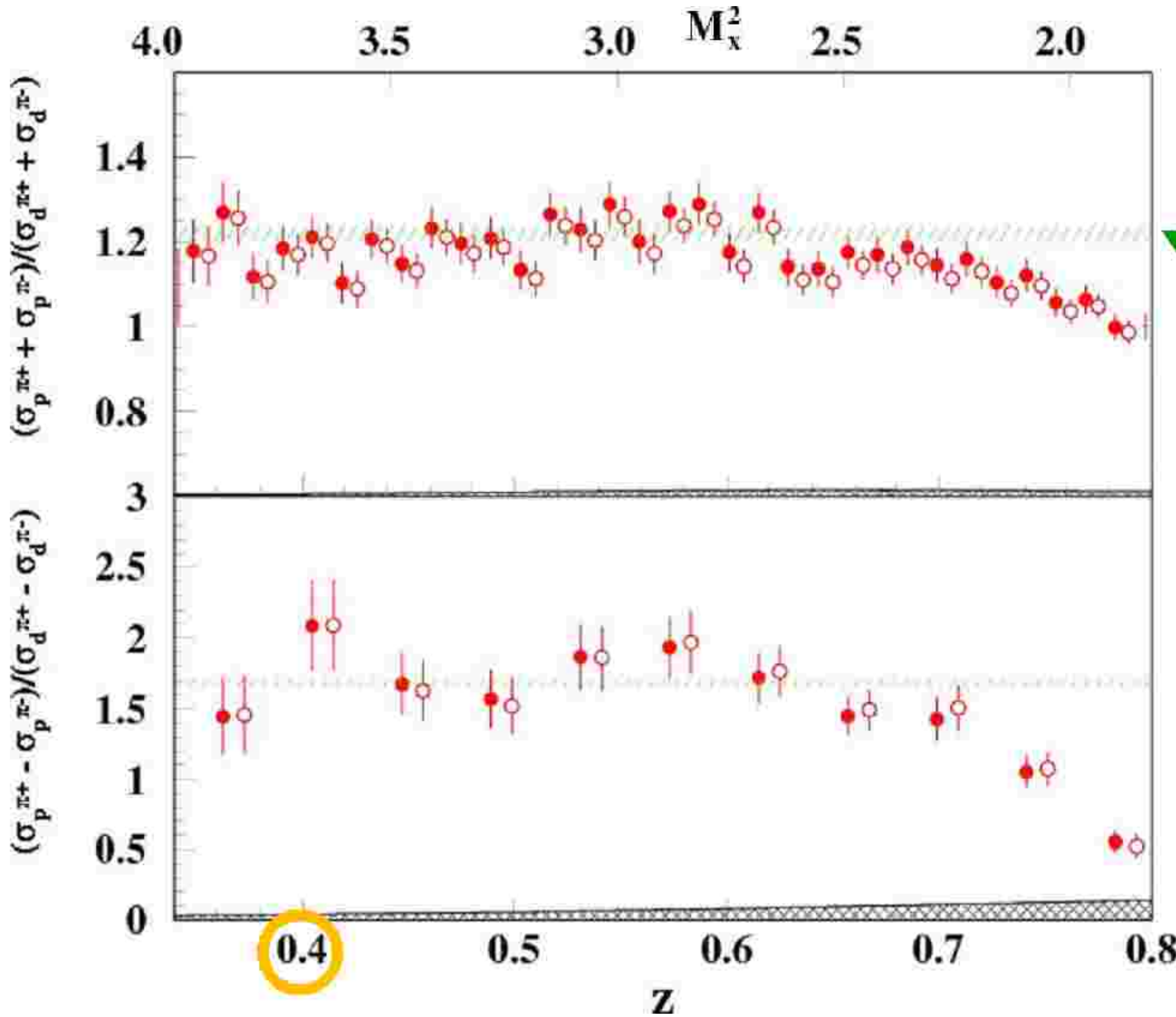
At W = 2.5 GeV: z > 0.4

(Typical JLab-6 GeV)

At W = 5 GeV: z > 0.2

(Typical HERMES)

E00-108: Leading-Order x-z factorization



$$\sum_q e_q^2 \, q(x) \, D_{q\to M}(z)$$

GRV & CTEQ, @ LO or NLO

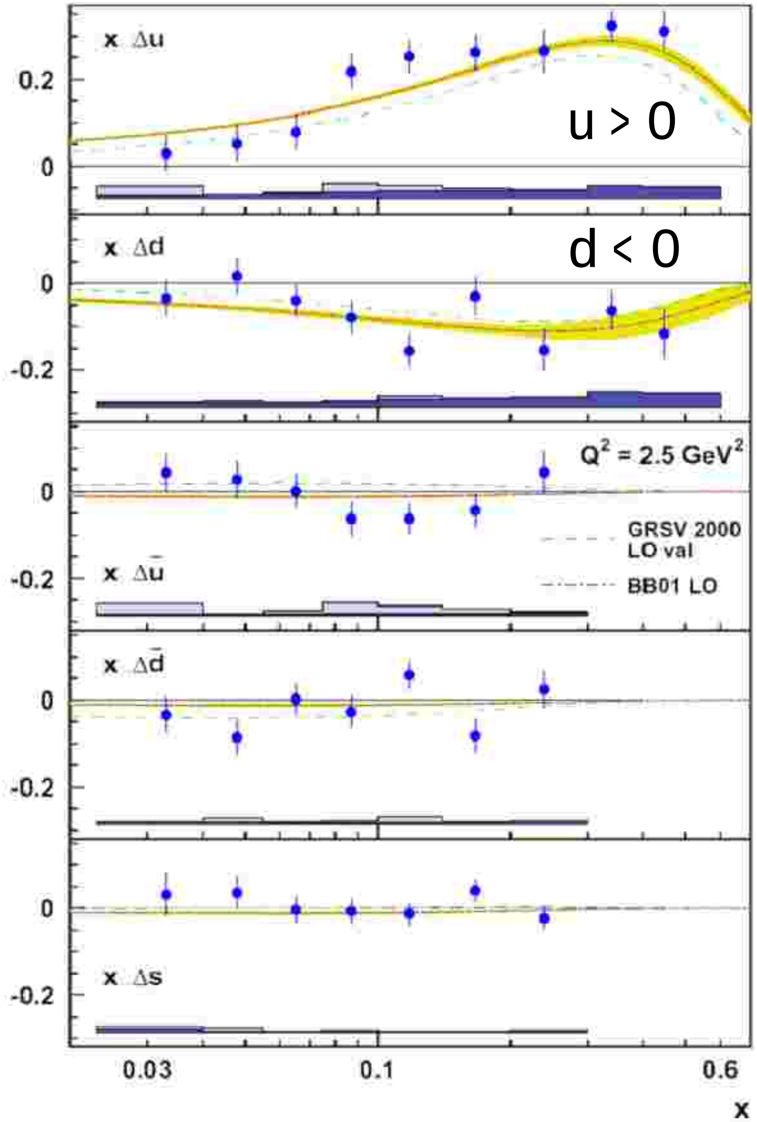
Good description for p and d targets for 0.4 < z < 0.65

(Note: z = 0.65 ~ M_x^2 = 2.5 GeV²)

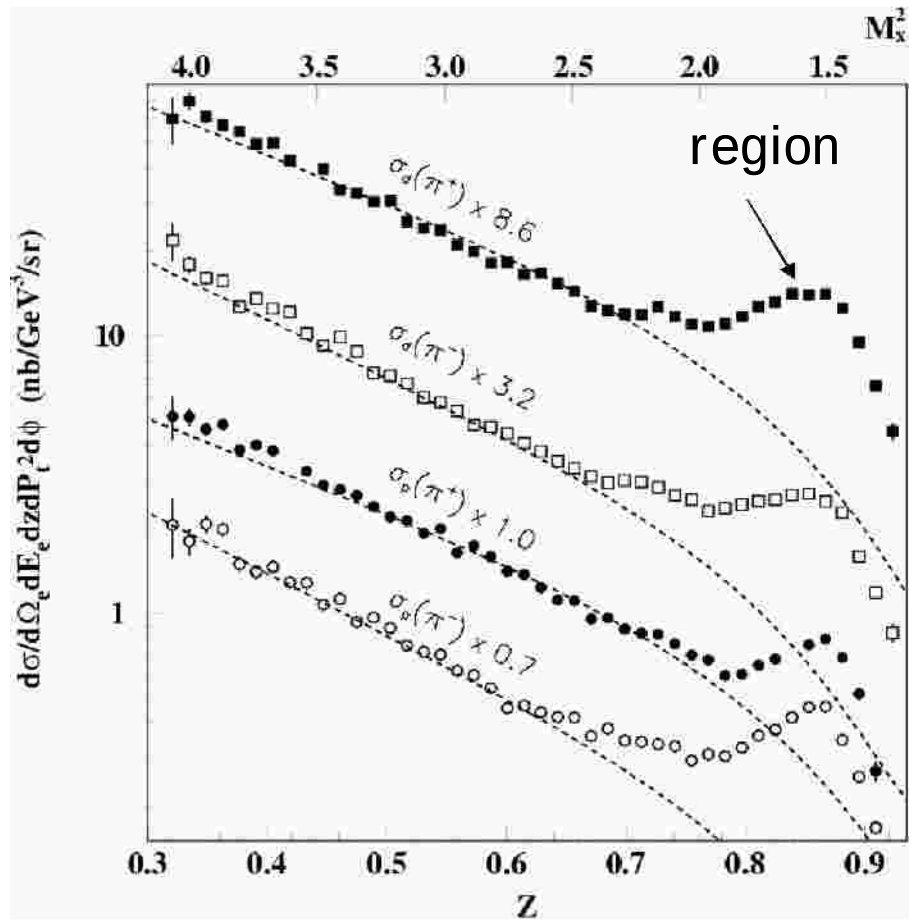
Closed (open) symbols reflect data after (before) events from coherent production are subtracted

Flavor Decomposition

v quark polarization q(x)
à first 5-flavor separation



Flavor decomposition within reach at 12-GeV JLab



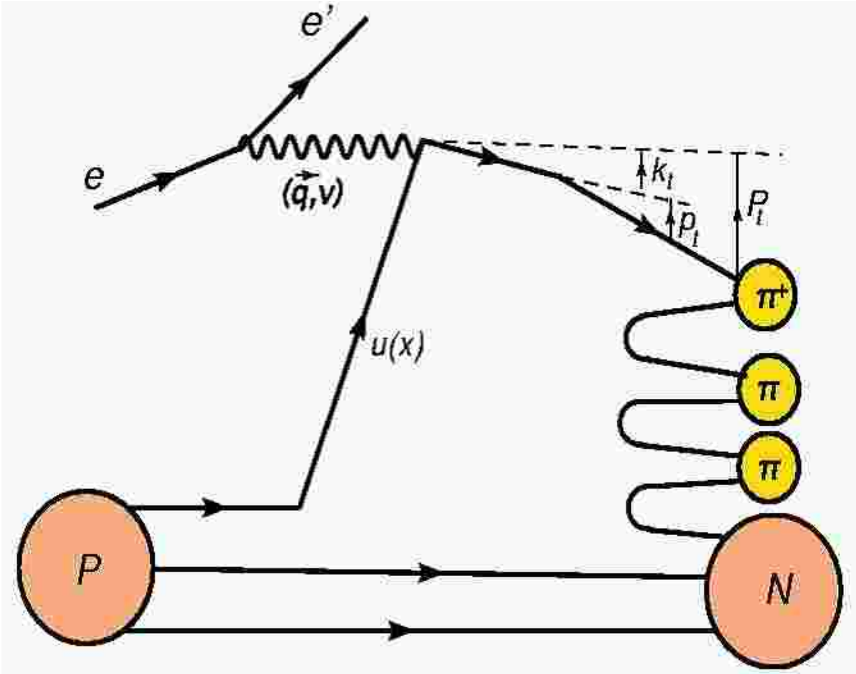
Solid lines: simple Parton Model prescription assuming factorization

Transverse Momentum Dependence of Semi-Inclusive Pion Production

- Not much is known about the orbital motion of partons
- Significant net orbital angular momentum of valence quarks implies significant transverse momentum of quarks

à Map the p_T dependence (p_T ~ < 0.5 GeV) of $^+$ and $^-$ production off proton and deuteron targets to study the k_T dependence of (unpolarized) up and down quarks.

SIDIS – k_T Dependence

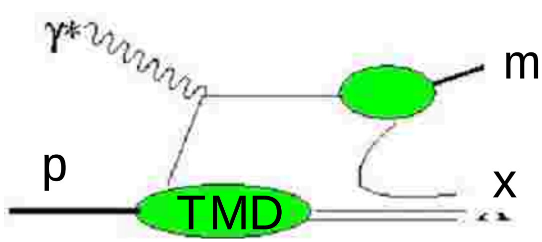


Final transverse momentum of the detected pion P_t arises from convolution of the struck quark transverse momentum k_t with the transverse momentum generated during the fragmentation p_t.

$$P_t = p_t + z\, k_t + O(k_t^2/Q^2)$$

Linked to framework of Transverse Momentum Dependent Parton Distributions

$$\sum e_q^2\, q(x)\, D_{q\rightarrow M}(z)$$



$\mathrm{TMD}^u(x,k_T)$

$f_1, g_1, f_{1T}^{\perp}, g_{1T}$

$h_1, h_{1T}^{\perp}, h_{1L}^{\perp}, h_1^{\perp}$

Transverse momentum dependence of SIDIS

General formalism for (e,e'h) coincidence reaction w. polarized beam:

[A. Bacchetta et al., JHEP 0702 (2007) 093]

$$\frac{ds}{dxdydy\,dzdf_h dP_{h,t}^2} = \frac{a^2}{xyQ^2}\frac{y^2}{2(1-e)}\left(1+\frac{g^2}{2x}\right)\Big\{F_{UU,T} + eF_{UU,L} +$$

$$\sqrt{2e(1+e)}\cos f_h F_{UU}^{\cos f_h} + e\cos(2f_h)F_{UU}^{\cos(2f_h)} + l_e\sqrt{2e(1+e)}\sin f_h F_{LU}^{\sin f_h}\Big\}$$

(= azimuthal angle of e' around the electron beam axis w.r.t. an arbitrary fixed direction)

<u>Unpolarized k_T-dependent SIDIS</u>: in framework of Anselmino et al. described in terms of convolution of quark distributions f and (one or more) fragmentation functions D, each with own characteristic (Gaussian) width.

Transverse momentum dependence of SIDIS

General formalism for (e,e'h) coincidence reaction w. polarized beam:

[A. Bacchetta et al., JHEP 0702 (2007) 093]

$$\frac{ds}{dxdydy\, dzdf_h\, dP_{h,t}^2} = \frac{a^2}{xyQ^2}\frac{y^2}{2(1-e)}\left(1+\frac{g^2}{2x}\right)\left\{F_{UU,T}+eF_{UU,L}+\right.$$

$$\left.\sqrt{2e(1+e)}\cos f_h F_{UU}^{\cos f_h} + e\cos(2f_h)F_{UU}^{\cos(2f_h)} + l_e\sqrt{2e(1+e)}\sin f_h F_{LU}^{\sin f_h}\right\}$$

(= azimuthal angle of e' around the electron beam axis w.r.t. an arbitrary fixed direction)

Azimuthal $_h$ dependence crucial to separate out kinematic effects (Cahn effect) from twist-2 correlations and higher twist effects.

data fit on EMC (1987) and Fermilab (1993) data assuming Cahn effect ? < $_0^2$> = 0.25 GeV^2

(assuming $_{0,u}$ = $_{0,d}$)

$$\left[1+(1-y)^2 - 4(2-y)\sqrt{1-y}\,\frac{2\mu_0^2|\mathbf{P}_{hT}|}{Q(\mu_D^2+\mu_0^2z^2)}\cos\varphi_h\right]\frac{\exp\left(-\frac{\mathbf{P}_{hT}^2}{\mu_D^2+\mu_0^2z^2}\right)}{\mu_D^2+\mu_0^2z^2}\sum_q e_q^2\, f_1^q(x)\, D_q^h(z)$$

Transverse momentum dependence of SIDIS

Constrain k_T dependence of up and down quarks *separately*

1) Probe $^{+}$ and $^{-}$ final states

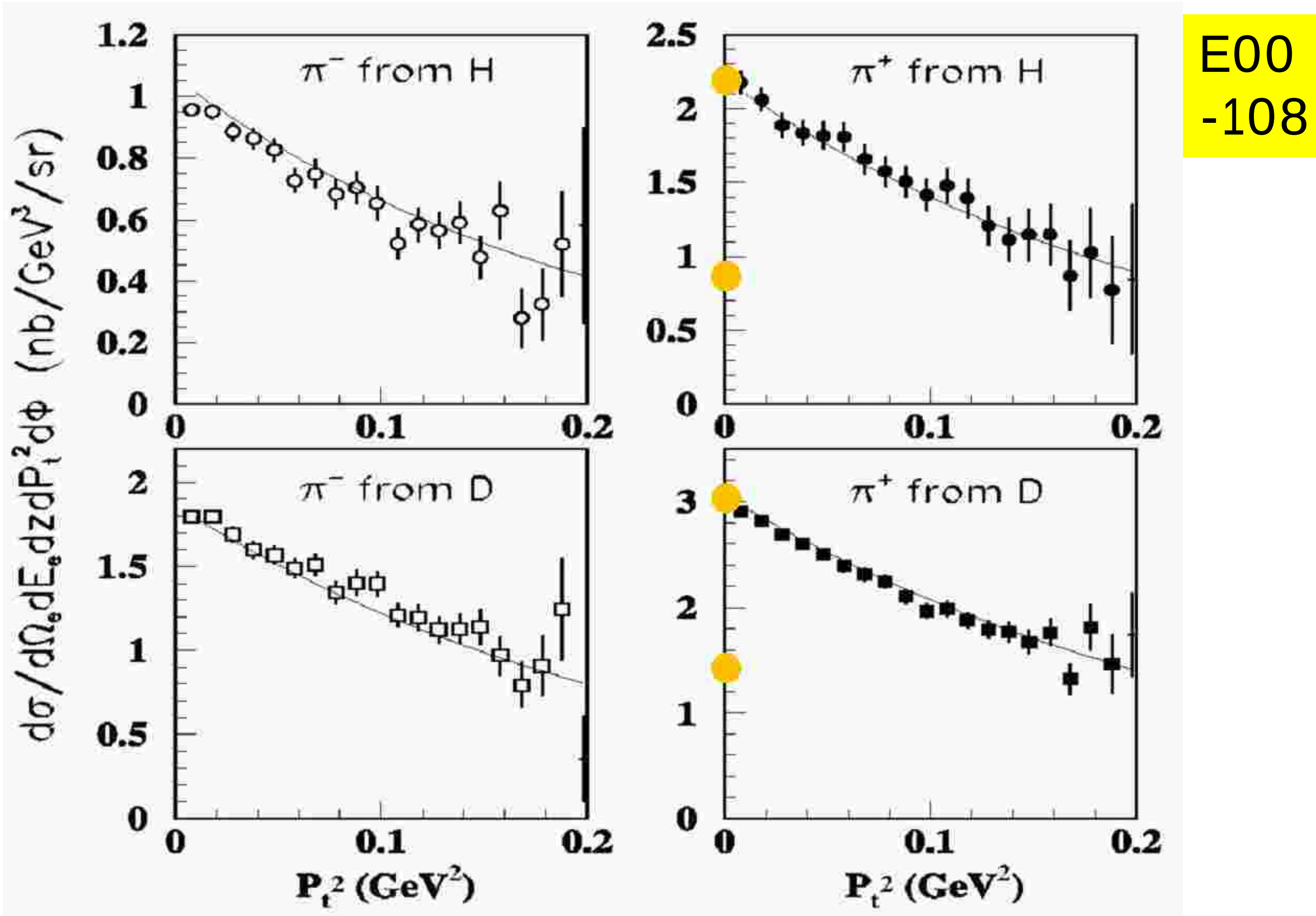
2) Use both proton and neutron (deuteron) targets

3) Combination allows, in principle, separation of quark width from fragmentation widths *(if sea quark contributions small)*

Spectrometers a <u>must</u> for such %-type (!) measurements: acceptance is a common factor to all measurements (and can use identical H and D cells)

First example from JLab at 6 GeV on next slide
(E00-108 experiment in Hall C,
H. Mkrtchyan, P. Bosted et al.,
Phys. Lett. B665 (2008) 20)

Transverse momentum dependence of SIDIS

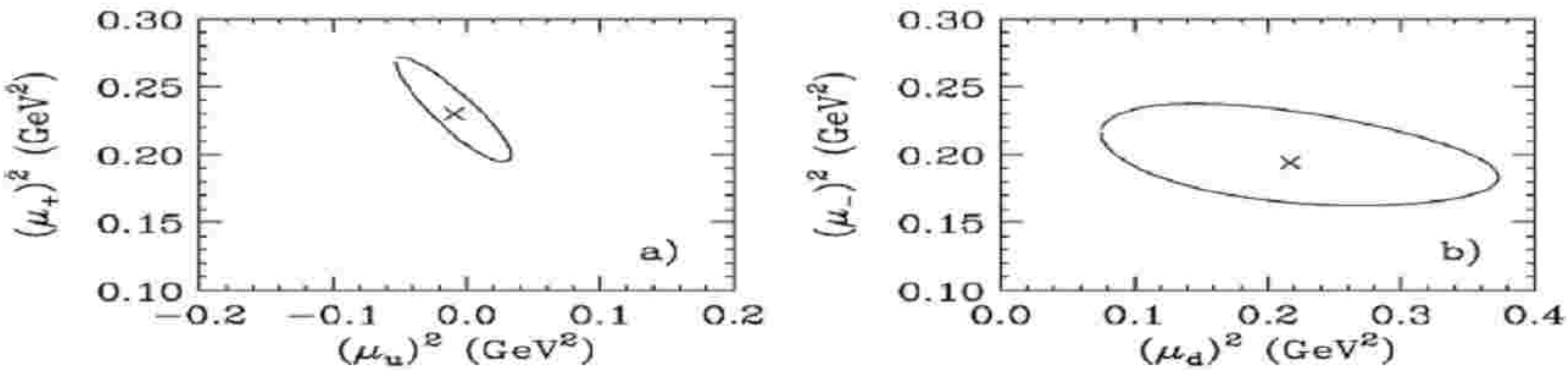


P_t dependence very similar for proton and deuterium targets, but deuterium slopes systematically smaller?

Transverse momentum dependence of SIDIS

Simple model, host of assumptions (factorization valid, fragmentation functions do not depend on quark flavor, transverse momentum widths of quark and fragmentation functions are gaussian and can be added in quadrature, sea quarks are negligible, assume Cahn effect, etc.) à

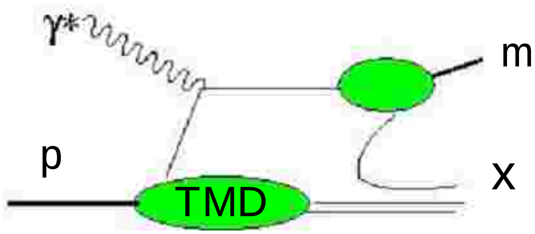
$(\ _{+})^2$ ~ width of $D^{+}(z,p_t)$, $(\ )^2$ ~ width of $D^{-}(z,p_t)$,
$(\ _{u})^2$ ~ width of $u(x,k_t)$, $(\ _{d})^2$ ~ width of $d(x,k_t)$

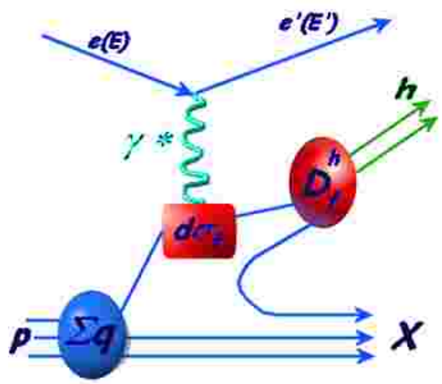


Many authors believe these widths to be of order 0.25 GeV^2 à these numbers are close! But … is $(\ _{u})^2 < (\ _{d})^2$?? Expected from diquark models… Or, is there something wrong in our simple model or the used SIDIS framework?

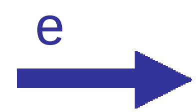
Transverse Momentum Dependent Parton Distributions

Accessible through Semi-Inclusive Reactions

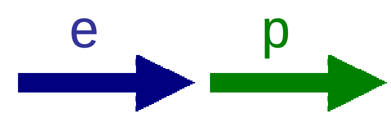




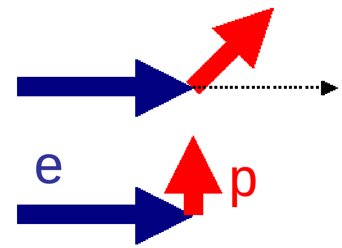
Unpolarized target



Longitudinally pol. target



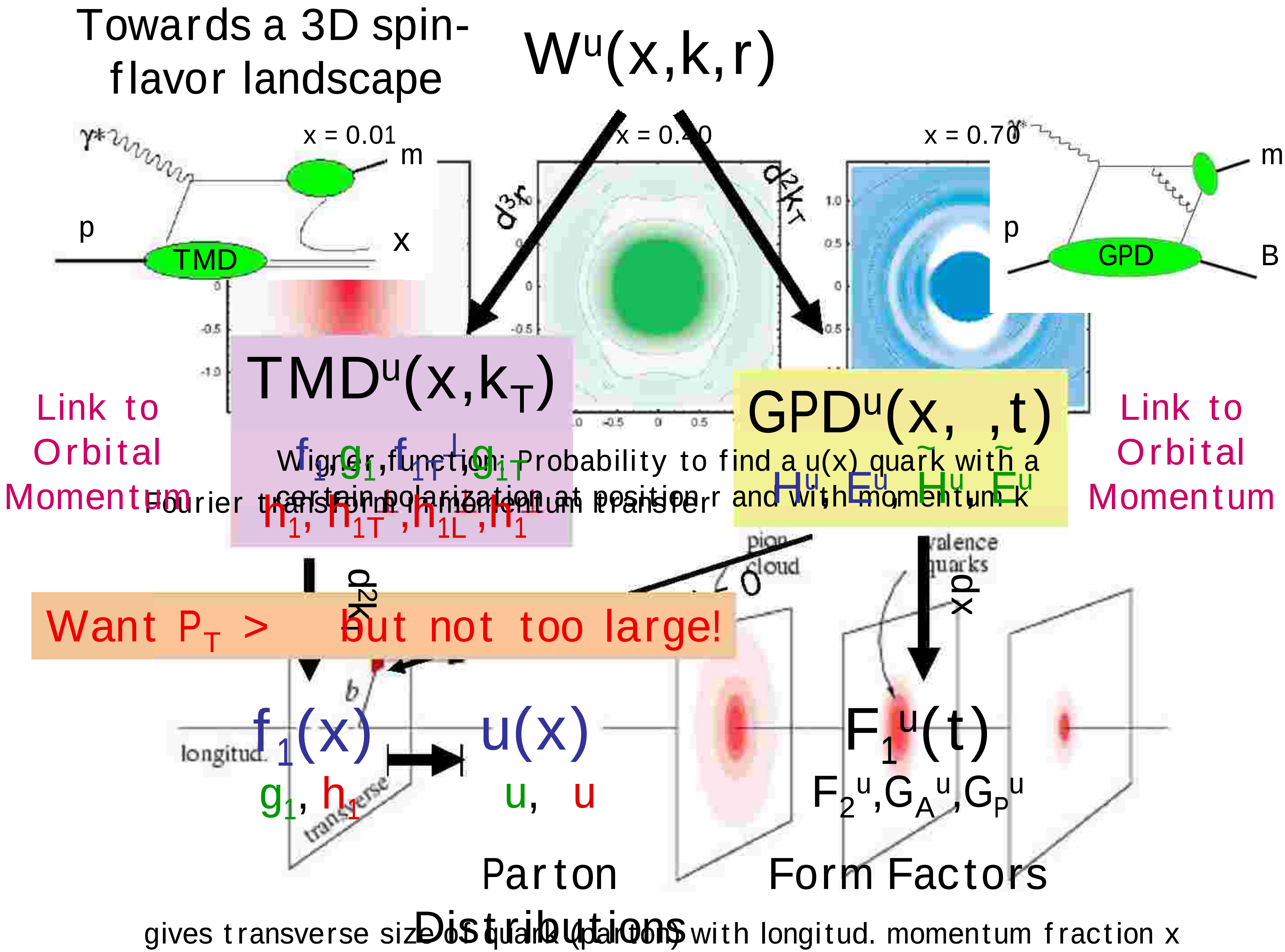
Transversely pol. target



$$d\ \ ^h \sim\ \ e_q^2 q(x)\ d\ \ _f\ D_f^h(z)$$

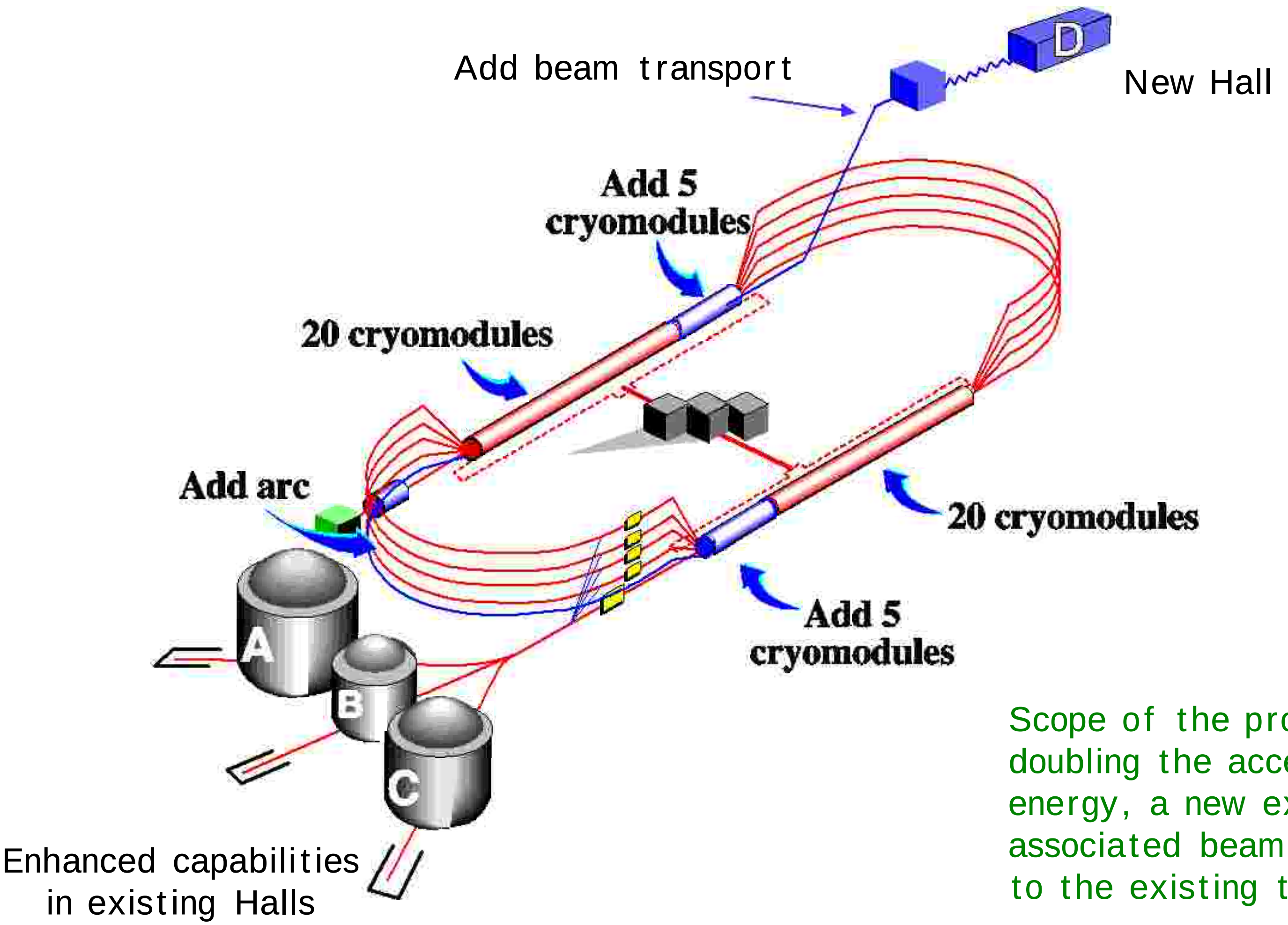
N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Off-diagonal PDFs related to interference between L=0 and L=1 light-cone wave functions


Towards a 3D spin-flavor landscape
W^u(x,k,r)
x = 0.01
x = 0.40
x = 0.70
d^3r
d^2k_T
p
m
x
TMD
GPD
B
TMD^u(x,k_T)
f_1, g_1, f_1T^⊥, g_1T
h_1, h_1T^⊥, h_1L^⊥, h_1^⊥
GPD^u(x, ,t)
H^u, E^u, H̃^u, Ẽ^u
Link to Orbital Momentum
Link to Orbital Momentum
Wigner function: Probability to find a u(x) quark with a certain polarization at position r and with momentum k
Fourier transform in momentum transfer
pion cloud
valence quarks
Want P_T > but not too large!
d^2k
dx
f_1(x)
u(x)
g_1, h_1
u, u
F_1^u(t)
F_2^u, G_A^u, G_P^u
longitud.
transverse
b
Parton Distributions
Form Factors
gives transverse size of quark (parton) with longitud. momentum fraction x

12 GeV Upgrade

Upgrade is designed to build on existing facility:
all accelerator and nearly all experimental equipment have continued use



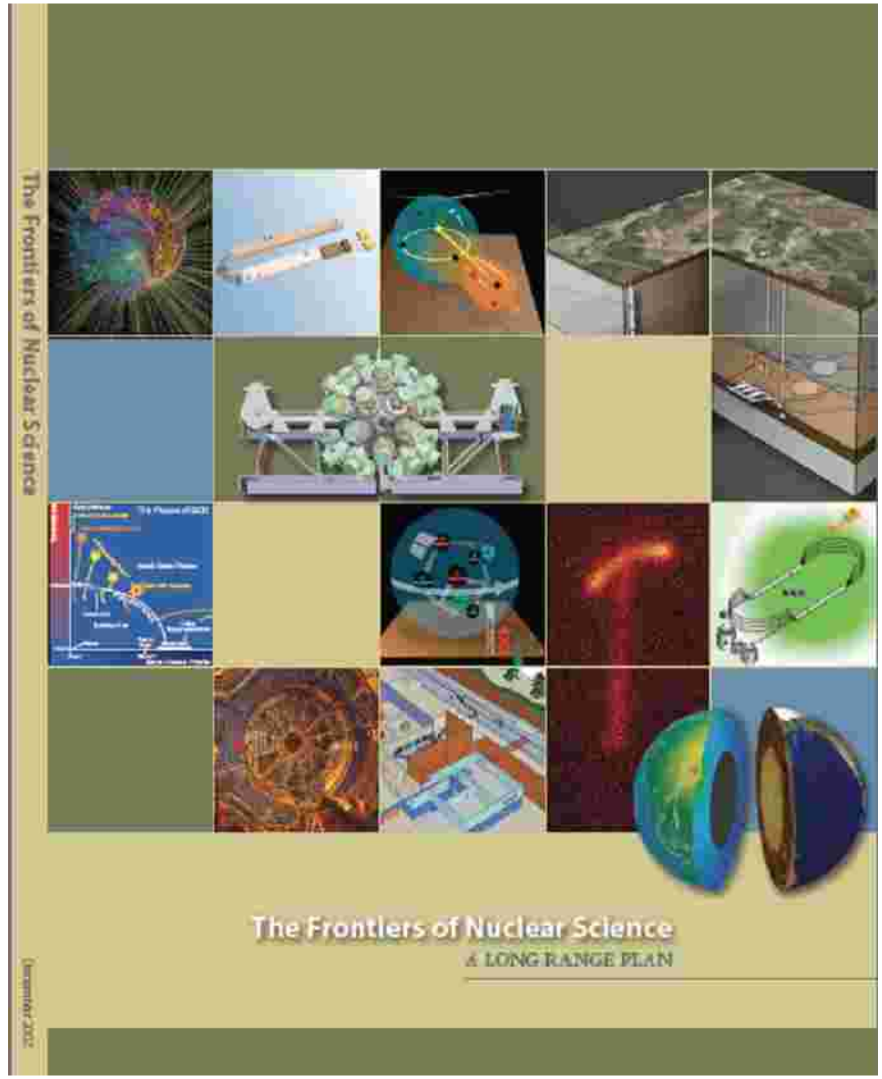
Scope of the proposed project includes doubling the accelerator beam energy, a new experimental Hall and associated beamline, and upgrades to the existing three experimental Halls.

NSAC 2007 Long Range Plan

Recommendation I

"We recommend completion of the 12 GeV Upgrade at Jefferson Lab. The Upgrade will enable new insights into the structure of the nucleon, the transition between the hadronic and quark/gluon descriptions of nuclei, and the nature of confinement."

A fundamental challenge for modern nuclear physics is to understand the structure and interactions of nucleons and nuclei in terms of QCD. Doubling the energy of the JLAB accelerator will enable three-dimensional imaging of the nucleon, revealing hidden aspects of the internal dynamics.



DOE CRITICAL DECISION SCHEDULE

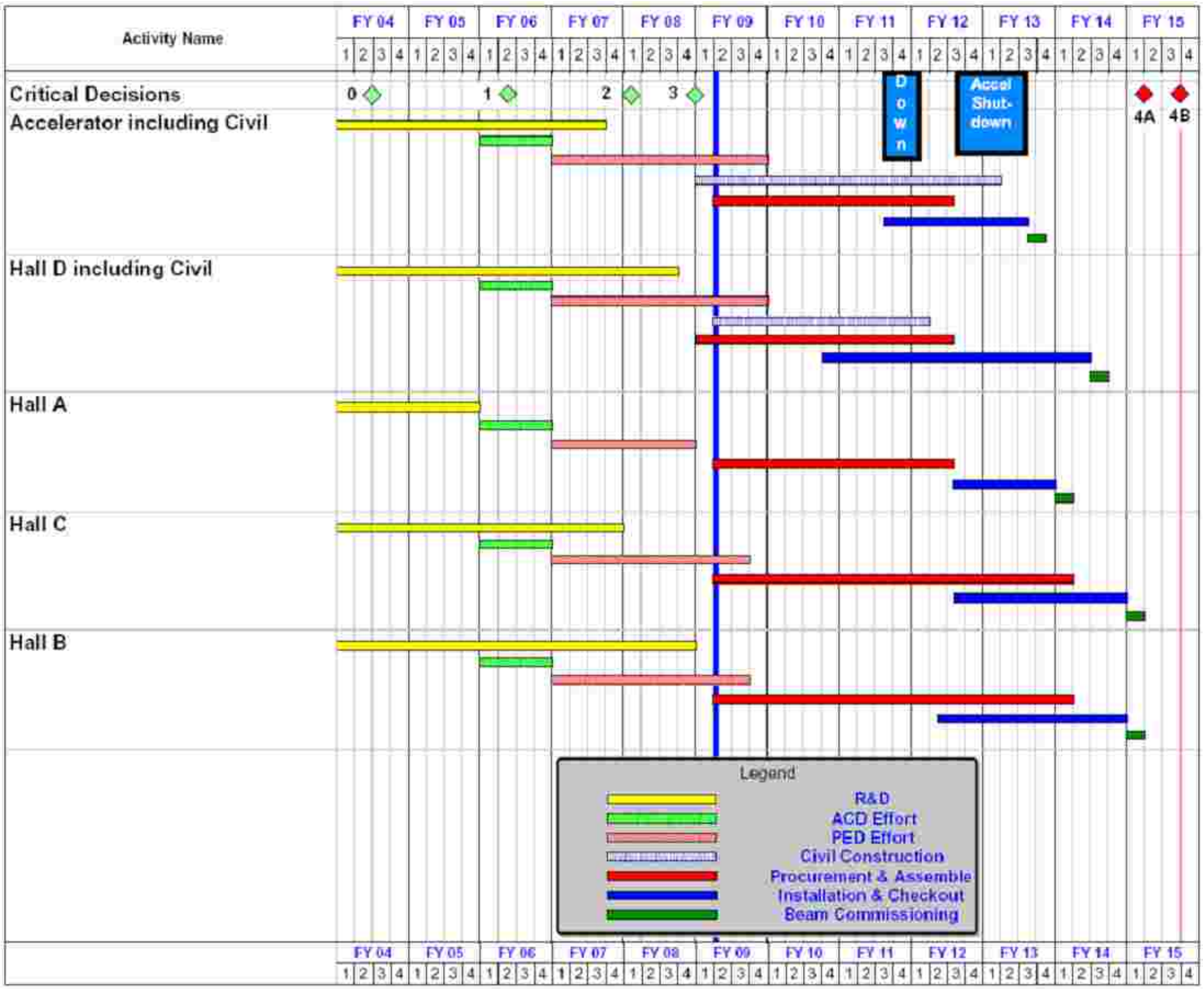
CD-0 Mission Need	MAR-2004 (A)
CD-1 Preliminary Baseline Range	FEB-2006 (A)
CD-2 Performance Baseline	NOV-2007 (A)
CD-3 Start of Construction	SEP-2008 (A)
CD-4A Accelerator Project Completion and Start of Operations	DEC-2014
CD-4B Experimental Equipment Project Completion and Start of Operations	JUN-2015

Now split in two to ease transition into operations phase

Note ? 6 to 18 months schedule float included

(A) = Actual Approval Date

12 GeV Schedule



May '11 - Oct '11: 6-month "down" for initial installations

Nov '11 - Apr '12: 6-month run 6 GeV

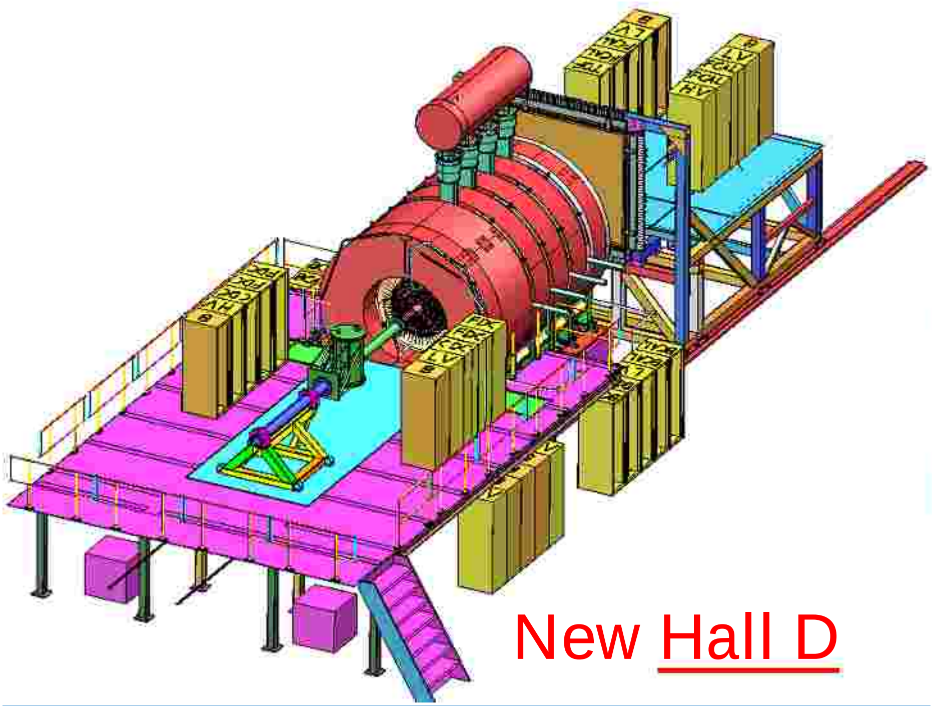
May '12 - May '13: 1-year "down" for major installation

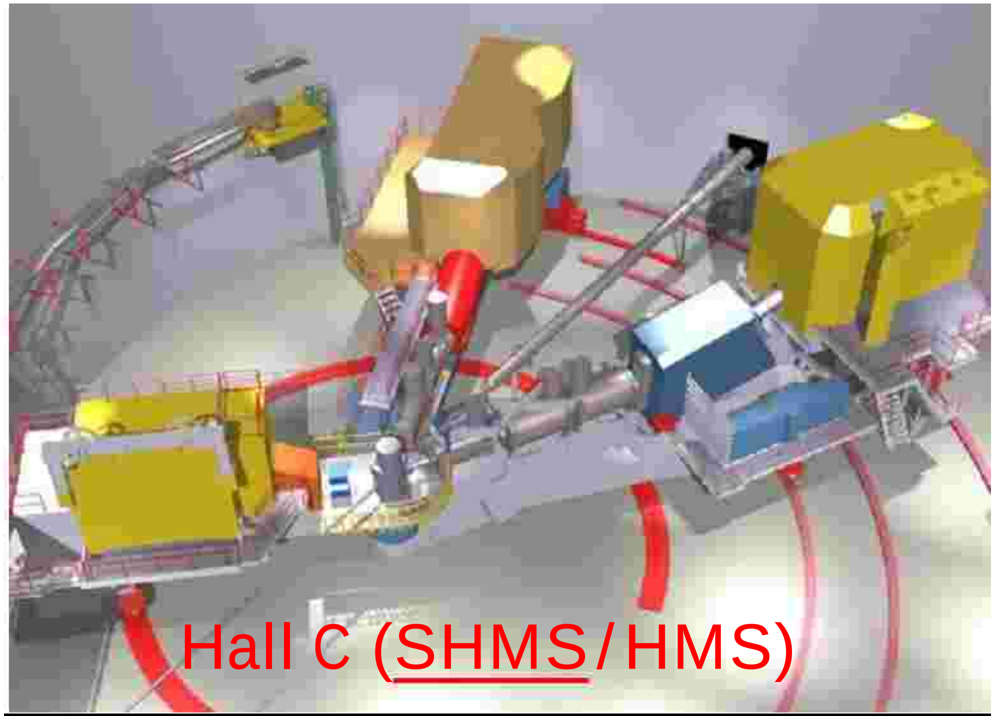
Jun '13 - Sep '13: Accelerator commissioning

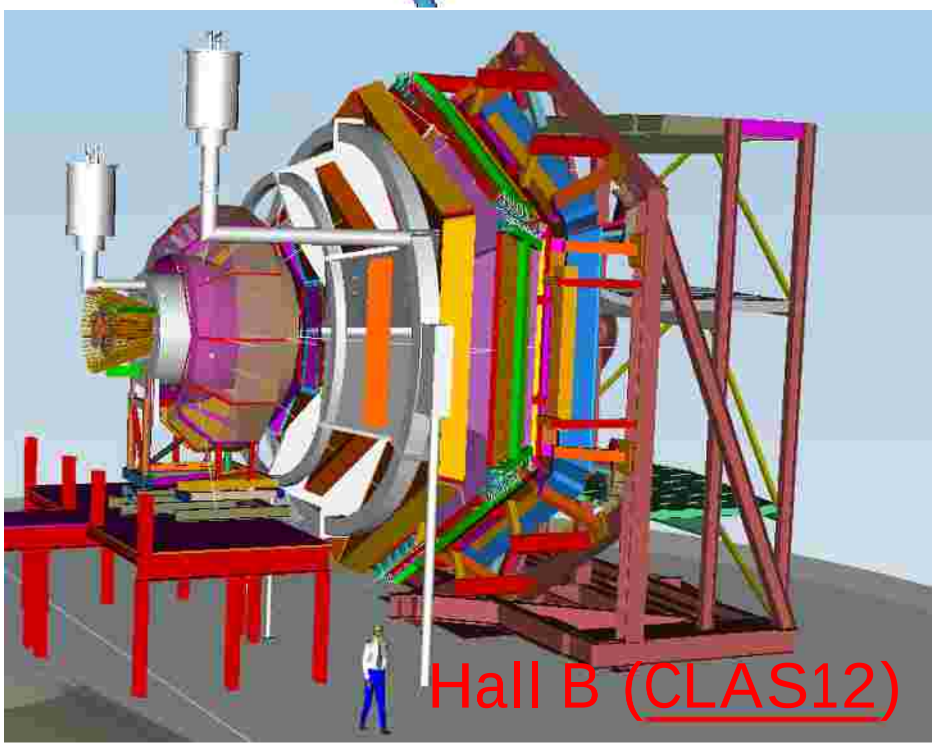
- Oct '13: Hall A commissioning start
- Apr '14: Hall D commissioning start
- Oct '14: Hall B & C commissioning start

6 GeV Experimental Instrumentation

12 GeV Scope:
Add new Hall D (photon beam)
Add SHMS to HMS
Upgrade CLAS to CLAS12
11 GeV Beam Capability to Hall A

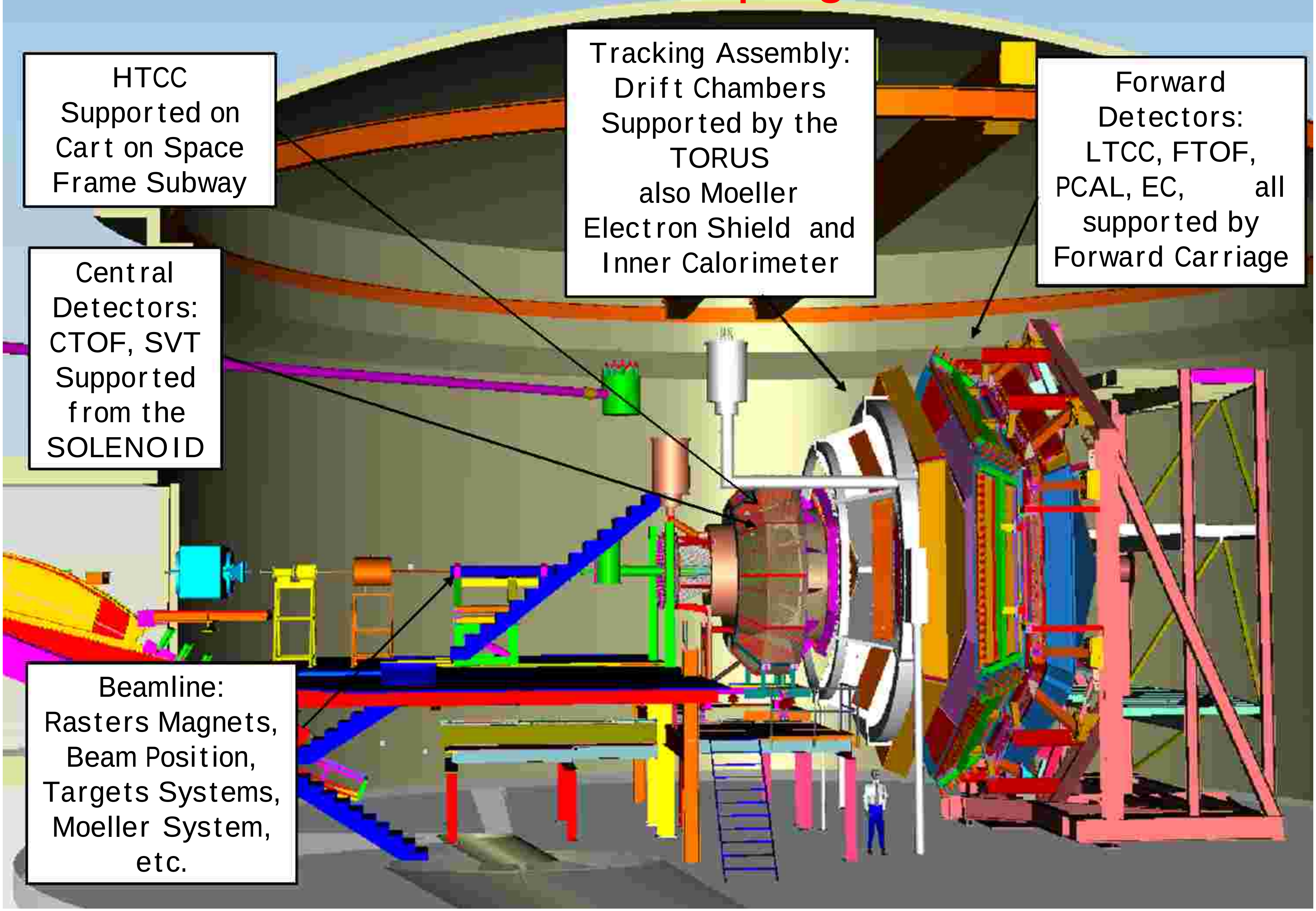

New Hall D


Hall C (SHMS/HMS)

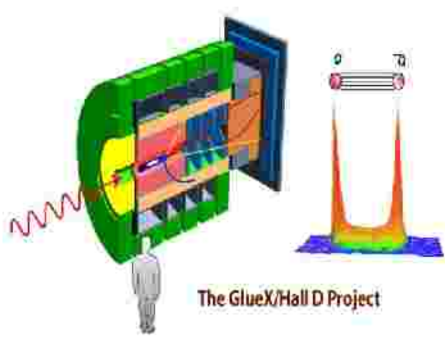

Hall B (CLAS12)

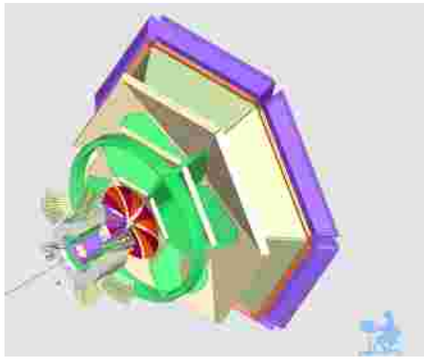

Hall A (2 HRS + 11 GeV beam
for large installations)

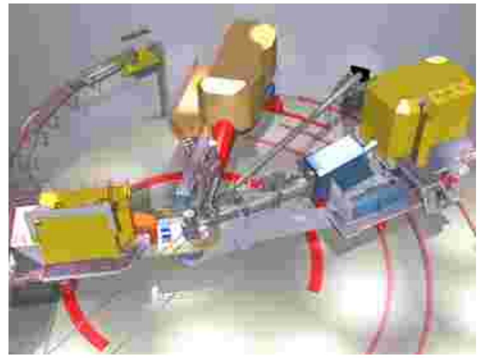
CLAS12 Detector Grouping - Breakdown

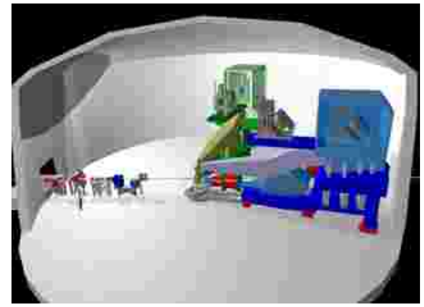


Overview of Technical Performance Requirements





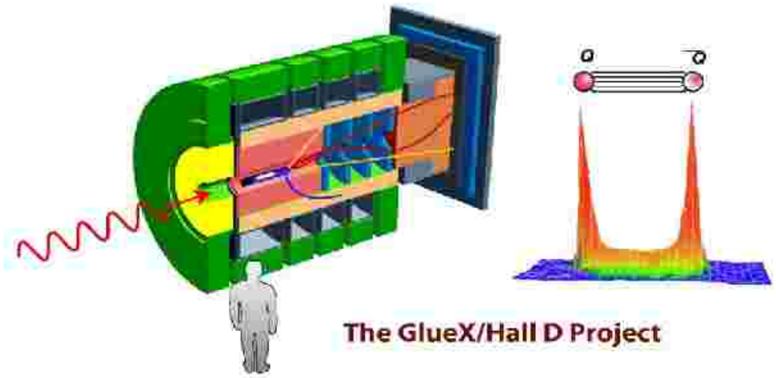


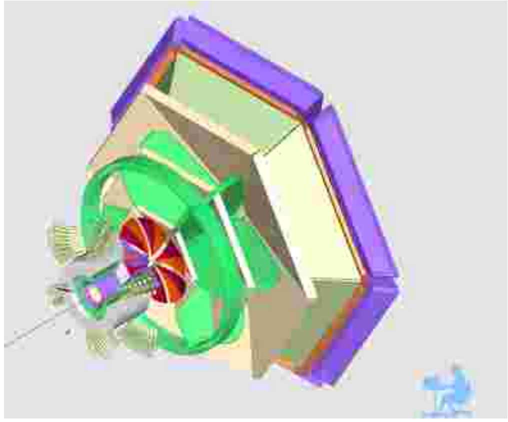


Hall D	Hall B	Hall C	Hall A
excellent hermeticity	luminosity 10×10^{34}	energy reach	installation space
polarized photons	hermeticity	precision	
E ~8.5-9 GeV		11 GeV beamline	
10^8 photons/s		target flexibility	
good momentum/angle resolution		excellent momentum resolution	
high multiplicity reconstruction		luminosity up to 10^{38}	
particle ID			

Overview of 12 GeV Physics Program

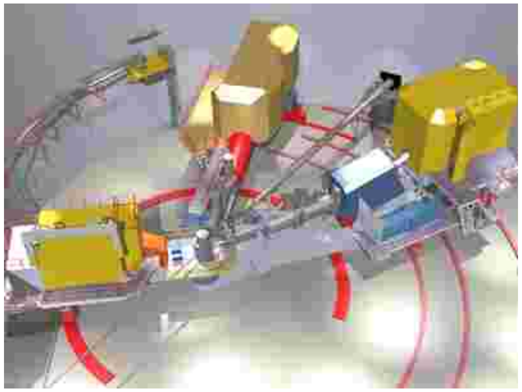
Hall D – exploring origin of confinement by studying exotic mesons

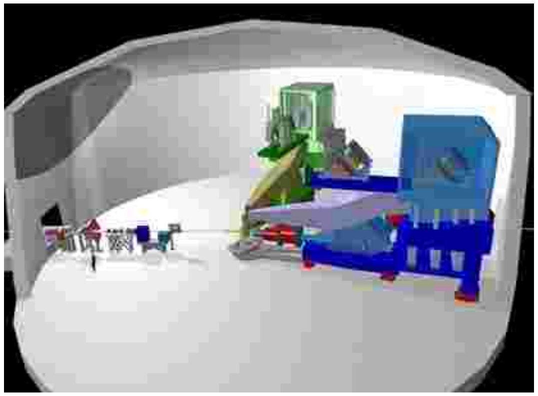




Hall B – understanding nucleon structure via generalized parton distributions

Hall C – precision determination of valence quark properties in nucleons and nuclei





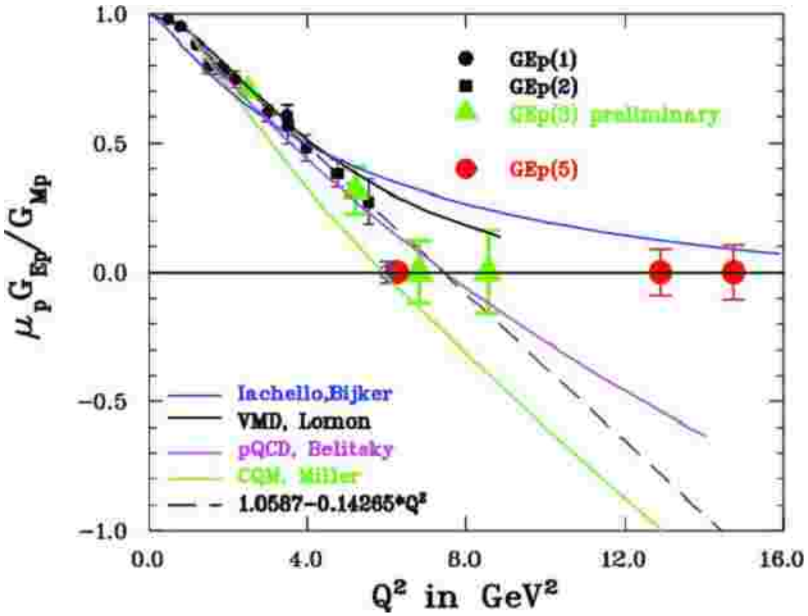
Hall A – short range correlations, form factors, hyper-nuclear physics, future new experiments

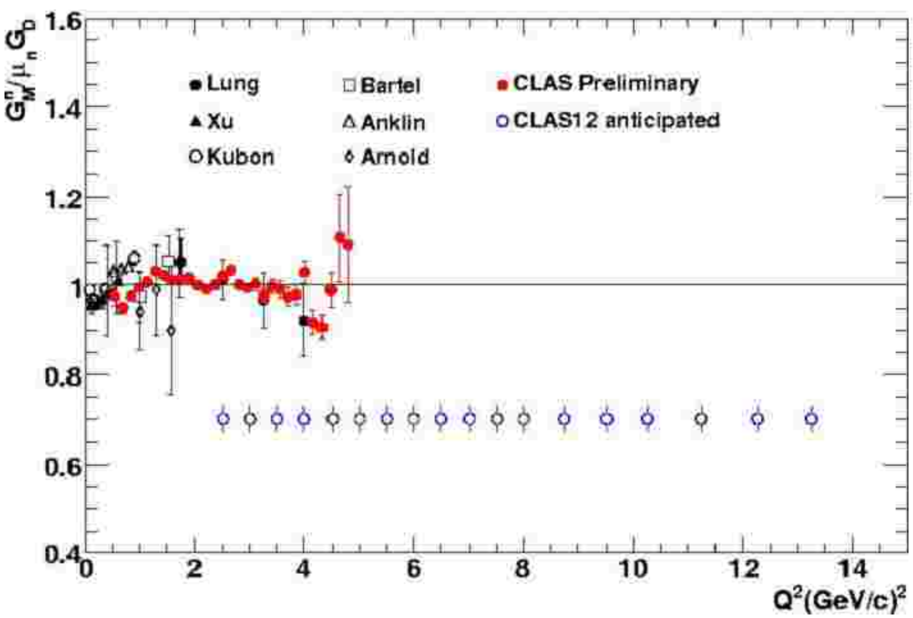
12 GeV Approved Science Program

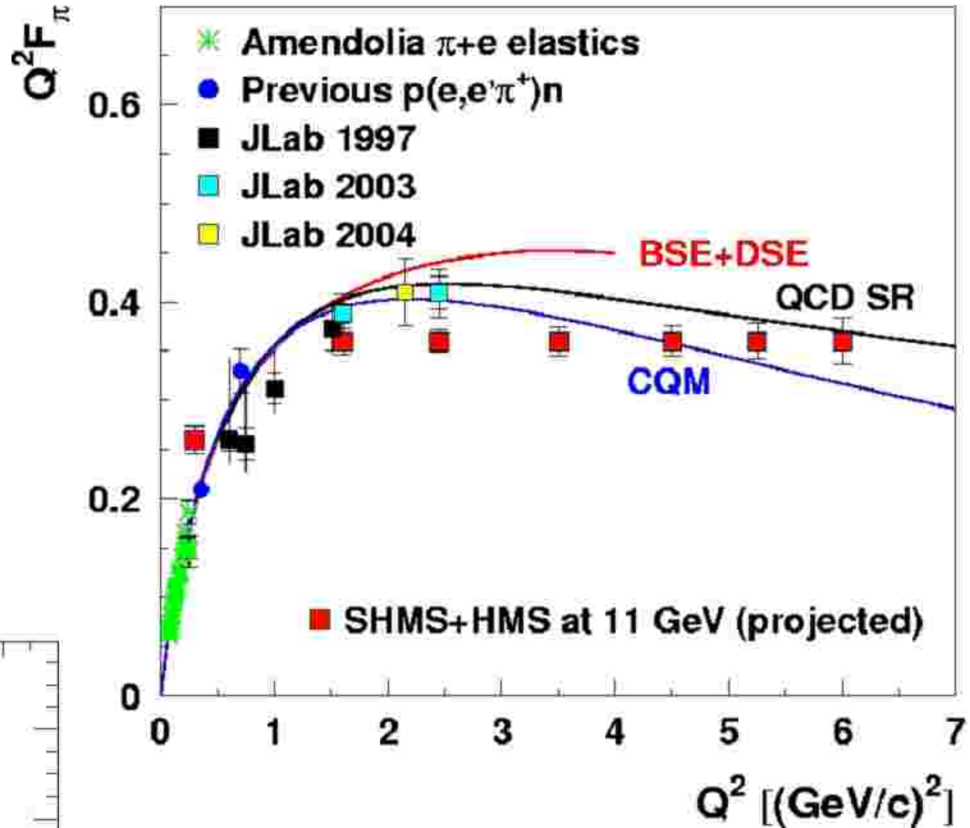
Six major science thrusts identified in the 12 GeV CDR:

- The Origin of Quark Confinement
- Form Factors – Constraints on the GPDs
- Valence Quark Structure and Parton Distributions
- Deep Exclusive Scattering and GPDs
- Hadron Structure in the Nuclear Medium
- Symmetry Tests in Nuclear Physics

Form Factors – Constraints on the GPDs





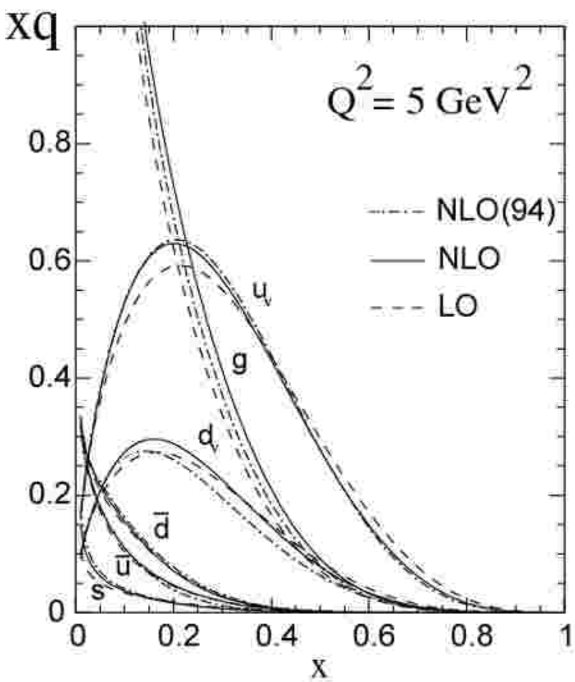


E12-07-109 (Hall A)
E12-07-104 (Hall B)
E12-06-101 (Hall C)

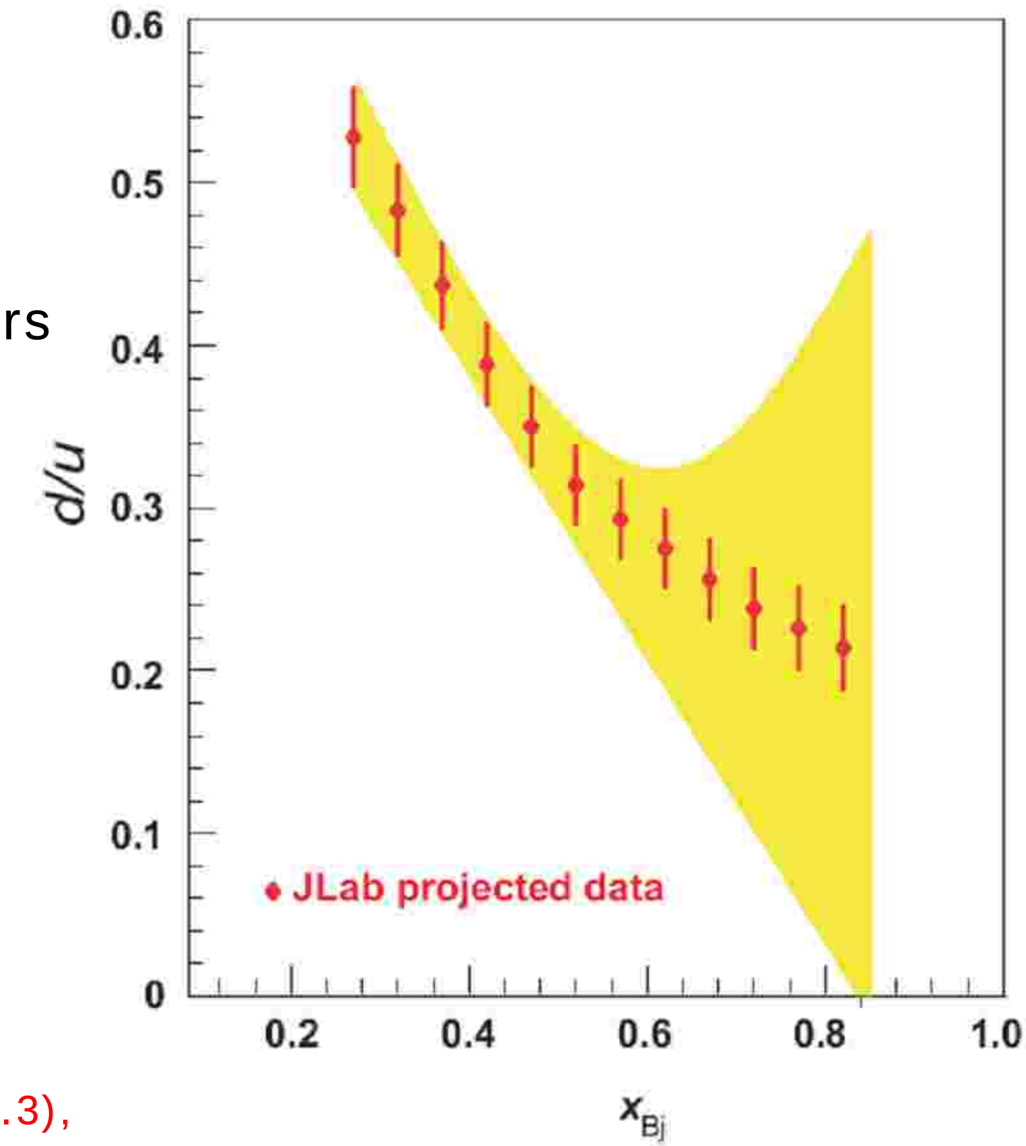
Measuring High-x Structure Functions

REQUIRES:

- High beam polarization
- High electron current
- High target polarization
- Large solid angle spectrometers

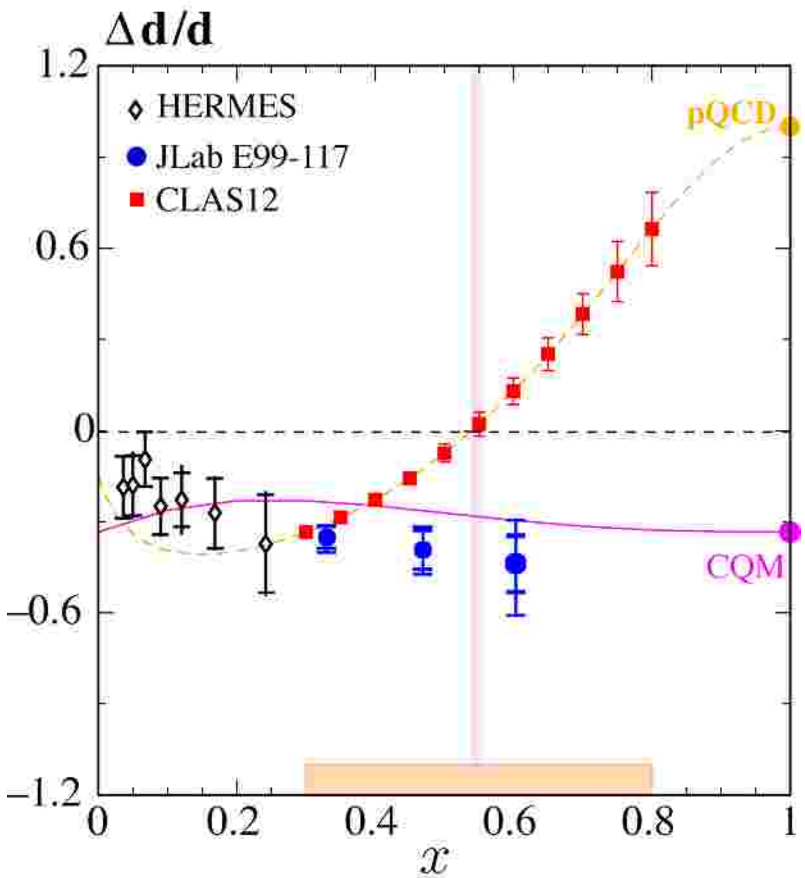


12 GeV will access the regime (x > 0.3), where valence quarks dominate

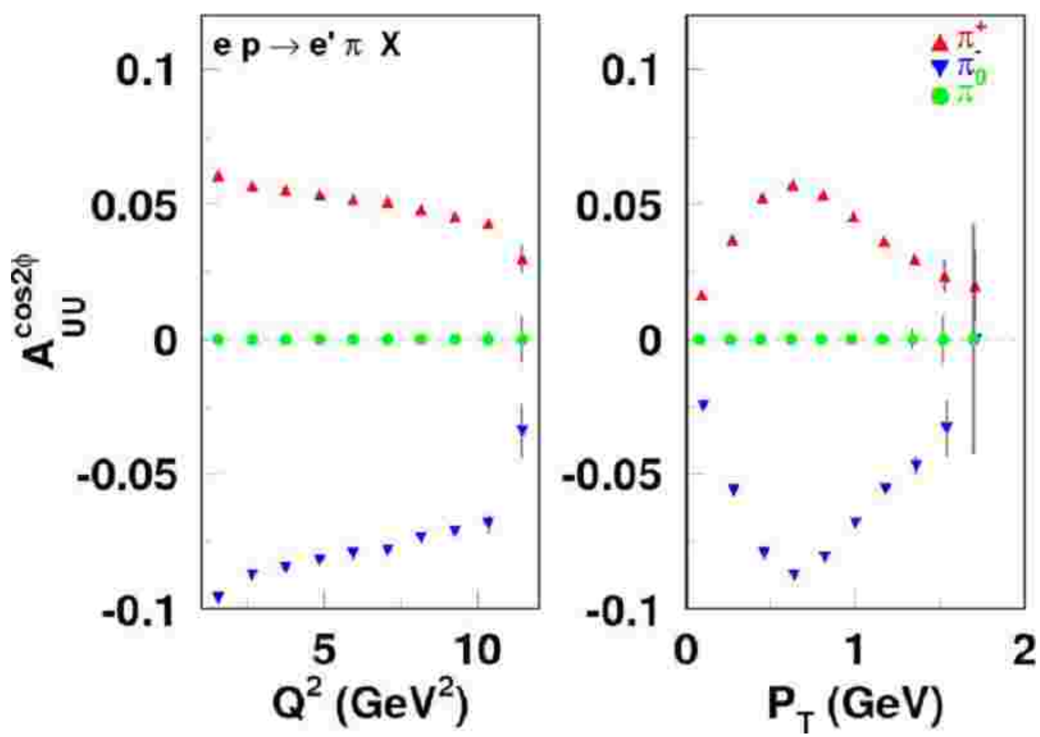


Valence Quark Structure and Parton Distributions

Access to valence quark region through DIS at large x will be augmented with a SIDIS program



Boer-Mulders asymmetry for pions as function of Q^2 and p_T



Transverse Momentum Dependence: E00-108 Summary

E00-108 results can only be considered suggestive at best:

- limited kinematic coverage
  - assume (P_T,) dependency ~ Cahn effect
- very simple model assumptions

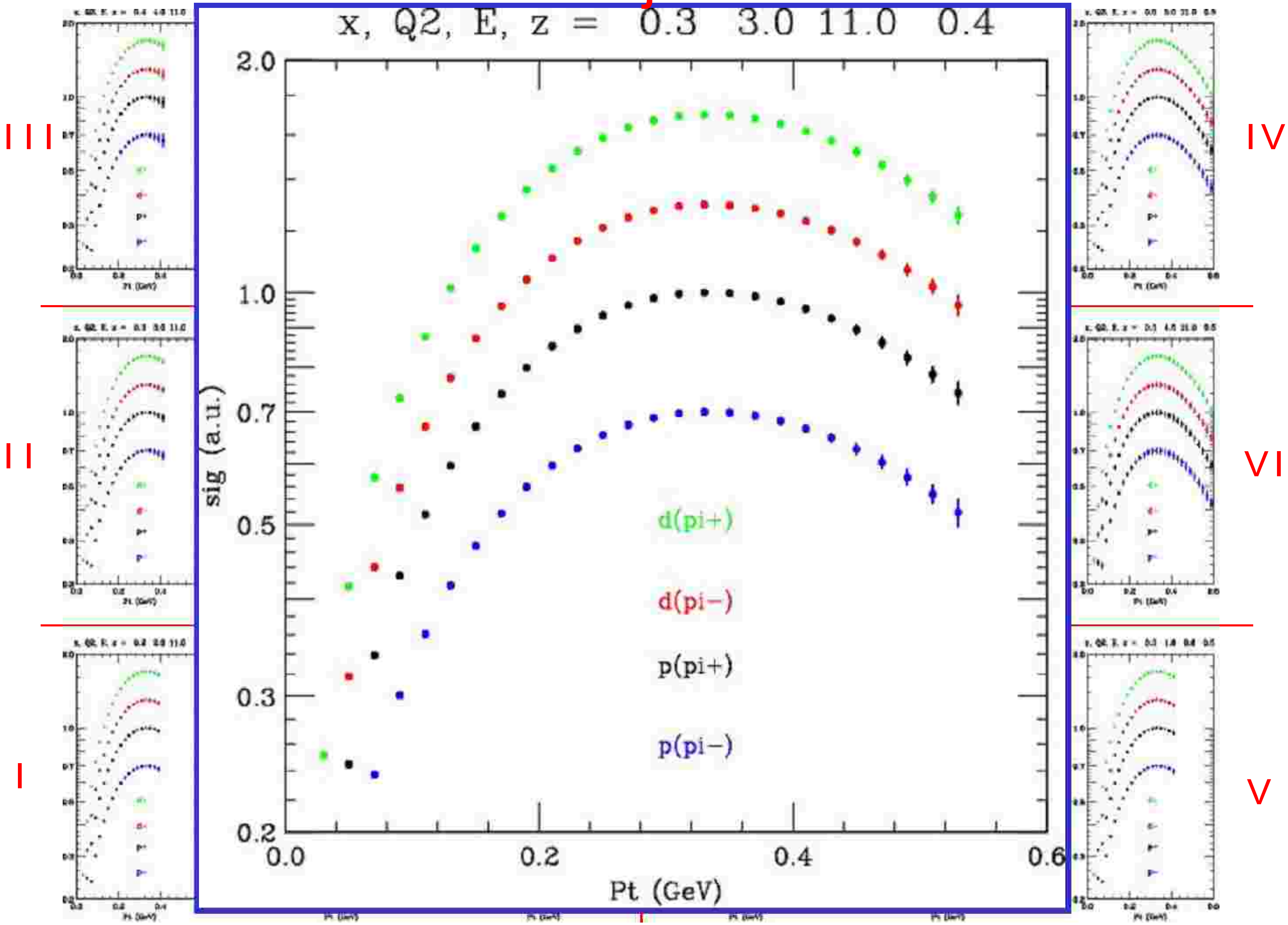
  but P_T dependence off D seems shallower than H

  and intriguing explanation in terms of flavor/k_T deconvolution

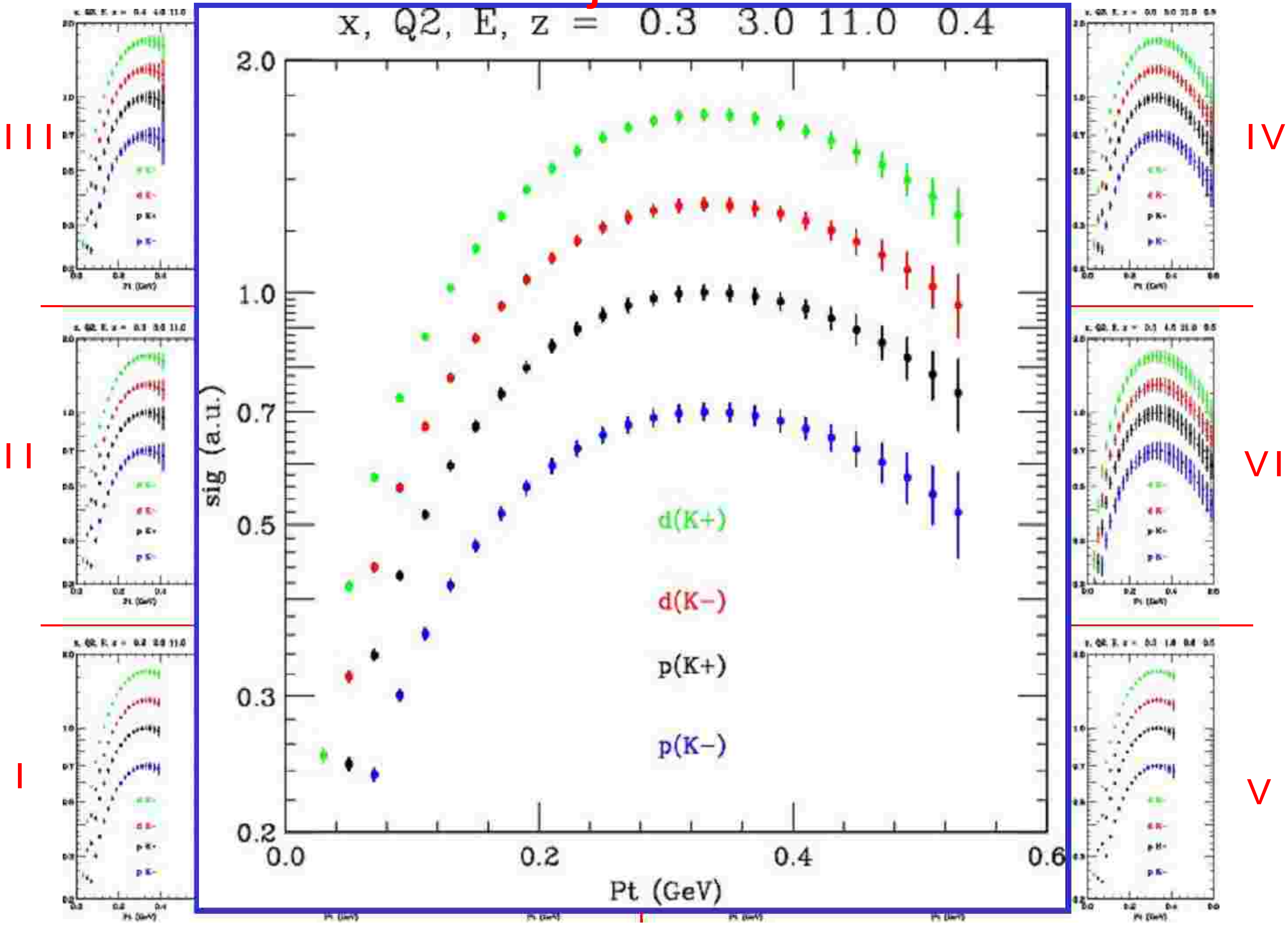
Many limitations could be removed with 12 GeV:

- wider range in Q^2
- improved/full coverage in (at low p_T)
- larger range in P_T
- wider range in x and z (to separate quark from fragmentation widths)
- possibility to check various model assumptions
  - Power of (1-z) for D^-/D^+
  - quantitative contribution of Cahn term
  - $D_u^+ = D_d^-$
  - Higher-twist contributions
  - consistency of various kinematics (global fit vs. single-point fits)

PR12-09-017 Projected Results - I



PR12-09-017 Projected Results - Kaons



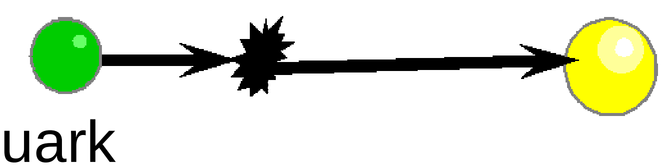
R = $_L$/ $_T$ in DIS and in (e,e') SIDIS

- R_{DIS} is in the naïve parton model related to the parton's transverse momentum: $R = 4(M^2x^2 + \langle k_T{}^2\rangle)/(Q^2 + 2\langle k_T{}^2\rangle)$.
- R_{DIS} à 0 at Q^2 à 8 is a consequence of scattering from free spin-½ constituents
- Of course, beyond this, at finite Q^2, R_{DIS} sensitive to gluon and higher-twist effects
- No distinction made up to now between diffractive and non-diffractive contributions in R_{DIS}
- $R_{DIS}{}^H = R_{DIS}{}^D$, to very good approximation

R = $_L$/ $_T$ in DIS and in (e,e’) SIDIS

- If integrated over z (and p_T, , hadrons), $R_{SIDIS} = R_{DIS}$
- R_{SIDIS} may vary with z
- At large z, there are known contributions from (semi-) exclusive channels: pions originating from à $^{+}$ $^{-}$
- R_{SIDIS} may vary with p_T
- Is R_{SIDIS} $^{+}$ = R_{SIDIS} $^{-}$? Is $R_{SIDIS}^{H} = R_{SIDIS}^{D}$?
- $R_{SIDIS} = R_{DIS}$ test of dominance of quark fragmentation

$$\sum e_q^2\, q(x)\, D_{q\rightarrow M}(z)$$



So what about R = $_L$/ $_T$ for pion electroproduction?

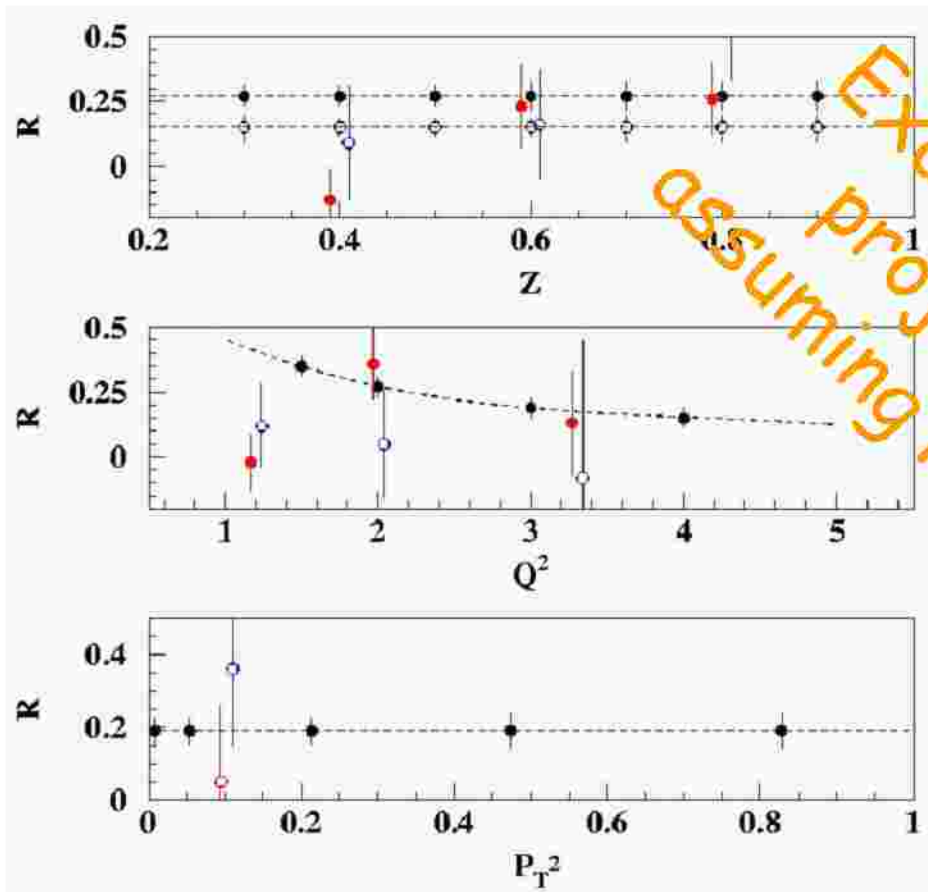
"Semi-inclusive DIS"

quark $\sum e_q^2 \, q(x) \; D_{q \to M}(z)$

R_{SIDIS} à R_{DIS} disappears with Q^2!

"Deep exclusive scattering" is the z à 1 limit of this "semi-inclusive DIS" process

Here, R = $_L$/ $_T$ ~ Q^2



Example of 12-GeV projected data assuming $R_{SIDIS} = R_{DIS}$

Not clear what R will behave like at large p_T

The path towards the extraction of GPDs

Use polarization!

A = =

$\vec{e}\, p \longrightarrow ep$

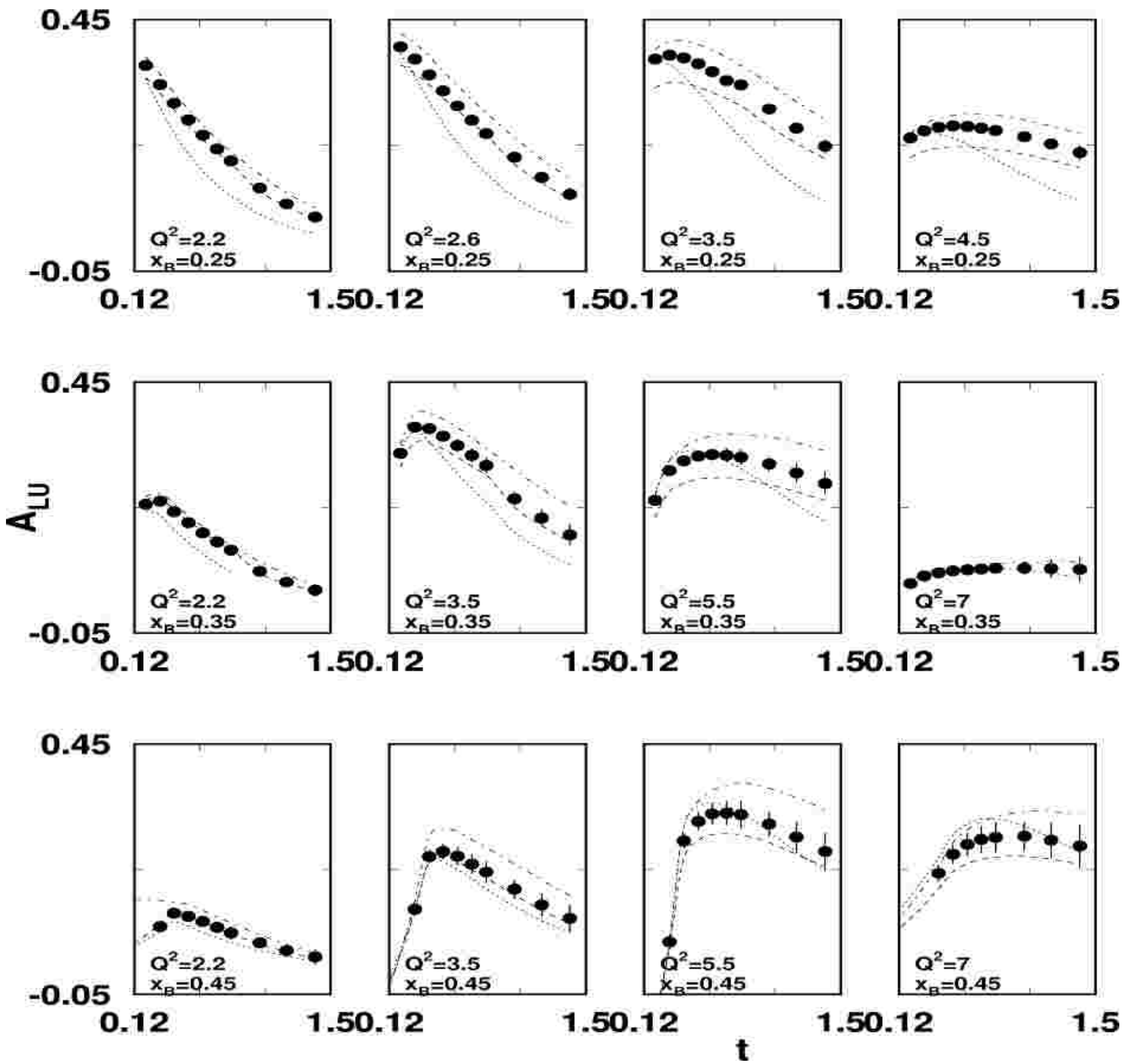
$_{LU}$ ~ sin Im{F_1H + }d

Kinematically suppressed

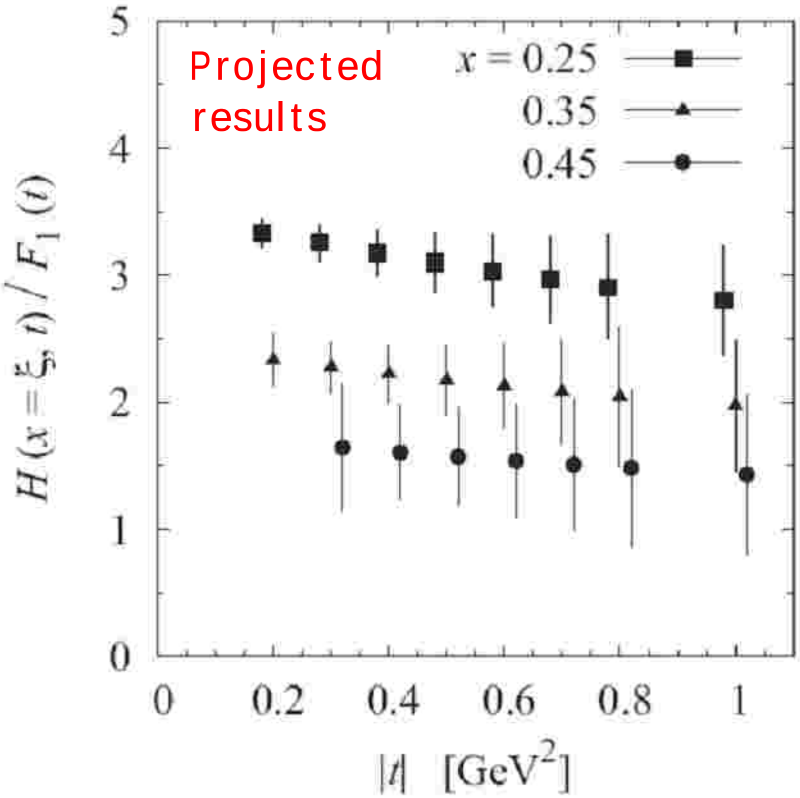
H(,t)

= $x_B/(2-x_B)$

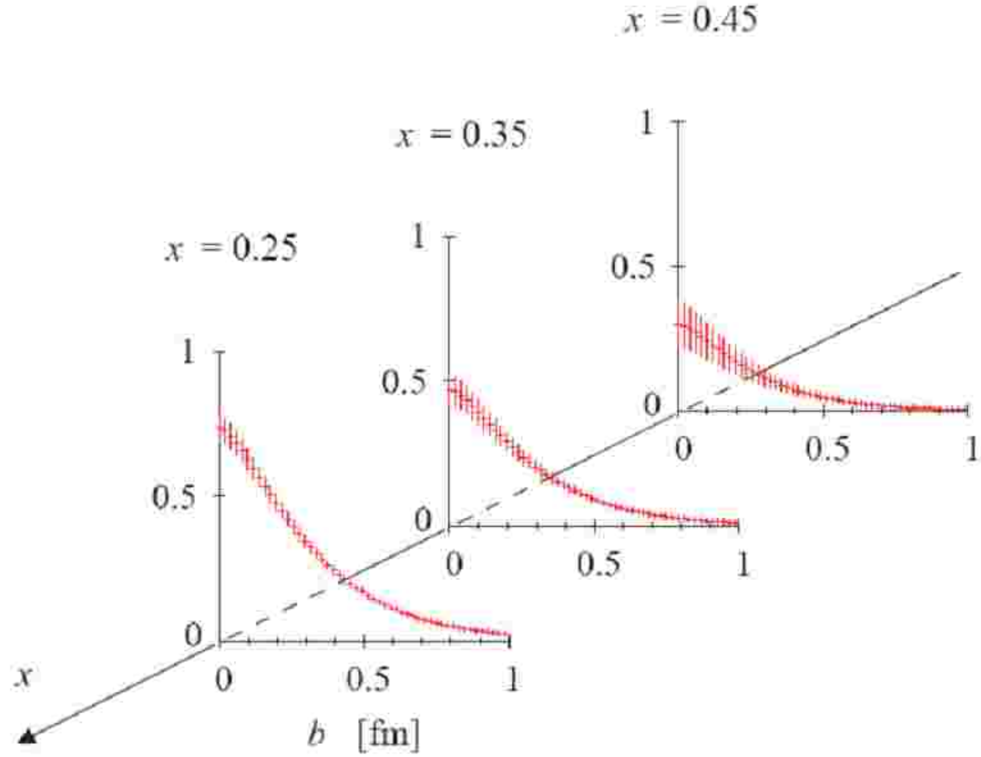
Subset of projected results



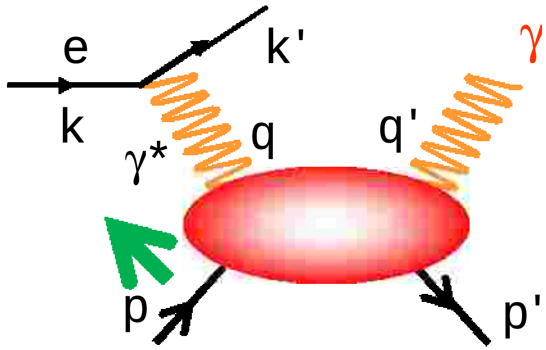
Projected precision in extraction of GPD H at x =



Spatial Image



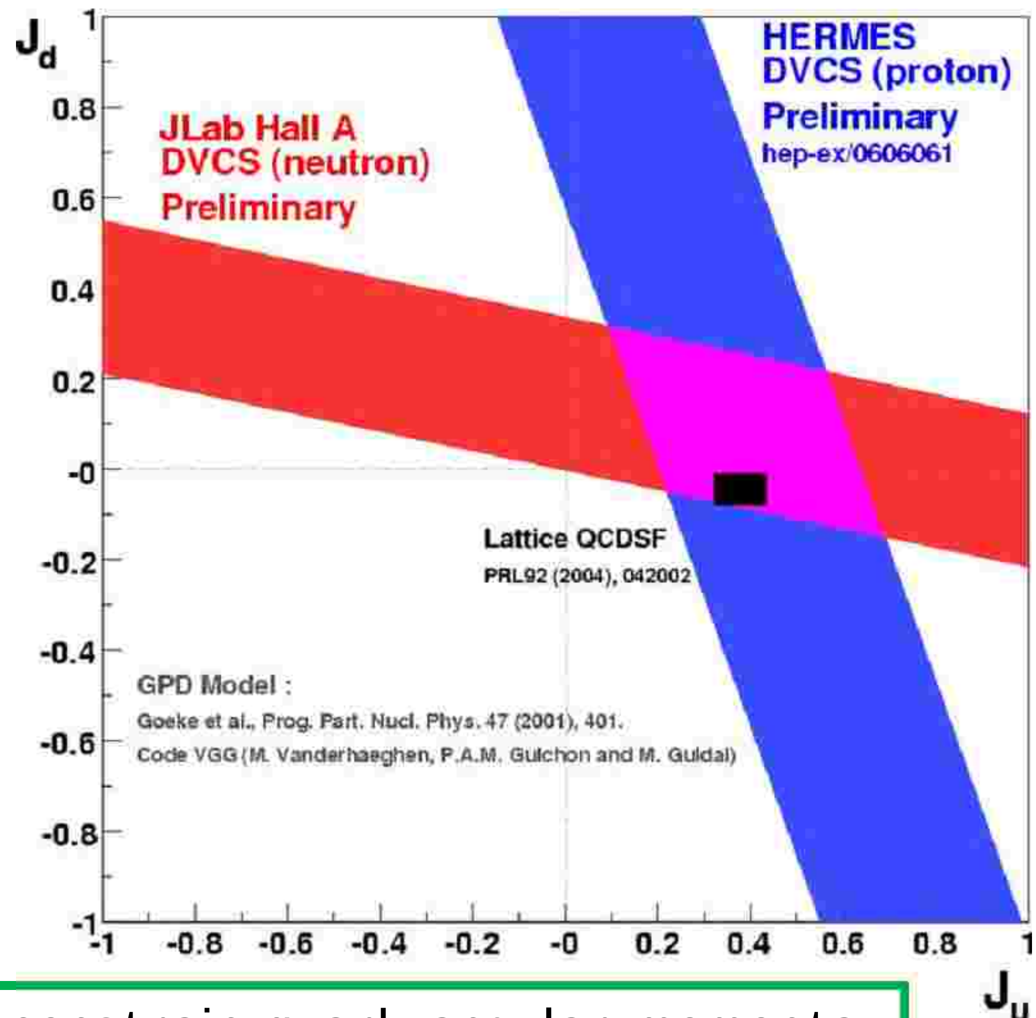
orbital angular momentum carried by quarks : solving the spin puzzle



At one value of x only

Ingredients:
1) GPD Modeling
2) HERMES $^1H(e,e'\ )p$
(transverse target spin asymmetry)
3) Hall A $^2H(e,e'\ n)p$

Or independent :
Lattice QCD!



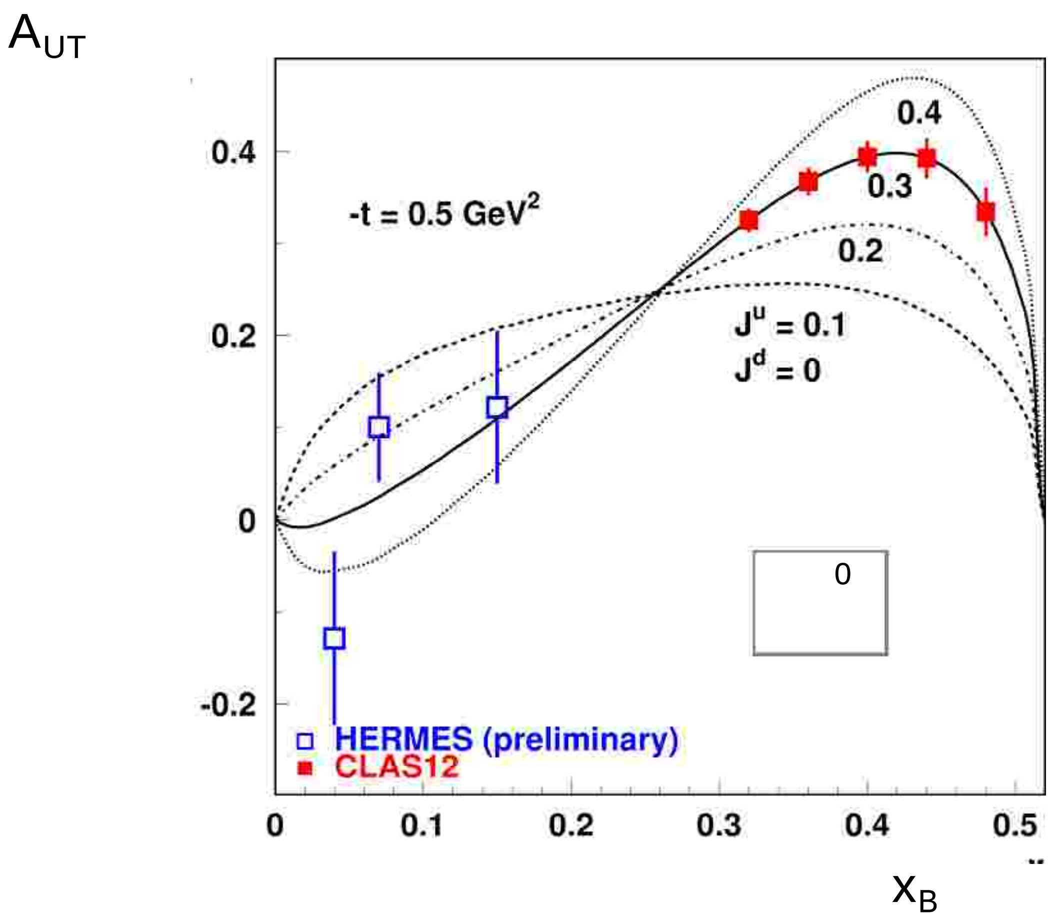
à Tremendous progress to constrain quark angular momenta
à 12 GeV will give final answers for quarks

Exclusive ^{0} production on transverse target

$$A_{UT} = - \frac{2 \quad _{\perp}(\mathrm{Im}(AB^*))/}{|A|^2(1- \;^2) - |B|^2(\;^2+t/4m^2) - \mathrm{Re}(AB^*)2 \;^2}$$

^{0}	$A \sim 2H^u + H^d$ $B \sim 2E^u + E^d$

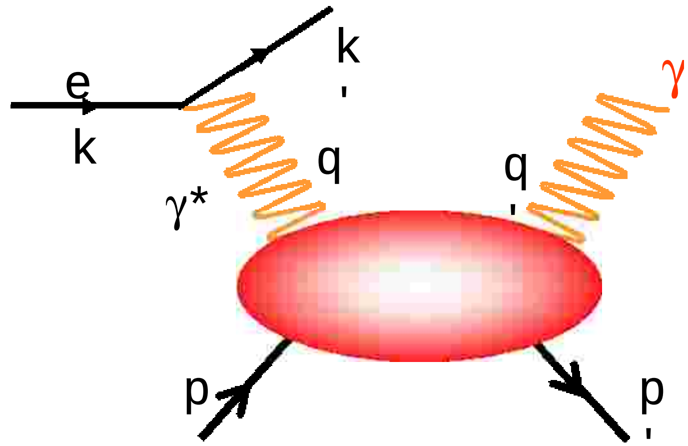
$^+$	$A \sim H^u - H^d$ $B \sim E^u - E^d$

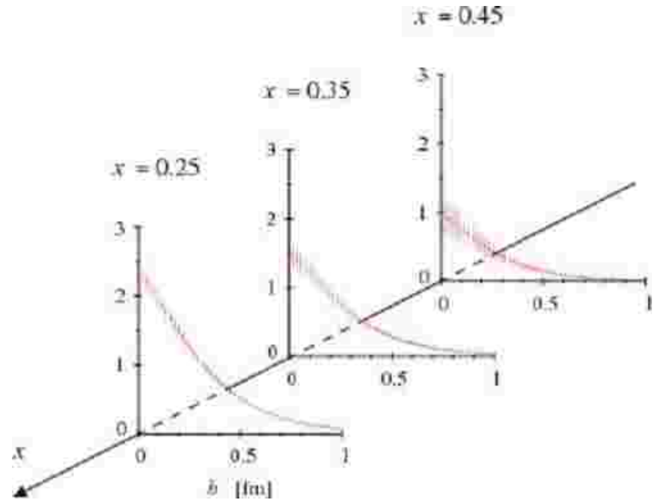


Asymmetry depends linearly on the GPD E, which enters Ji's sum rule.

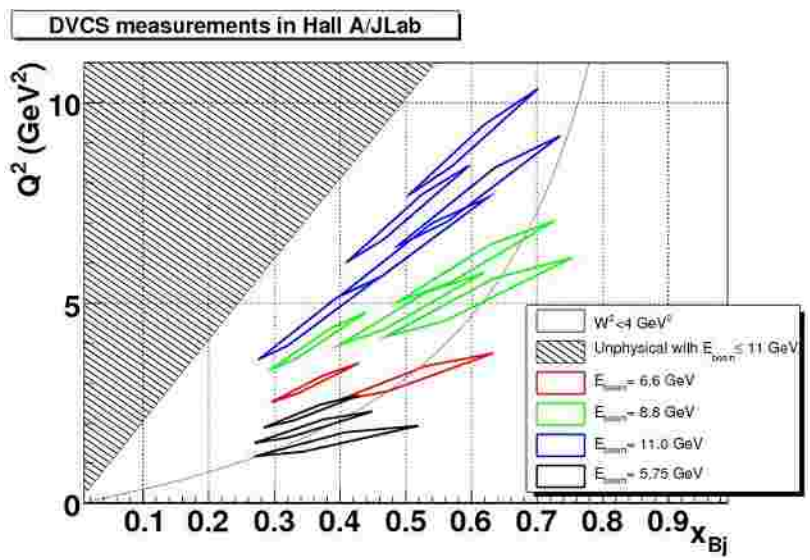
K. Goeke, M.V. Polyakov, M. Vanderhaeghen, 2001

Deep Exclusive Scattering and GPDs

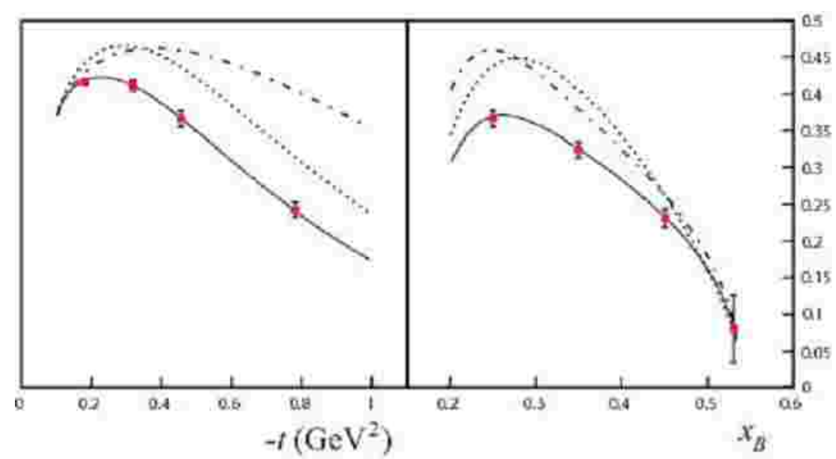




Rich DVCS program in Halls A (E12-06-114) and Hall B (E12-06-118) to start nucleon tomography, including cross section measurements and beam spin asymmetries

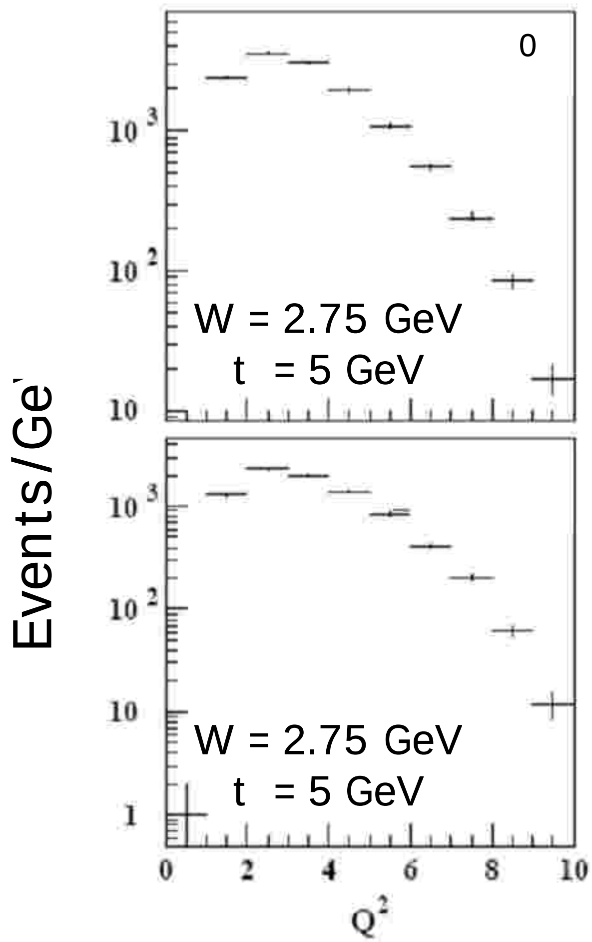


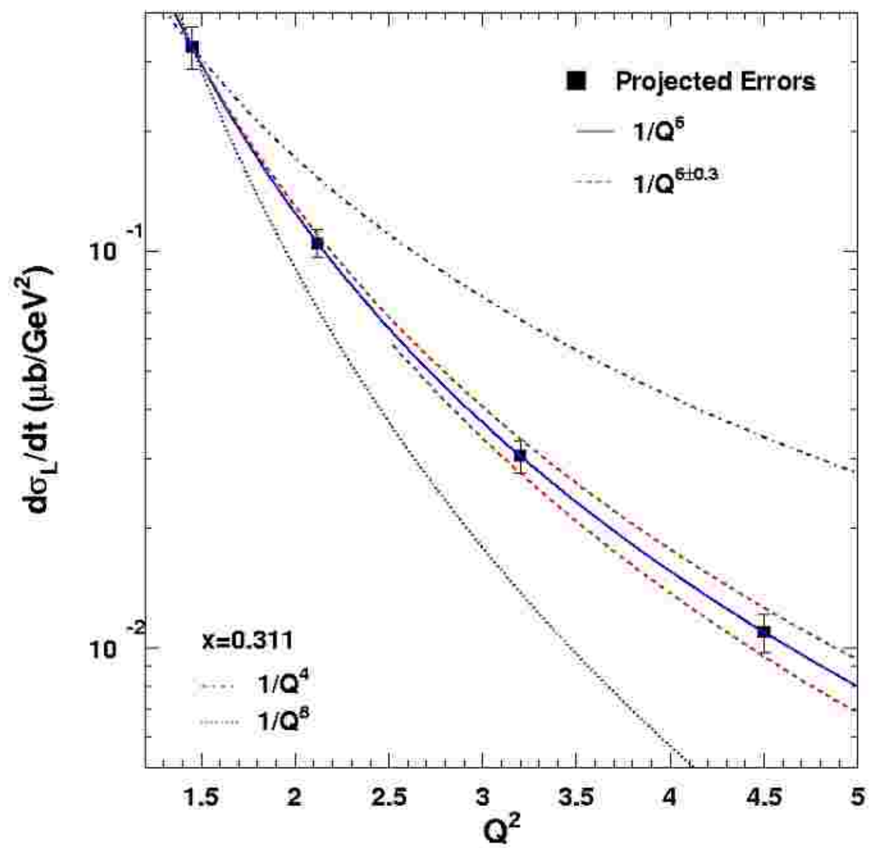
Example of beam spin asymmetry in Hall B



Deep Exclusive Scattering and GPDs

Augmented with dedicated experiments to study onset of scaling in meson electroproduction channels: ^{0}/ in E12-06-108 (Hall B) and $^+$ in E12-07-105 (Hall C)





DOE Office of Nuclear Physics Science Review of the proposed 12 GeV CEBAF Upgrade rated the scientific merit of program very highly & foresaw potential for discovery in at least two areas

General: "… these programs … will not be possible at any other known facility in the foreseeable future … have a high probability for discoveries that may lead to significant paradigm shifts."

Quark confinement: "… an impressive framework of research directed towards one of the top frontiers of contemporary science: the exploration of confinement, a unique phenomenon of the Strong Interaction …"

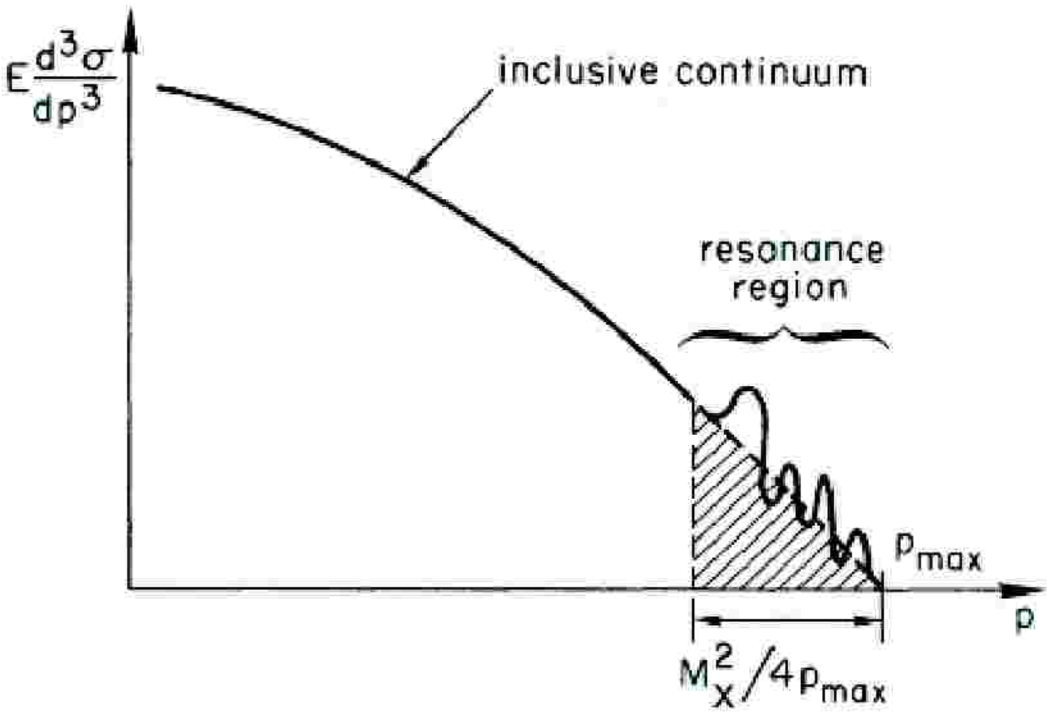
Proton structure: "… have a significantly advanced theoretical understanding of quark-gluon systems sufficient to allow scientists to construct the first "3D" images of the nucleon, and relate the quarks structure of the nucleon to its global properties."

Quark structure of nuclei: "… address the fundamental quest for the role of quarks in the nuclear many body system at an unprecedented level of precision."

Fundamental symmetries: "… provides a unique opportunity to use the electroweak interaction to search for physics beyond the Standard Model by performing low energy precision parity-violating measurements that are complementary to and comparable to searches at the LHC. … make possible a set of measurements that cannot be done elsewhere in the world until the construction of the ILC."

The Inclusive-Exclusive Connection

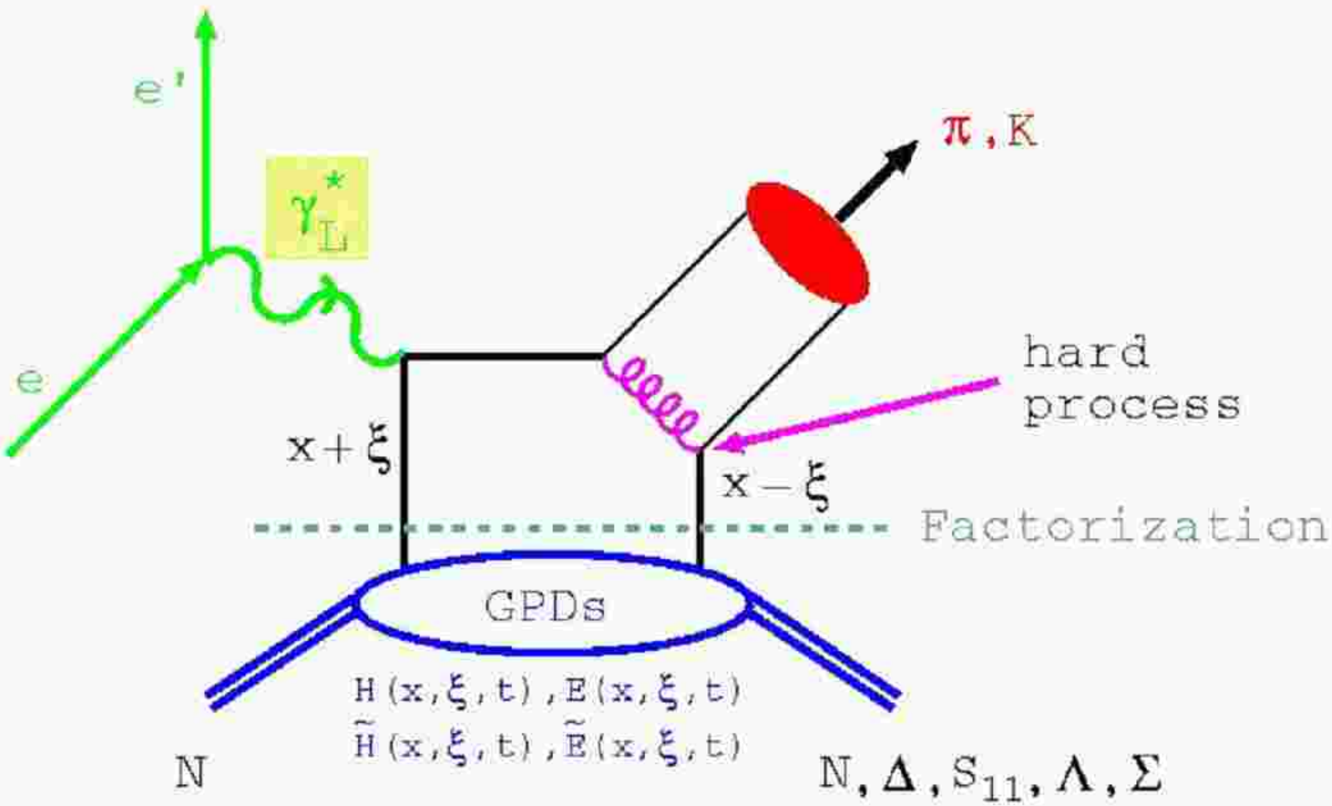
Inclusive-Exclusive connection: Bjorken and Kogut impose "correspondence principle": demanding continuity of the dynamics from one region of kinematics to the other à relates exclusive cross sections at low energy to inclusive production at high energies



Momentum Spectrum of produced hadrons in an inclusive reaction *N à MX

Duality is embedded in the quantum field theory (but we don't exactly know how).

Generalized Parton Distributions and Pion Electroproduction



For factorization to be strictly valid

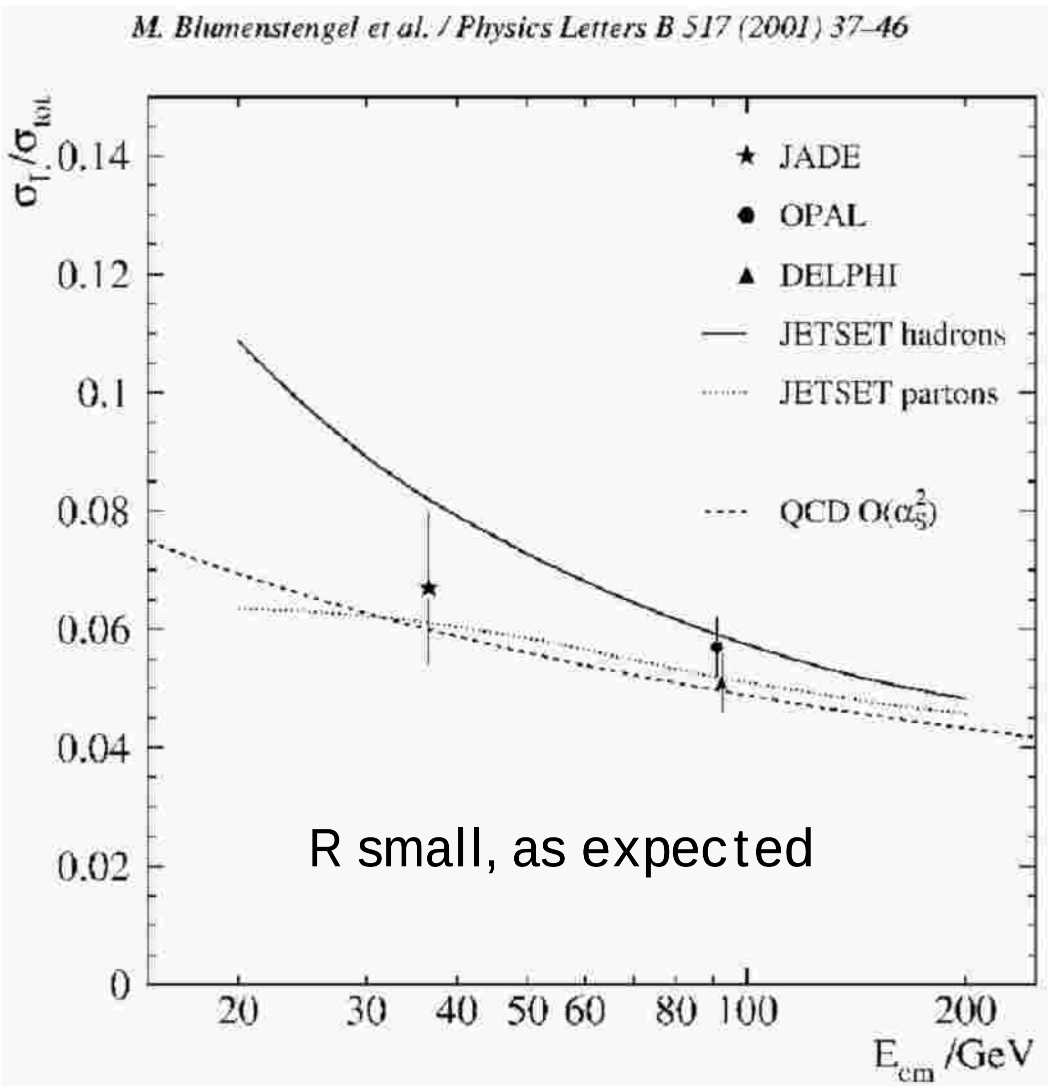
→ $\sigma_L \sim Q^{-6}, \sigma_T \sim Q^{-8}$

and

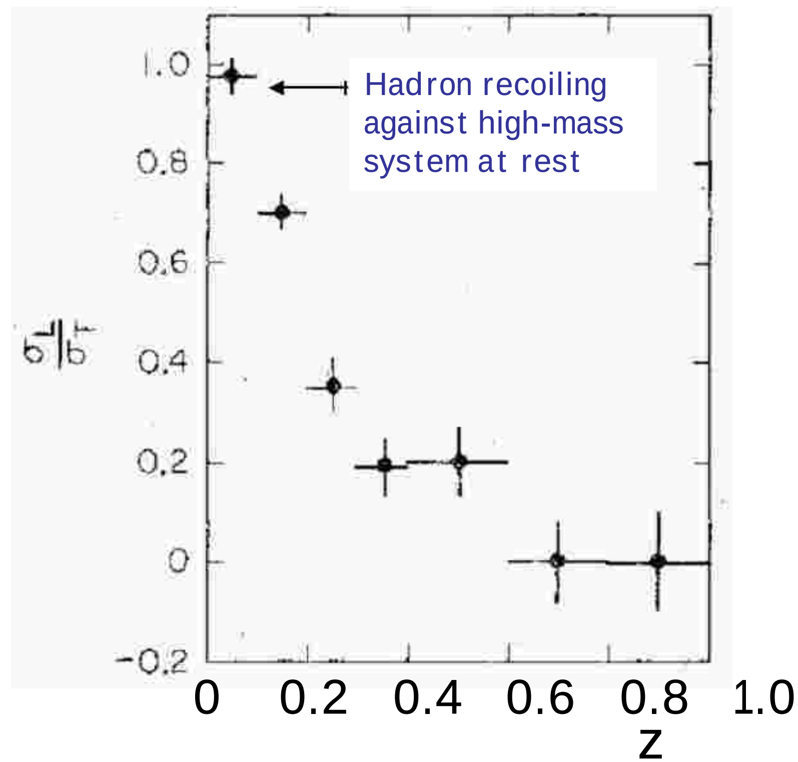
Onset of Color Transparency

$_L$/ $_T$ and Fragmentation at lower energies: e^+e^- à hadrons

$$d\sigma/d\Omega = \sigma_T + \sigma_L + (\sigma_T - \sigma_L)\cos^2\theta \; + P^2(\sigma_T - \sigma_L)\sin^2\theta\cos 2\varphi$$

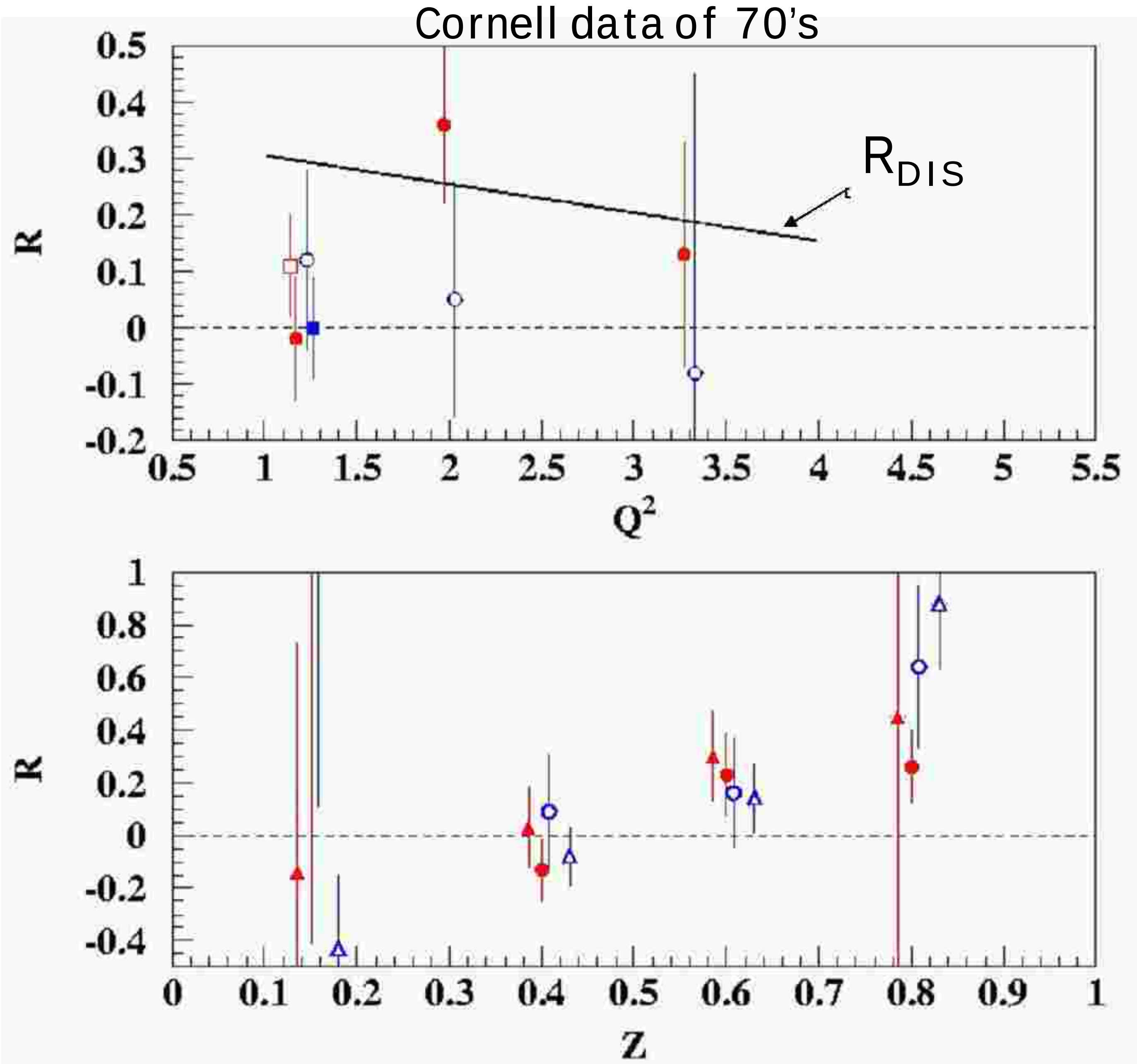


And at $s^{½}$ = 7.4 GeV?



Conclusion: the observed hadrons (pions) are only emitted by spin-½ partons at large z.

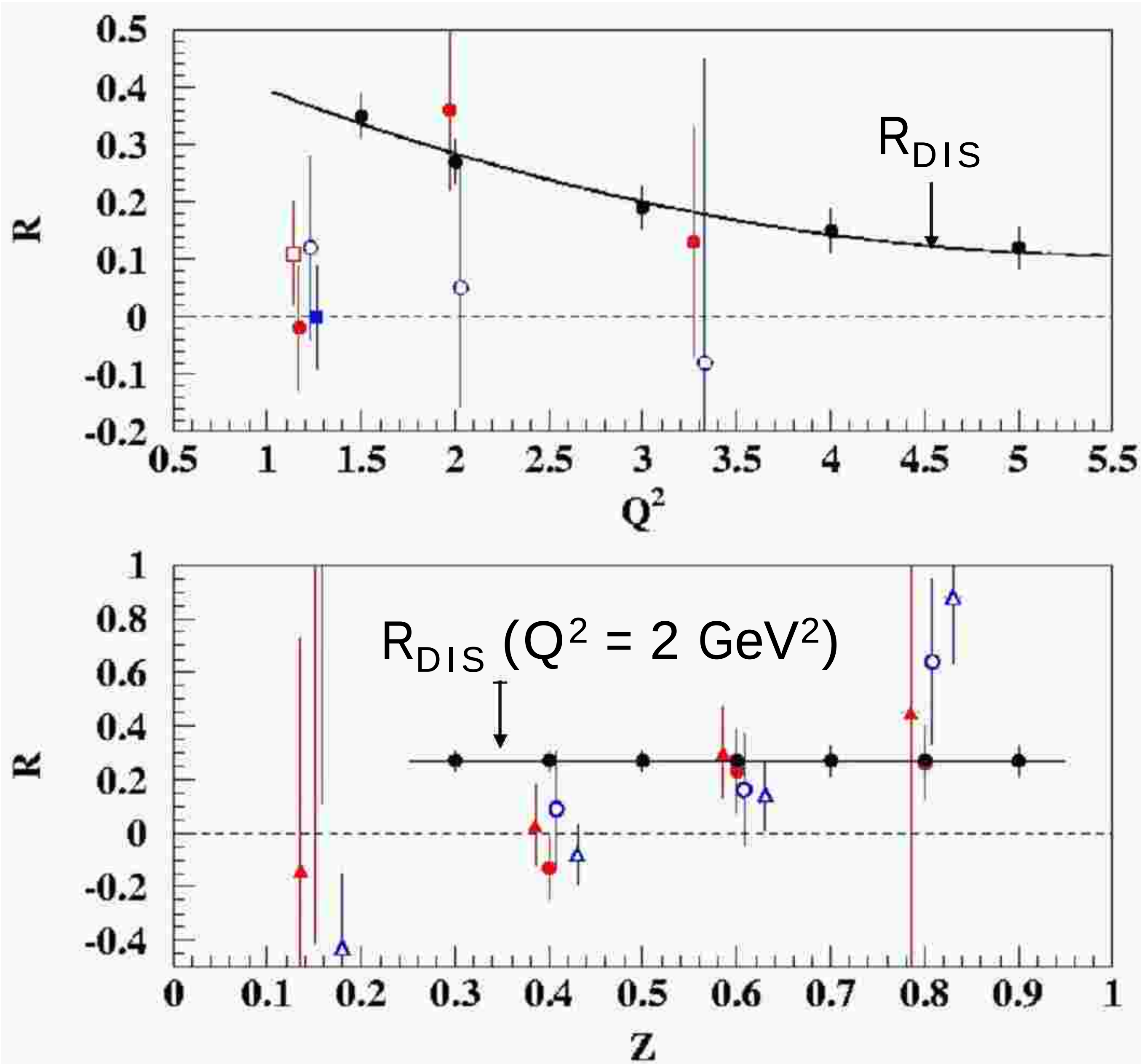
$_L$/ $_T$ and Fragmentation at lower energies: ep à e'hX



Conclusion: "data both consistent with R = 0 and R = R_{DIS}"

Most precise data at Q^2 = 1.2 GeV^2 are from mid-z region. Hint of larger R at large z?

For comparison, quality of proposed data



Proposed data cover range Q^2 = 1.5 – 5.0 GeV^2, with data for both H and D at Q^2 = 2 GeV^2

Scans in z are proposed at Q^2 = 2.0 (x = 0.2) and 4.0 GeV^2 (x = 0.4) à should settle the behavior of $_L/\ _T$ for large z