The longitudinal spin structure of the nucleon

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Contents

Deep inelastic scattering

Reactions of the type

$$lepton(k, s) + nucleon(P, S) \rightarrow lepton(k') + X$$

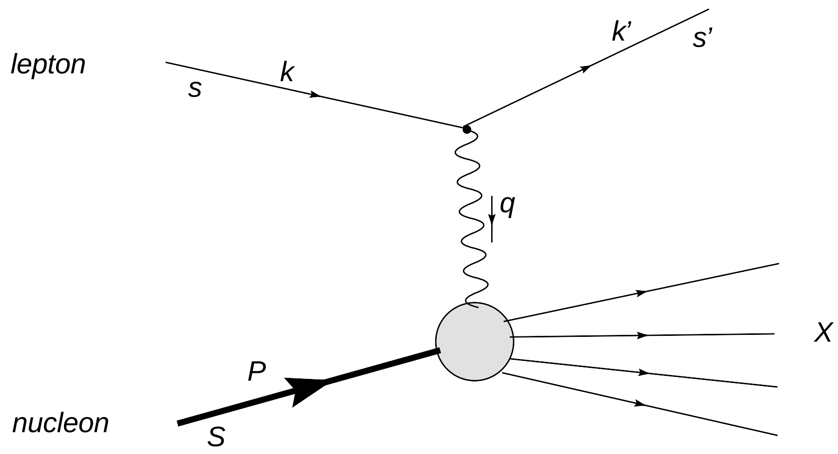
- Played a seminal role in the development of our present understanding of the substructure of elementary particles.

- Bjorken scaling (late nineteen-sixties) suggested that elementary particles contain almost pointlike constituents $\Rightarrow$ the Parton Model.

- Existence of missing constituents.... gluons.

- Testing of QCD.

One photon exchange approximation



$m =$ lepton mass, $M =$nucleon mass $s \cdot s = -1$

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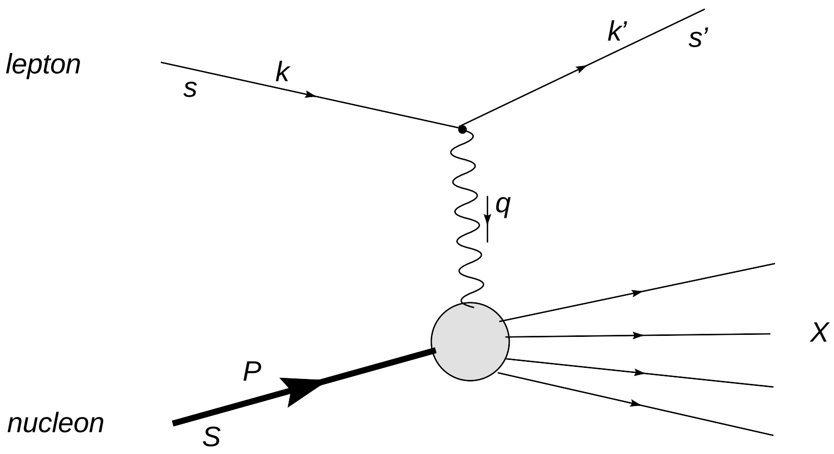

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Differential cross-section for detecting the final lepton in the solid angle $d\Omega$ and in the final energy range $(E', \ E' + dE')$ in the laboratory frame, $P = (M, \mathbf{0})$, $k = (E, \boldsymbol{k})$, $k' = (E', \boldsymbol{k}')$:

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{2Mq^4}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu}$$

where $q = k - k'$ and α is the fine structure constant.

The leptonic tensor $L_{\mu\nu}$ is given by

$$L_{\mu\nu}(k, s; k',) = \sum_{s'} [\bar{u}(k', s')\ \gamma_\mu\ u(k, s)]^*\ [\bar{u}(k', s')\ \gamma_\nu\ u(k, s)]$$

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Can split into symmetric (S) and antisymmetric (A) parts under μ,ν interchange:

$$L_{\mu\nu}(k,s;k',) = 2\{L^{(S)}_{\mu\nu}\ (k;k') + iL^{(A)}_{\mu\nu}\ (k,s;k')\}$$

where

$$\begin{aligned} L^{(S)}_{\mu\nu}\ (k;k') &= k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}\ (k\cdot k' - m^2) \\ L^{(A)}_{\mu\nu}\ (k,s;k') &= m\ \varepsilon_{\mu\nu\alpha\beta}\ s^\alpha\ q^\beta \end{aligned}$$

The unknown hadronic tensor $W_{\mu\nu}$: describes interaction between the virtual photon and the nucleon.
Depends upon four scalar structure functions: Unpolarized functions $W_{1,2}$; spin-dependent functions $G_{1,2}$.
Can only be functions of the scalars q^2 and $q \cdot P$.

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Usually work with

$$Q^2 \equiv -q^2 \quad \text{and} \quad x_{Bj} \equiv Q^2/2q \cdot P = Q^2/2M\nu$$

where $\nu = E - E'$ is the energy of the virtual photon in the Lab frame.

x_{Bj} is known as "x-Bjorken", and we shall simply write it as x.

$$W_{\mu\nu}(q;P,S) = W^{(S)}_{\mu\nu}(q;P) + i\ W^{(A)}_{\mu\nu}(q;P,S)$$

with

$$\frac{1}{2M}\ W^{(S)}_{\mu\nu}(q;P) =$$
$$\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\ W_1(P\cdot q, q^2)$$
$$+\ \ \left[\left(P_\mu - \frac{P\cdot q}{q^2}\ q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}\ q_\nu\right)\right]\frac{W_2(P\cdot q, q^2)}{M^2}$$

$$\frac{1}{2M}\ W^{(A)}_{\mu\nu}(q;P,S) =$$
$$\varepsilon_{\mu\nu\alpha\beta}\ q^\alpha\left\{MS^\beta G_1(P\cdot q, q^2)\right.$$
$$\left.+\ \ [(P\cdot q)S^\beta - (S\cdot q)P^\beta]\ \frac{G_2(P\cdot q, q^2)}{M}\right\}.$$

These expressions are electromagnetic gauge-invariant:

$$q^\mu W_{\mu\nu} = 0$$

The Bjorken limit, or Deep Inelastic Scattering (DIS) regime,

$$-q^2 = Q^2 \to \infty \qquad \nu = E - E' \to \infty$$

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{2M\nu}\,,\ \text{fixed}$$

Introduce scaling functions:

$$\begin{aligned}\lim_{Bj}\ MW_1(P\cdot q, Q^2) &= F_1(x, Q^2)\\ \lim_{Bj}\ \nu W_2(P\cdot q, Q^2) &= F_2(x, Q^2)\,,\end{aligned}$$

$$\begin{aligned}\lim_{Bj}\ \frac{(P\cdot q)^2}{\nu}\ G_1(P\cdot q, Q^2) &= g_1(x, Q^2)\\ \lim_{Bj}\ \nu\ (P\cdot q)\ G_2(P\cdot q, q^2) &= g_2(x, Q^2)\,.\end{aligned}$$

where $F_{1,2}$ and $g_{1,2}$ vary *very* slowly with Q^2 at fixed x.....they approximately *scale*.

Expression for $W_{\mu\nu}^{(A)}$ becomes

$$W_{\mu\nu}^{(A)}(q;P,s) = \frac{2M}{P\cdot q}\ \varepsilon_{\mu\nu\alpha\beta}\ q^{\alpha}\Bigg\{S^{\beta} g_1(x,Q^2) + \left[S^{\beta} - \frac{(S\cdot q)\,P^{\beta}}{(P\cdot q)}\right] g_2(x,Q^2)\Bigg\}.$$

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What can we measure?

Unpolarized scattering:

$$\frac{d^2\sigma}{dx\,dy} = \frac{4\pi\alpha^2 s}{Q^4}[xy^2F_1 + (1-y)F_2]$$

where

$$y \equiv \frac{\nu}{E} = \frac{P\cdot q}{P\cdot k} \qquad s = (P+k)^2$$

Lepton and target nucleon polarized longitudinally

$$\frac{d^2\sigma^{\overset{\longrightarrow}{\Leftarrow}}}{dx\,dy} - \frac{d^2\sigma^{\overset{\longrightarrow}{\Rightarrow}}}{dx\,dy} = \frac{16\pi\alpha^2}{Q^2}\left[(1-\frac{y}{2})\,g_1 - \frac{2M^2xy}{Q^2}g_2\right].$$

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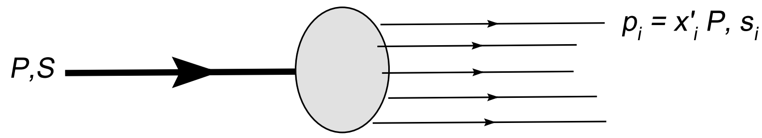
In principle can measure both g_1 and g_2, but the transverse asymmetry much smaller and therefore much more difficult to measure. Only in past few years have information on g_2 which turns out to be smaller than g_1.

The simple parton model

In frame where the proton is moving very fast, say along the OZ axis, it can be viewed as a beam of *parallel-moving* partons,



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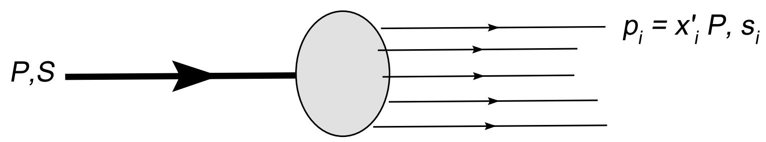
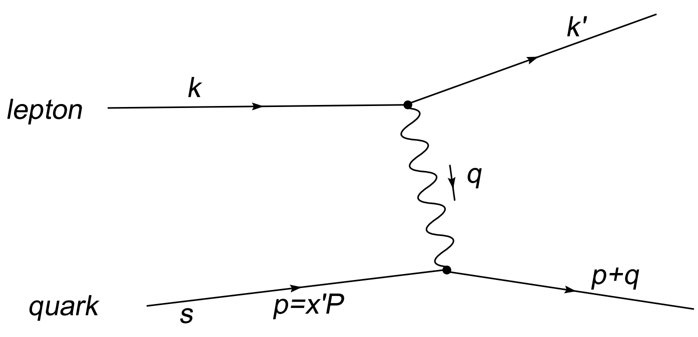


In the hard interaction with the photon, the quark-partons are treated as free, massless particles with momentum $x'\boldsymbol{P}$,



Find antisymmetric part of the hadronic tensor is given by

$$W_{\mu\nu}^{(A)}(q:P,S) = \sum_{f,s} e_f^2 \frac{1}{2P\cdot q} \int_0^1 \frac{dx'}{x'} \delta(x'-x) \, n_f(x';s,S)\, w_{\mu\nu}^{(A)}(x';q,s)$$

where $w_{\mu\nu}^{(A)}(x';q,s)$ = quark tensor, just like leptonic tensor $L_{\mu\nu}^{(A)}$ since quarks are treated as point-like particles; sum is over flavours f and spin orientations s of struck quark.

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Delta-function forcing $x' = x$ arises from treating the quarks as “free” particles on mass shell i.e. taking

$p^2 = (x'P)^2 = 0 \qquad (q+p)^2 = (q+x'P)^2 = 0$

so that

$$q^2 + 2x'q\,.\,P = 0 \quad \Rightarrow \quad -Q^2 + Q^2\frac{x}{x'} = 0$$

i.e. $x' = x = x_{Bjorken}$.

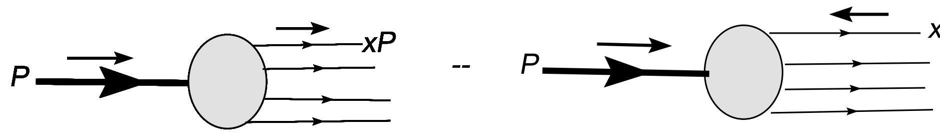
Longitudinal polarization

Fast moving proton, momentum along OZ, and polarized along OZ. Find

$$g_1(x) = \frac{1}{2} \sum_f e_f^2 \triangle q_f(x)$$

$$\triangle q(x) = q_{(+)}(x) - q_{(-)}(x)$$

where $q_{(\pm)}(x)$ are the number densities of quarks whose spin orientation is parallel or antiparallel to the spin direction of the proton .



Usual (unpolarized) parton density is

$$q(x) = q_{(+)}(x) + q_{(-)}(x)$$

What about $g_2(x)$?

There are many different, inconsistent results for $g_2(x)$ in the literature, including this beautiful one

$$g_2(x) = \frac{1}{2}\sum e_f^2 \left(\frac{m_q}{xM} - 1\right) \Delta q(x)$$

due to Anselmino and myself, which, alas, should not be taken seriously. There is no exact parton model result for $g_2(x)$. The only reliable result is the Wandzura-Wilcczek approximate relation

$$g_2(x) \simeq -g_1(x) + \int_x^1 \frac{g_1(x')}{x'} \, dx'$$

which was originally derived as an approximation in an operator product expansion approach, but which has recently been shown to be derivable directly in the simple parton model.

The spin crisis in the parton model

Expression for g_1 completely analogous to F_1, with $q(x) \to \Delta q(x)$.

$$g_1(x) = \frac{1}{2}\left\{\frac{4}{9}\Delta u(x) + \frac{1}{9}\Delta d(x) + \frac{1}{9}\Delta s(x) + \Delta \bar{q}s\right\}$$

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Define combinations with specific transformation properties under the group of flavour transformations $SU(3)_F$:

$$\Delta q_3 = (\Delta u + \Delta\overline{u}) - (\Delta d + \Delta\overline{d})$$

$$\Delta q_8 = (\Delta u + \Delta\overline{u}) + (\Delta d + \Delta\overline{d}) - 2(\Delta s + \Delta\overline{s})$$

$$\Delta\Sigma = (\Delta u + \Delta\overline{u}) + (\Delta d + \Delta\overline{d}) + (\Delta s + \Delta\overline{s})$$

which transform respectively as the third component of an isotopic spin triplet, the eighth component of an $SU(3)_F$ octet and a flavour singlet.

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First moment of this yields

$$\Gamma_1 \equiv \int_0^1 g_1(x)dx = \frac{1}{9}\left[\frac{3}{4}a_3 + \frac{1}{4}a_8 + a_0\right]$$

where

$$\begin{aligned} a_3 &= \int_0^1 dx\ \Delta q_3(x) \\ a_8 &= \int_0^1 dx\ \Delta q_8(x) \\ a_0 &= \Delta\Sigma \equiv \int_0^1 dx\ \Delta\Sigma(x) \end{aligned}$$

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Via the Operator Product Expansion these moments can be related to hadronic matrix elements of an octet of currents which are *measurable* in *other* processes.

These currents control the β-decays of the neutron and of the octet of hyperons which implies that the values of a_3 and a_8 are known from β-decay measurements.

$$a_3 = 1.2670 \pm 0.0035 \qquad a_8 = 0.585 \pm 0.025$$

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Why startling???

Consider the physical significance of $\Delta\Sigma(x)$. Since $q_\pm(x)$ count the number of quarks of momentum fraction x with spin component $\pm\frac{1}{2}$ along the direction of motion of the proton (say the z-direction), the total contribution to J_z coming from a given flavour quark is

$$\begin{aligned} S_z &= \int_0^1 dx \left\{ \left(\frac{1}{2}\right) q_+(x) + \left(\frac{-1}{2}\right) q_-(x) \right\} \\ &= \frac{1}{2}\int_0^1 dx\ \Delta q(x)\,. \end{aligned}$$

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where S_z^{quarks} is the contribution to J_z from the spin of all quarks and antiquarks. The EMC result for the value of a_0 implied that

$$\left(S_z^{quarks}\right)_{Exp} = 0.03 \pm 0.06 \pm 0.09 \; .$$

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In a more realistic relativistic model one expects $2\langle S_z^{quarks}\rangle \approx 0.6$, which is far from the EMC value.

This discrepancy between the contribution of the quark spins to the angular momentum of the proton, as measured in DIS and as computed in both non-relativistic and relativistic constituent models of the proton, was termed a “spin crisis in the parton model” .

The parton model in QCD

In QCD with quark and gluon fields as the fundamental fields, there are interaction dependent modifications of the parton model formulae for DIS.

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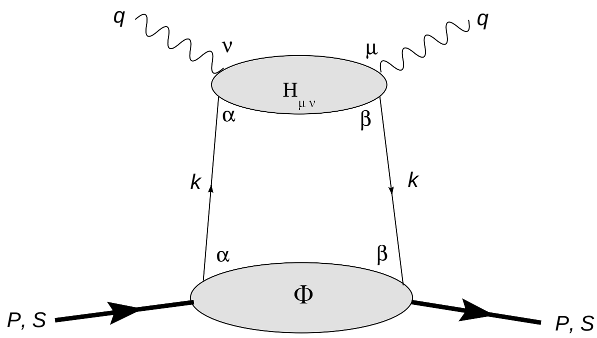
The description of nucleon structure becomes much more complicated, involving twelve functions. The parton model number densities $q(x)$, $\Delta q(x)$ (and the analogue for transversely spinning nucleons $\Delta_T q(x)$ are only the principal, so called "leading twist" members of this set.

The reaction is visualized as follows



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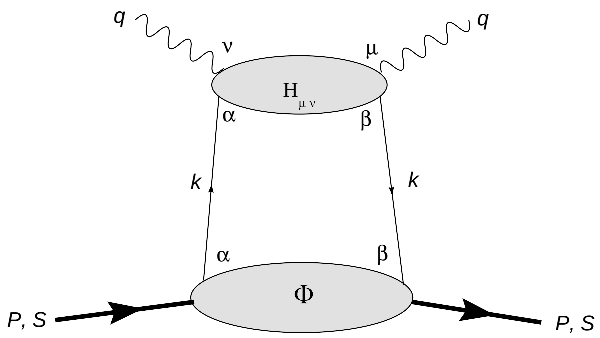

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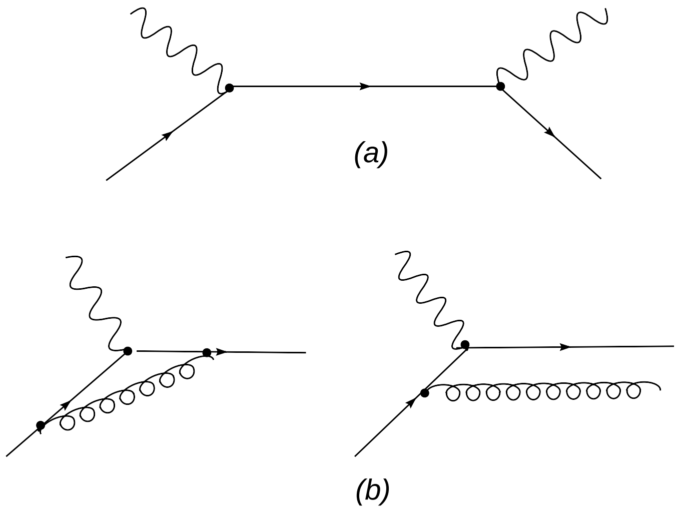
The main impact of the QCD interactions will be twofold:
1) to introduce a mild, calculable logarithmic Q^2 dependence in the parton densities
2) to generate a contribution to g_1 arising from the polarization of the gluons in the nucleon

Source of these effects:

1) **QCD corrections and evolution.** Born term for the interaction of the virtual photon with a quark (the hard blob), and simplest correction terms:



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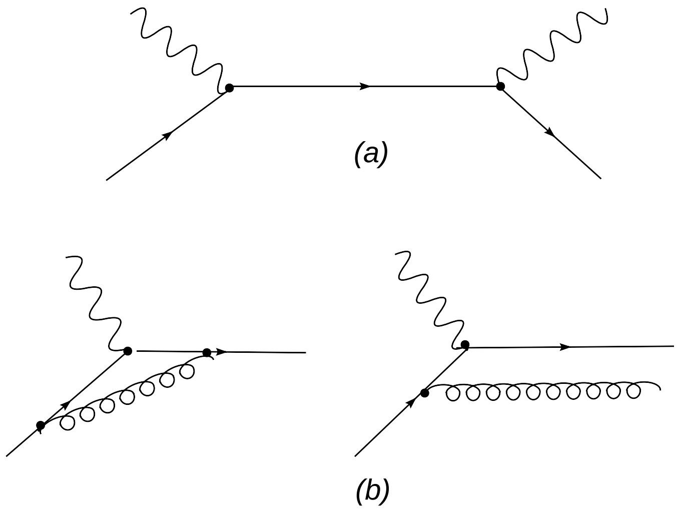

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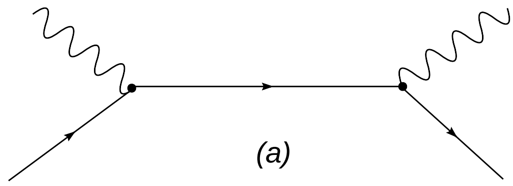
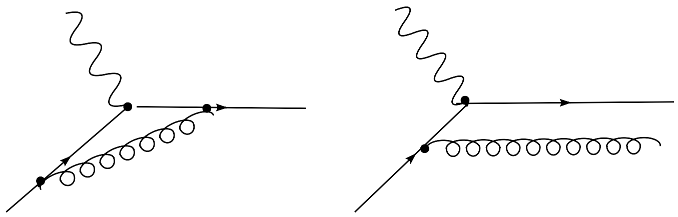

(a)


(b)

Correction terms are infinite: collinear divergences because of masslessness of the quarks. Removed by a process known as factorization. Reaction is factorized (separated) into a hard and soft part and the infinity is absorbed into the soft part, which cannot be calculated and has to be parametrized and studied experimentally.

The point at which this separation is made is the *factorization scale* μ^2. Terms like $\alpha_s \, ln \, \frac{Q^2}{m_q^2}$ are split:

$$\alpha_s \, ln \, \frac{Q^2}{m_q^2} = \alpha_s \, ln \, \frac{Q^2}{\mu^2} + \alpha_s \, ln \, \frac{\mu^2}{m_q^2}$$

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However the parton density has an extra label μ^2 specifying our choice.Also we never calculate to all orders in perturbation theory, so can make a difference what value we choose. An optimal choice is $\mu^2 = Q^2$, so the parton densities now depend on both x and Q^2 i.e. we have $q(x, Q^2)$ and $\Delta q(x, Q^2)$, and perfect Bjorken scaling is broken.

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$$\alpha_s \, ln \, \frac{Q^2}{m_q^2} = \alpha_s \, ln \, \frac{Q^2}{\mu^2} + \alpha_s \, ln \, \frac{\mu^2}{m_q^2}$$

Absorb first term on the RHS into the hard part and the second into the soft part. μ^2 is an arbitrary number, like the renormalization scale, and, in an *exact* calculation, physical results *cannot* depend on it.

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The variation with Q^2 is gentle (logarithmic), and can be calculated via what are called the *evolution equations*

Scheme dependence

In handling these divergences use the technique of *dimensional regularization*. Straightforward in unpolarized case. Ambiguities in the polarized case.

Hence several different factorization schemes. *Crucial*, when presenting results on the parton densities, to specify scheme.

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At present there are three schemes in use:

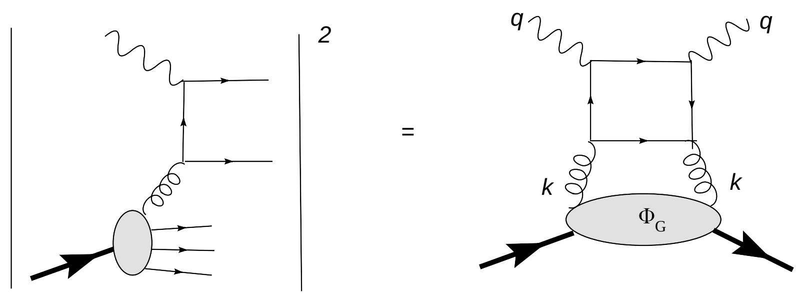
i) $\overline{MS} - MNV$ (abbreviated as $\overline{MS}$). In this scheme a_3 and a_8 are independent of Q^2.

ii) AB Here also the first moment $\Delta\Sigma$ is independent of Q^2.

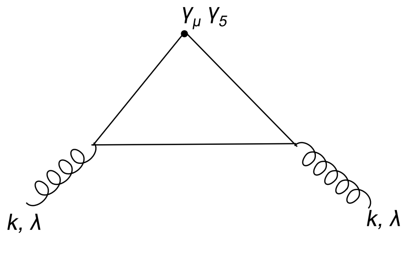
iii) $JET = Chiral Invariant scheme$. Here a_3 and a_8 are independent of Q^2 as is $\Delta\Sigma$, but it can be argued that the JET scheme is superior to the others in that *all* hard effects are included in $H_{\mu\nu}$.

2) **The gluon contribution to g_1.**

In NLO gluon-initiated contribution to DIS:



In the Bjorken limit, for the longitudinal polarized case, involves the gluonic version of the Adler and Bell and Jackiw anomalous triangle diagram:



Result: an anomalous gluonic contribution to the flavor singlet a_0

$$\begin{aligned} a_0^{gluons}(Q^2) &= -3\,\frac{\alpha_s(Q^2)}{2\pi}\int_0^1 dx\ \Delta G(x,Q^2) \\ &\equiv -3\,\frac{\alpha_s(Q^2)}{2\pi}\,\Delta G(Q^2). \end{aligned}$$

Note: factor 3 corresponds to the number of *light* flavors i.e. u, d, s. Heavy flavors do not contribute.

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Note: factor 3 corresponds to the number of *light* flavors i.e. u,d,s. Heavy flavors do not contribute.

So, there exists potentially a *gluonic* contribution to the first moment of g_1:

$$\Gamma_1^{gluons}(Q^2) = -\frac{1}{3}\,\frac{\alpha_s(Q^2)}{2\pi}\,\Delta G(Q^2)\,! \qquad (4)$$

This result is of fundamental importance. It implies that the simple parton model formula for a_0 (and hence for Γ_1^p) in terms of the Δq_f is incomplete. Instead,

$$a_0 = \Delta\Sigma - 3\,\frac{\alpha_s}{2\pi}\ \Delta G. \qquad (5)$$

A subtlety: result actually depends on the factorization scheme. Correct in AB and JET schemes, but gluon contribution to a_0 is zero in the $\overline{MS}$ scheme.

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Fundamental conclusion: small measured value of a_0 does not necessarily imply that the physically meaningful, invariant $\Delta\Sigma$ is small. A resolution of the spin crisis???

Expression for g_1 in NLO

The expression for $g_1(x, Q^2)$ now becomes

$$
\begin{aligned}
g_1(x,Q^2) &= \frac{1}{2} \sum_{flavours} e_q^2 \Big\{ \Delta q(x,Q^2) + \Delta\bar{q}(x,Q^2) \\
&+ \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \{ \Delta C_q(x/y) \, [\Delta q(y,Q^2) \\
&+ \Delta\bar{q}(y,Q^2)] \\
&+ \Delta C_G(x/y) \, \Delta G(y,Q^2) \} \Big\}
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\end{aligned}
$$

where ΔC_G and ΔC_q are Wilson coefficients evaluated from the hard part calculated beyond the Born approximation. Depend on the *scheme*!

Note that very often these equations are written using the *convolution notation*, for example,

$$\Delta C_q \otimes \Delta q \equiv \int_x^1 \frac{dy}{y} \Delta C_q(x/y)\, \Delta q(y) \qquad (7)$$

Extraction of parton densities from DIS data

1) Parametrize densities at some Q_0^2 e.g.

$$\Delta q(x, Q_0^2) = C\, x^{\alpha}(1-x)^{\beta} q(x, Q_0^2)$$

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3) Calculate $\Delta q(x, Q^2)$ for $Q^2 \neq Q_0^2$ via evolution equations.

4) Determine parameters by χ^2 minimization.

Evolution equations

For the polarized densities the evolution equations are

$$\frac{d}{d\,lnQ^2}\,\Delta q(x,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}$$
$$\{\Delta P_{qq}(x/y)\Delta q(y,Q^2) + \Delta P_{qG}(x/y)\,\Delta G(y,Q^2)\}$$

$$\frac{d}{d\,lnQ^2}\,\Delta G(x,Q^2) = \frac{\alpha_s(Q^2}{2\pi}\int_x^1 \frac{dy}{y}$$
$$\{\Delta P_{Gq}(x/y)\,\Delta q(y,Q^2) + \Delta P_{GG}(x/y)\,\Delta G(y,Q^2)\}$$

where $\Delta G(x)$ is analogous to $\Delta q(x)$

$$\Delta G(x) = G_+(x) - G_-(x).$$

The ΔP are the polarized *splitting functions* and are calculated perturbatively

$$\Delta P(x) = \Delta P^{(0)}(x) + \frac{\alpha_s}{2\pi}\,\Delta P^{(1)}(x)$$

where the superscripts (0) and (1) refer to LO and NLO contributions.

Behaviour as $x \to 1$

Perturbative QCD argument:

$$q_{\pm}(x) \to (1-x)^{2n-1+(1\mp1)}$$

where n is the number of *spectator quarks*.

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Implies

$$\frac{\Delta q(x)}{q(x)} \to 1 \qquad \text{as} \qquad x \to 1$$

Not clear whether parton densities obey this. Not imposed in parametrization.

Problem with polarized data

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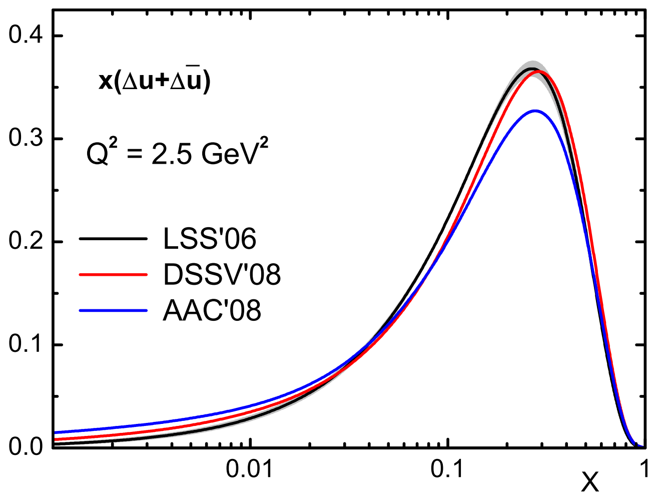
Most polarized data at relatively small values of Q^2.

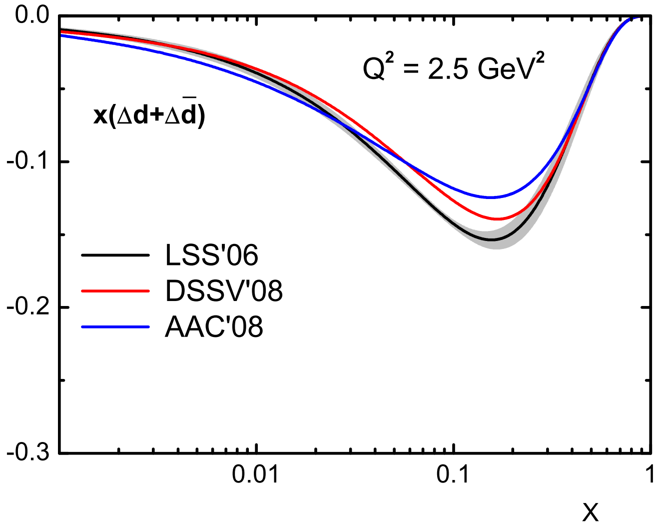
Solution: Add *higher twist* terms:$\frac{h(x)}{Q^2}$. Parametrize $h(x)$.

Status of the polarized parton densities

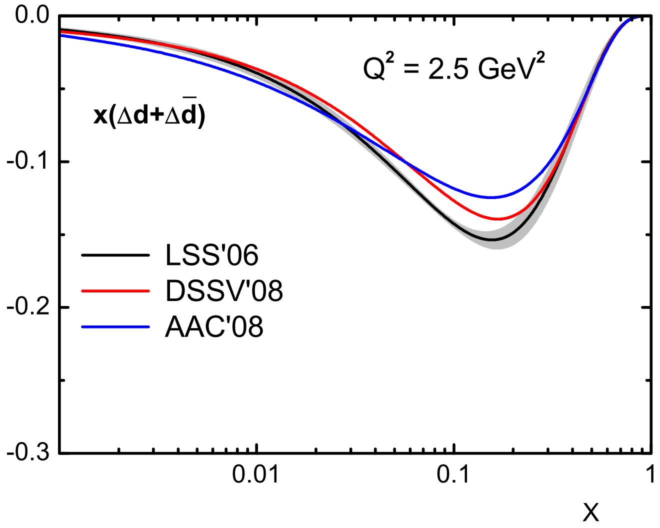
1) **The light quark densities**

Broad agreement between the various analyses for the $\Delta u(x) + \Delta\bar{u}(x)$ and $\Delta d(x) + \Delta\bar{d}(x)$ parton densities.





Early data demanded negative values of $\Delta d(x)+\Delta\bar{d}(x)$ and continued to do so even when the measured region was extended to $x = 0.6$ at Jefferson Laboratory .



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Note that enforcing

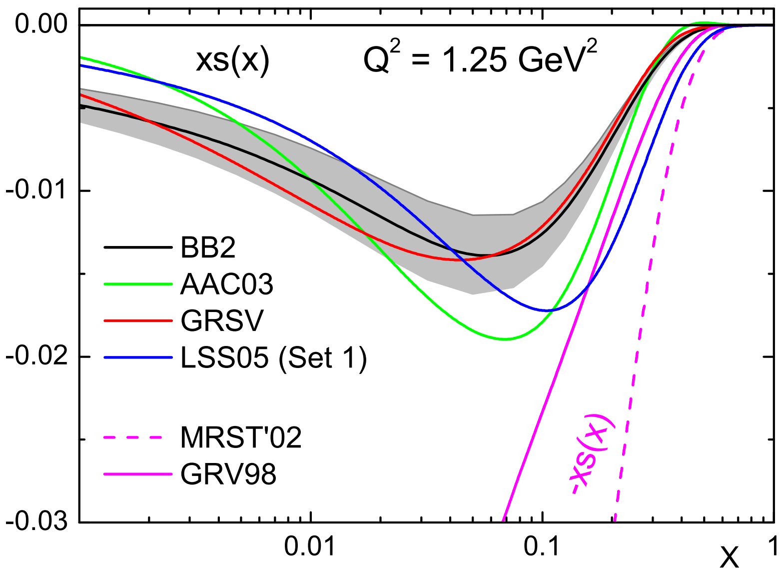
$$\frac{\Delta q(x)}{q(x)} \longrightarrow 1 \qquad \text{as} \qquad x \longrightarrow 1$$

led to a $\Delta d(x)+\Delta\bar{d}(x)$ which became positive just beyond $x = 0.6$. With the 12 GeV upgrade at Jefferson it should be possible to explore out to $x = 0.8$ and to settle the matter.

2) **The polarized strange quark density.**

A controversial issue at present. All analyses of purely DIS data have found negative values for $\Delta s(x) + \Delta\bar{s}(x)$. But shapes little different.

Reason: positivity



An important quantity:

$$\Delta S \equiv \int_0^1 dx[\Delta s(x) + \Delta \bar{s}(x)].$$

LSS'06 give

$\Delta S_{\overline{MS}} = -0.126 \pm 0.010$ at $Q^2 = 1\, GeV^2$

Can show: positive value $\Rightarrow$ huge breaking of $SU(3)_F$ invariance.

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Why? Not clear. Need fragmentation function. LO???

$$q \to K + X$$

More puzzling. Recent NLO combined analysis of DIS, SIDIS and $pp \to \pi^0$ or jet $+ X$ by the DSSV group also finds positive values for $\Delta s(x) + \Delta\bar{s}(x)$ for $x \geq 0.03$.

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Caused by need to satisfy $SU(3)_F$ symmetry??

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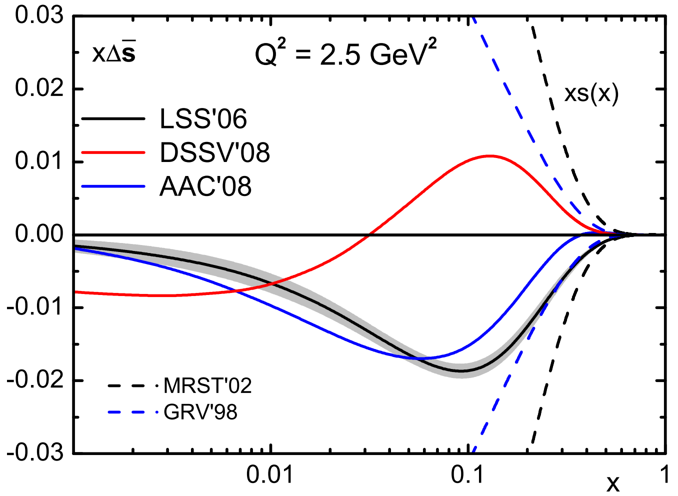
Why, in χ^2 analysis, does it come out almost zero???

Recent AAC analysis (2008) of DIS plus the π^0 production data from RHIC finds a negative strange quark density.



3) **The flavor singlet first moment $\Delta\Sigma$.**

All modern global analyses obtain compatible values for $\Delta\Sigma$. In the $\overline{MS}$ scheme, where $a_0(Q^2) = \Delta\Sigma(Q^2)$ they find at $Q^2 = 4$ GeV^2:

LSS'06	COMPASS'06	AAC'08	DSSV
0.24 ± 0.04	0.29 ± 0.01	0.25 ± 0.05	0.24

4) **The polarized gluon density.**

Hoped for resolution of "spin crisis in the parton model":

$$a_0 = \Delta\Sigma - 3\,\frac{\alpha_s}{2\pi}\;\Delta G.$$

Get small value of a_0 via cancellation between relatively large $\Delta\Sigma$ and ΔG

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Get small value of a_0 via cancellation between relatively large $\Delta\Sigma$ and ΔG

Present day estimates are $a_0 \approx 0.25$. Thus demanding $\Delta\Sigma \approx 0.6$ requires, for the first moment,

$$\Delta G \approx 1.7 \qquad \text{at} \qquad Q^2 = 1\,GeV^2$$

The question is whether this is compatible with what we know about the polarized gluon density.

Three ways to access $\Delta G(x)$:

i) via polarized DIS

ii) via the measurement of the asymmetry A_{LL} in SIDIS production of charmed quarks or high p_T jets

iii) via the measurement of the asymmetry A_{LL} in semi-inclusive polarized pp reactions at RHIC.

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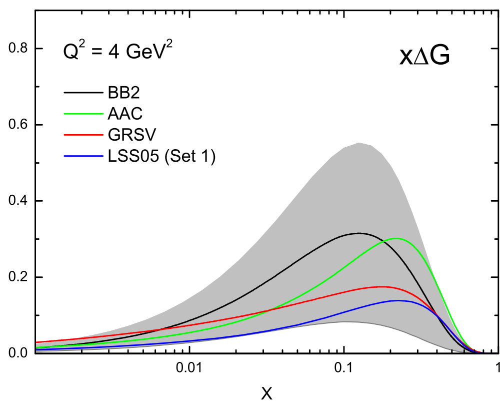
i) *DIS*

In fits to data on $g_1(x, Q^2)$ main role of the gluon is in the evolution with Q^2. But range of Q^2 very limited. Hence determination of $\Delta G(x)$ is imprecise.

Pre 2005 all analyses seemed to indicate that $\Delta G(x)$ was a positive function of x.



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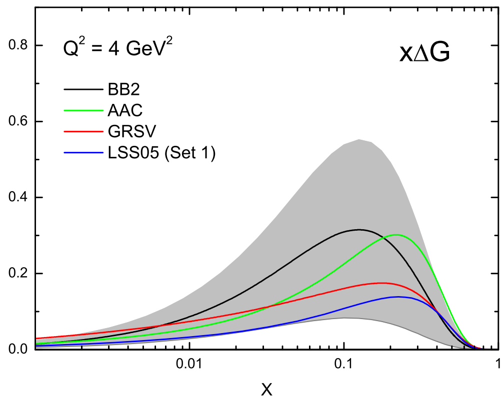
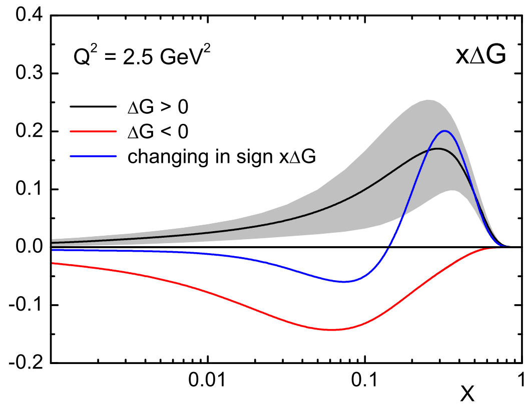


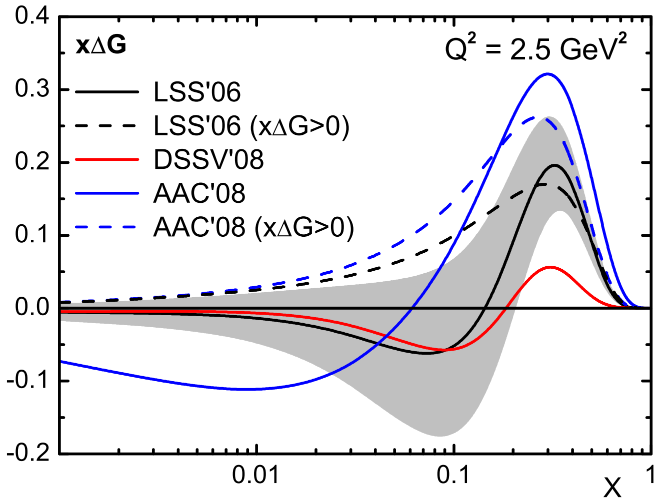
With recent data: LSS'06 good fits with positive, negative and sign-changing $\Delta G(x)$, *provided* higher twist terms included.



0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
Q^2 = 2.5 GeV^2
xΔG
ΔG > 0
ΔG < 0
changing in sign xΔG
0.01
0.1
1
x

45

The present situation:



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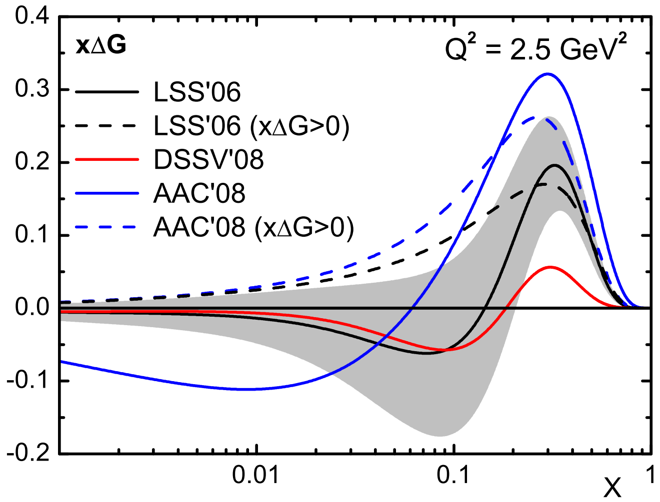
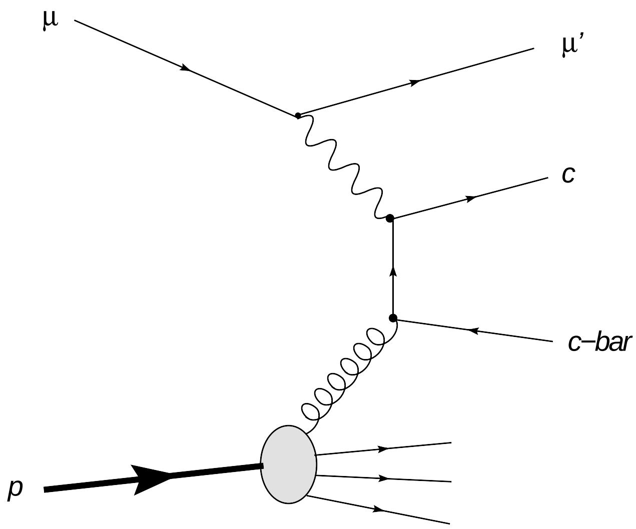


At present, in *all* fits, irrespective of the form of the gluon density, the magnitude is very small. Typically $|\Delta G| \approx 0.29 \pm 0.32$, *much* smaller than the desired 1.7 !

ii) *Asymmetry A_{LL} in SIDIS production of charmed quarks or high p_T jets (HERMES, COMPASS)*

No charmed quarks in nucleon. Hence produced via



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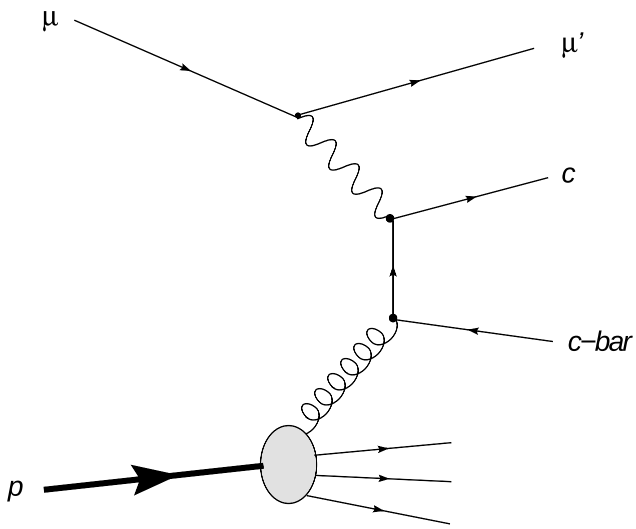

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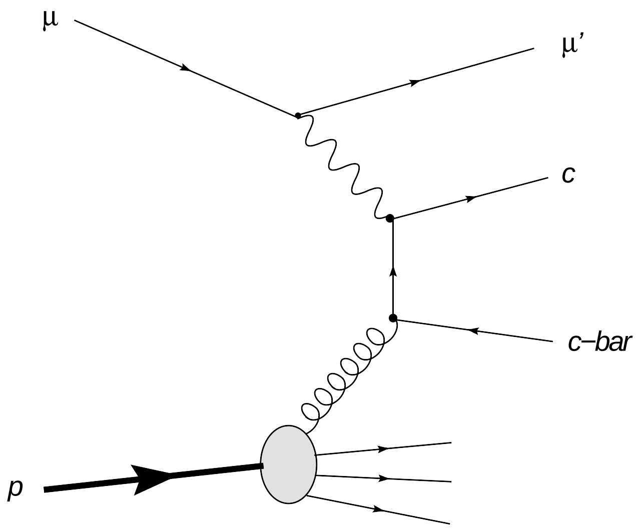

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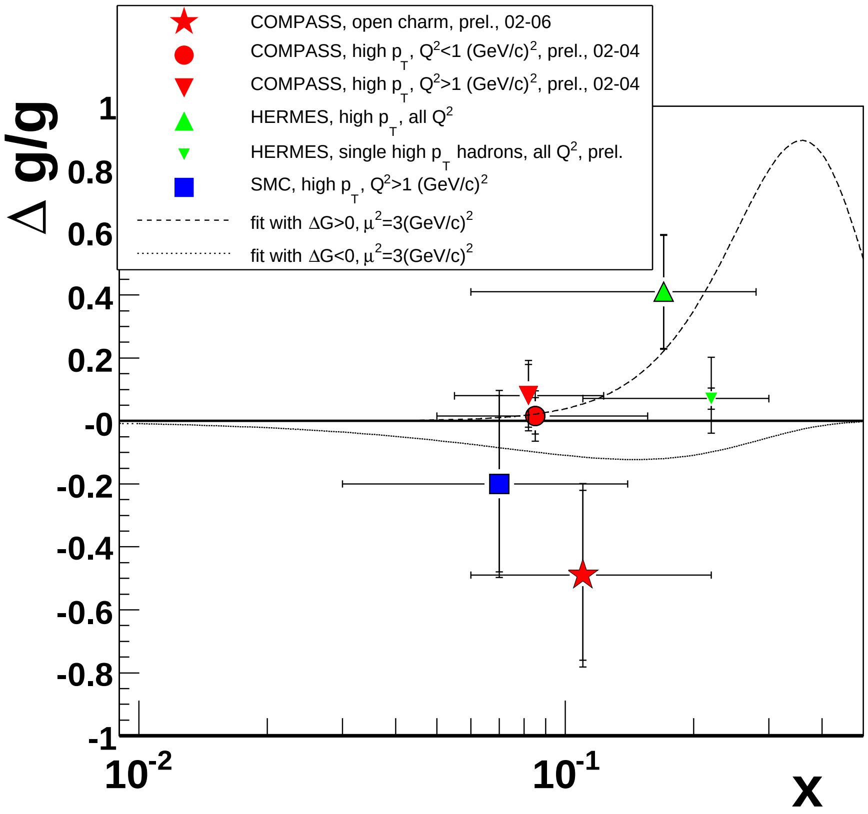
Hence detect single, charmed meson with large transverse momentum or jets.

Results (some preliminary)



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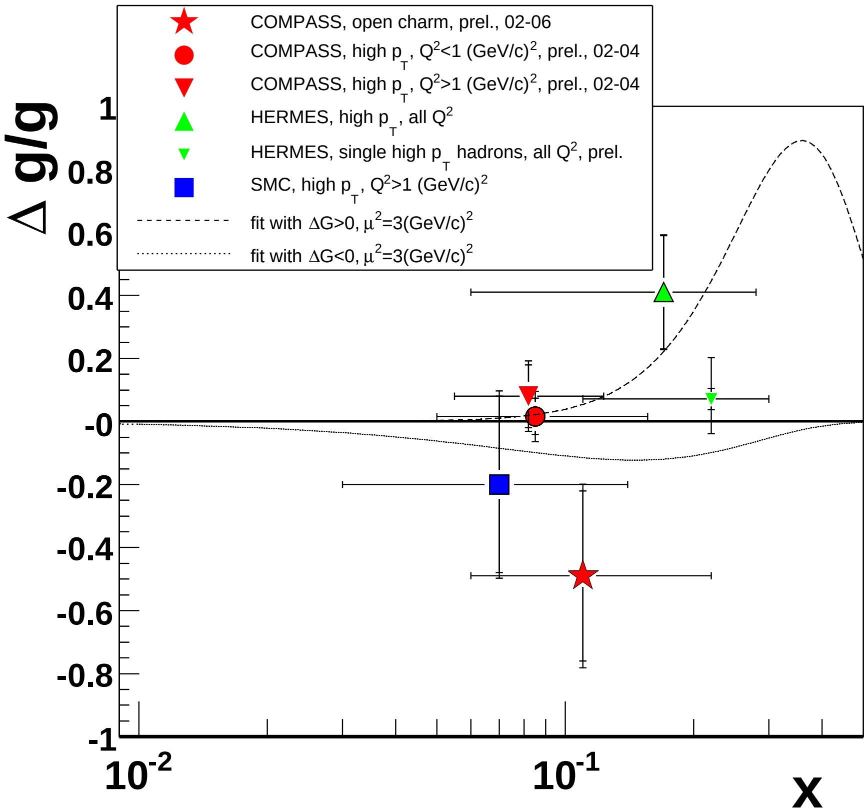
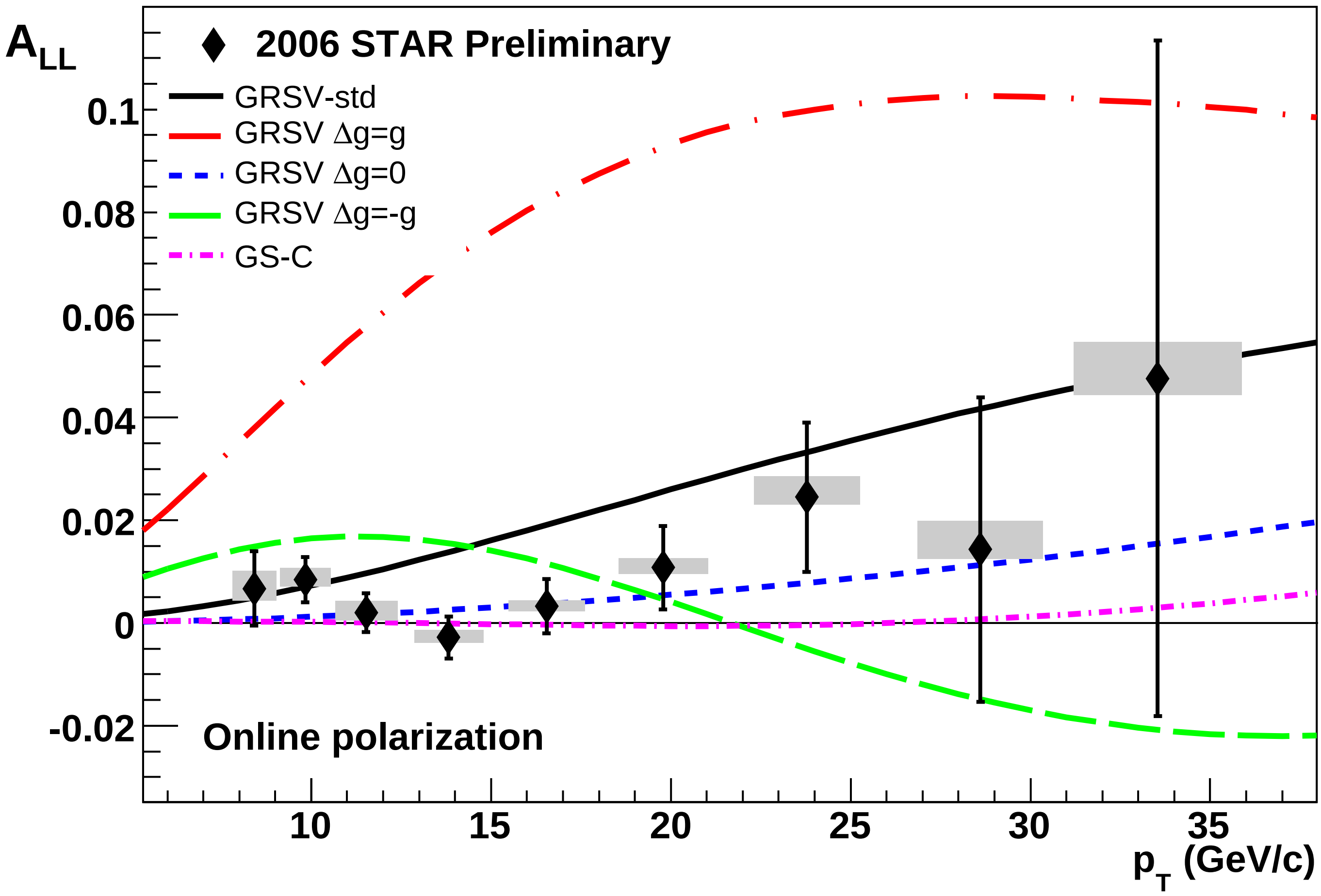


Compatible with very small $\Delta G(x)$ even *zero*.

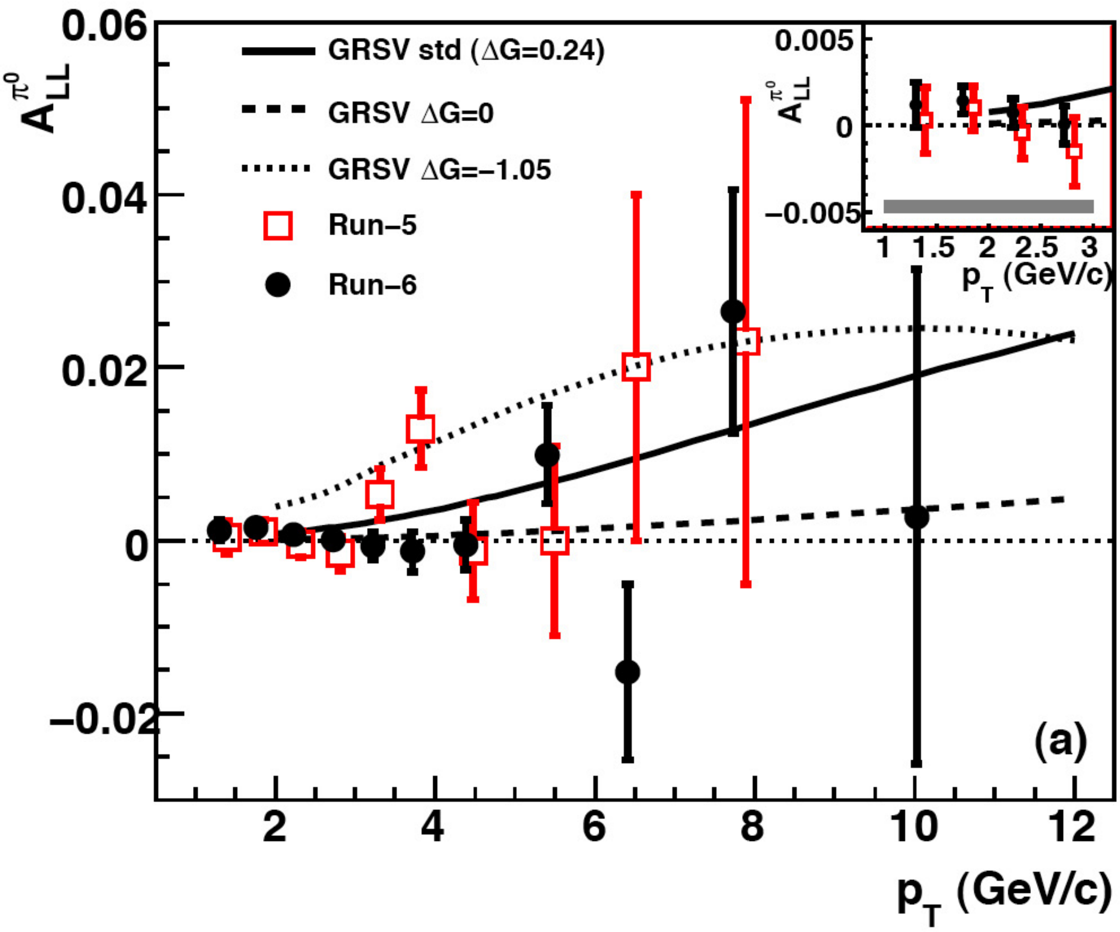
iii) *Asymmetry* A_{LL} *in semi-inclusive polarized* pp *reactions at RHIC.*

$$\overrightarrow{p} + \overrightarrow{p} \rightarrow (hadrons, jets, gammas) + X$$

Preliminary results from STAR and PHENIX



A_{LL}
2006 STAR Preliminary
GRSV-std
GRSV Δg=g
GRSV Δg=0
GRSV Δg=-g
GS-C
0.1
0.08
0.06
0.04
0.02
0
-0.02
Online polarization
10
15
20
25
30
35
p_T (GeV/c)



PHENIX

Again compatible with $\Delta G(x) = 0$!!!!!!!

Outlook on the "spin crisis"

Large polarized gluon density to resolve "spin crisis" no longer tenable.

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From known magnitude of $\boldsymbol{k}_T \Rightarrow$ enough L_z to satisfy the longitudinal angular momentum sum rule.

Against this explanation is the intuitive, but *probably incorrect*, argument that in quark models of hadrons the nucleon appears as an s-wave ground state i.e. with zero orbital angular momentum.

Other possibilities. e.g. *Modified Cloudy Bag Model* (Thomas et al):

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nucleon made up of valence quarks and a pion cloud

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In this approach there is no dramatic "spin crisis".

The future

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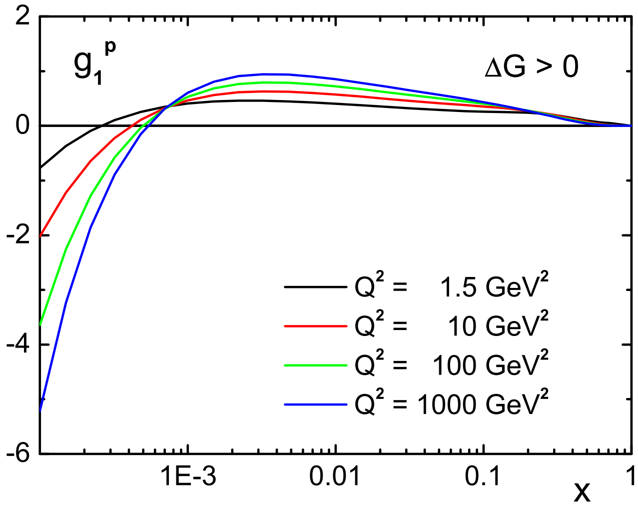
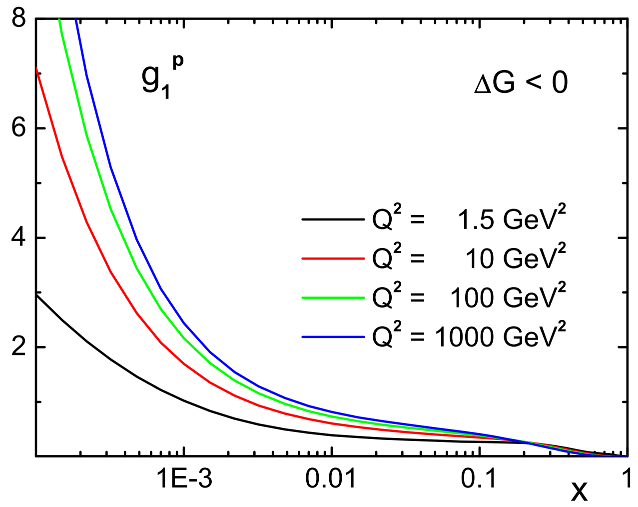
Major improvement in knowledge of $\Delta G(x)$....
determine sign



g1p
ΔG > 0
Q² = 1.5 GeV²
Q² = 10 GeV²
Q² = 100 GeV²
Q² = 1000 GeV²
1E-3
0.01
0.1
1
x



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Gauge invariance depends on polarization state

Let us for the moment take $p^2 = m^2$ and $p'^2 = m'^2$. One finds that

$$w_{\mu\nu}^{(A)} = 2\,\varepsilon_{\mu\nu\alpha\beta}\,m' s^\alpha \Big[(1 - \frac{m}{m'})\,p^\beta - \frac{m}{m'}\,q^\beta\Big]$$

Note that because of the term in round brackets the result is not gauge invariant i.e. $q^\mu\,w_{\mu\nu} \neq 0$ *unless* $m' = m$. But for *longitudinal* polarization, $s^\alpha = s_L^\alpha$, we have

$$m'\,s_L^\alpha \to \pm p^\alpha \qquad \text{for} \qquad \frac{m'}{p} \ll 1$$

and therefore the non-gauge invariant term vanishes because of the antisymmetry of the ε symbol.

OPE

The hadronic tensor $W^{\mu\nu}$ is given by the Fourier transform of the nucleon matrix elements of the commutator of electromagnetic currents $J_\mu(x)$:

$$W_{\mu\nu}(q;P,S) = \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle P,S|[J_\mu(x),J_\nu(0)]|P,S\rangle$$

where S^μ is the covariant spin vector specifying the nucleon state of momentum P^μ.

In hard processes, $x^2 \simeq 0$ is important, so we can use the Wilson expansion.

The OPE gives moments of $g_{1,2}$ in terms of hadronic matrix elements of certain operators multiplied by perturbatively calculable coefficient functions. The a_i are hadronic matrix elements of the octet of quark $SU(3)_F$ axial-vector currents $J^j_{5\mu}$ $(j = 1, ..., 8)$ and the flavour singlet axial current $J^0_{5\mu}$.

The octet currents are

$$J^j_{5\mu} = \bar{\psi}\gamma_\mu\gamma_5\left(\frac{\lambda_j}{2}\right)\psi \qquad (j = 1, 2, ..., 8)$$

where the λ_j are the usual Gell-Mann matrices and ψ is a column vector in flavour space

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \\ \psi_s \end{pmatrix},$$

and the flavour singlet current is

$$J^0_{5\mu} = \bar{\boldsymbol{\psi}}\gamma_\mu\gamma_5\boldsymbol{\psi}\,.$$

The forward matrix elements of the $J^j_{5\mu}$ can only be proportional to S^μ, and the a_j are defined by

$$\begin{aligned} \langle P, S|J^j_{5\mu}|P, S\rangle &= Ma_jS_\mu \\ \langle P, S|J^0_{5\mu}|P, S\rangle &= 2Ma_0S_\mu\,. \end{aligned}$$

Analogously one introduces an octet of vector currents

$$J_\mu^j = \bar{\psi}\gamma_\mu\left(\frac{\lambda_\mathbf{j}}{2}\right)\psi \qquad (\mathbf{j = 1, ..., 8})$$

which are *conserved currents* to the extent that $SU(3)_F$ is a symmetry of the strong interactions.

Problems with dimensional renormalization in spin case

It turns out to be crucial in handling these divergences to use the technique of *dimensional regularization*, which is straightforward in the unpolarized case, but which runs into a snag in the polarized case. The problem is that the generalization of γ_5 in more than 4-dimensions is ambiguous. In 4-dimensions we have

$$\{\gamma^{\mu}, \gamma_5\} = 0 \qquad \mu = 0, 1, 2, 3$$

If we try

$$\{\gamma^n\,,\,\gamma_5\} = 0 \qquad n = 4, 5, \ldots$$

it leads to a contradiction when using

$$Tr[ABC---X] = Tr[XABC---]$$

There is also a problem with the generalization of $\varepsilon^{\mu\nu\rho\sigma}$. 't Hooft and Veltman and Breitenlohner and Maison suggested using

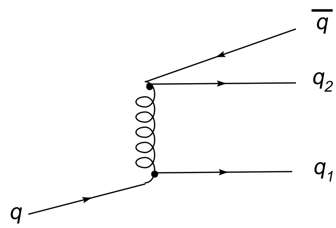
$$\{\gamma^{\mu}, \gamma_5\} = 0 \qquad \mu = 0, 1, 2, 3$$

$$[\gamma^{n}, \gamma_5] = 0 \qquad n = 4, 5, \ldots$$

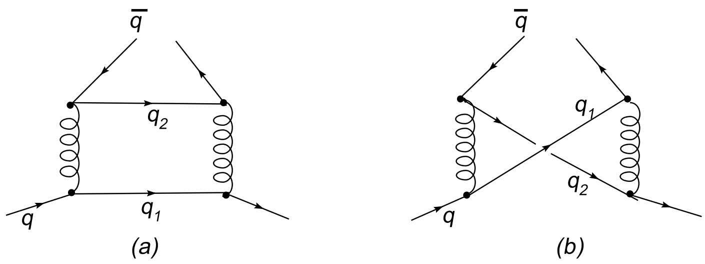
This gives rise to the $\overline{MS} - HVBM$ *renormalization scheme*, which, however, has a problem. The third component of the isovector axial current $J^3_{\mu 5}$ is NOT conserved, implying that a_3 depends on Q^2. It turns out that this feature is linked to how the factorization between hard and soft parts is implemented and can be remedied.

Non-singlets in NLO

Note that in LO flavor combinations like $q_f - q_{f'}$ (e.g., $u(x) - d(x)$) and valence combinations like $q_f - \bar{q}_f$ (e.g., $u(x) - \bar{u}(x)$) are *non-singlet* and evolve in the same way, without the ΔG term. (There is no splitting in LO from a q to a $\bar{q}$, nor from, say, a u to a d.) However, in NLO flavor non-singlets like $u(x) - d(x)$ and charge-conjugation non-singlets like $u(x) - \bar{u}(x)$ evolve differently. The origin of this difference can be seen in Figs.



Take modulus squared of this. Obtain two possibilities



Shows two possible contributions to $\Delta P_{q\bar{q}}$ from taking the modulus squared of this amplitude. In (a) the contribution is pure flavor singlet and involves only gluon exchange, whereas in (b) the contribution is non-singlet. However, if we try to do something similar for a flavor changing splitting function e.g. ΔP_{du} we find that we cannot construct the non-singlet diagram.

Bound on positivity of strange quark density

Consider the following constraint on the first moment

$$\delta_s \equiv [\Delta s + \Delta \bar{s}]$$

We can rewrite the expression for Γ_1^p as

$$\Gamma_1^p(Q^2) = \frac{1}{6}\Big[\frac{1}{2}a_3 + \frac{5}{6}a_8 + 2\delta_s(Q^2)\Big]$$

or

$$a_8 = \frac{6}{5}\Big[6\Gamma_1^p(Q^2) - \frac{1}{2}a_3 - 2\delta_s(Q^2)\Big]$$

We know a_3 very accurately. Using the measured values of $\Gamma_1^p(Q^2)$ we show that $\delta_s(Q^2) \geq 0$ implies an unacceptable value for a_8.

We have to decide what value to use for $\Gamma_1^p(Q^2)$, since the result depends on the extrapolation to $x = 0$. We take two extremes:

(i) Assume perturbative QCD holds at small x as done by SLAC experiment E155 etc. This yields

$$\Gamma_1^p(Q^2 = 5) = 0.118 \pm 0.004 \pm 0.007$$

(ii) Assume Regge behaviour at small x as utilized by SLAC experiment E143 etc. This gives

$$\Gamma_1^p(Q^2 = 3) = 0.133 \pm 0.003 \pm 0.009$$

Results: If δ_s is *positive* we find:

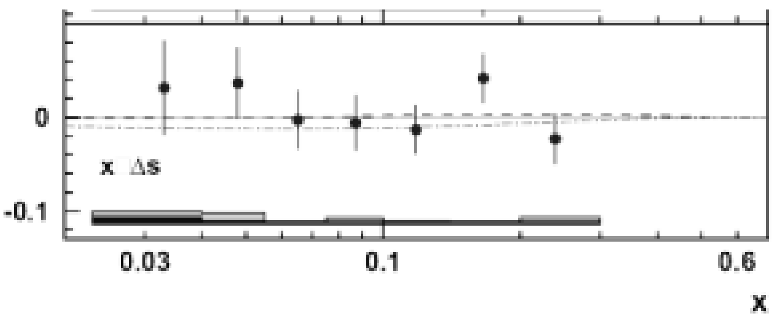
(i) $a_8 \leq 0.089 \pm 0.058$

(ii) $a_8 \leq 0.197 \pm 0.068$

Now to the best of our knowledge hyperon β-decay is adequately described by $SU(3)_F$ and this leads to $a_8 = 0.585 \pm 0.025$

Thus $\delta_s(Q^2) \geq 0$ implies a dramatic breaking of $SU(3)_F$, and we conclude that it is *almost* impossible to have $\delta_s(Q^2) \geq 0$.

Now HERMES has extracted $\Delta s(x) + \Delta \bar{s}(x)$ from a study of SIDIS . The results are shown below.



Within errors the results are consistent with zero, and HERMES quote

$$\delta_s(Q^2 = 2.5) = 0.028 \pm 0.033 \pm 0.009$$

The previous discussion suggests that the central value *cannot* be the true value unless we have totally failed to understand the connection between DIS and SIDIS . If the latter is not the case, how can we understand the HERMES results?

I think it is important to remember that HERMES uses a LO method based on so-called *purities*. I suspect that such an approach is unreliable at the values of Q^2 involved, and that the errors on the purities are somewhat underestimated in their analysis. So I strongly believe that this new ‘strange quark crisis’ will prove to be illusory.