Transverse Momentum Dependent (TMD) Parton Distribution Functions in a Spectator Diquark Model

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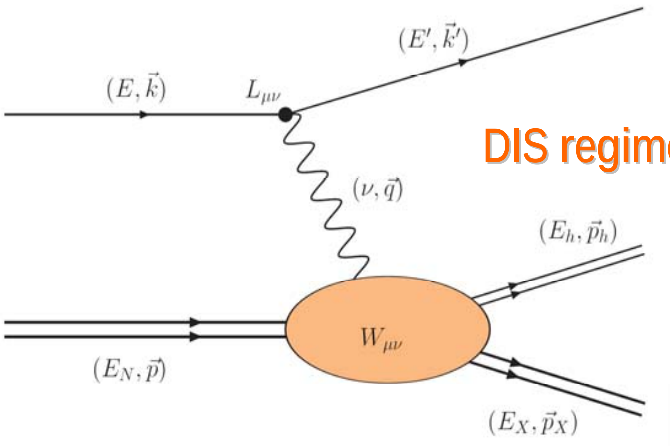
Nucleon Spin Structure

usefulness of an expansion in powers of 1/Q, besides that in powers of α_s (pQCD): TWIST

Deep Inelastic Scattering:

$$ep \longrightarrow e'X$$



DIS regime:

$$Q^2 = -(\nu, \mathbf{q})^2 \gg M^2$$
$$\nu = E - E' \gg M$$
$$x_B = Q^2/2M\nu \qquad \text{fixed}$$

Leptonic tensor: known at any order in pQED

$$\sigma \sim L_{\mu\nu} W^{\mu\nu}$$

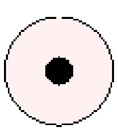
Hadronic tensor: hadron internal dynamics (low energy ⇔ non-pert. QCD), in terms of *structure functions*, with SCALING properties (Q-INdependence)

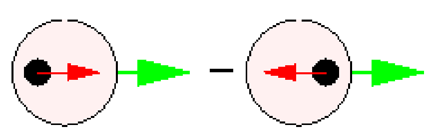
PARTON MODEL:

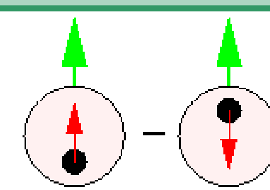
Asymptotic Freedom / Confinement

- incoherent sum of interactions on almost free (on shell) pointlike partons
- hard/soft factorization theorems: convolution between *hard* elementary cross sections and *soft* (non-pert.) and universal parton distribution functions ⇔ PDF

Parton distributions = Probability densities of finding a parton with x momentum fraction in the target hadron (NO intrinsic transverse momentum ⇔ Collinear factorization)

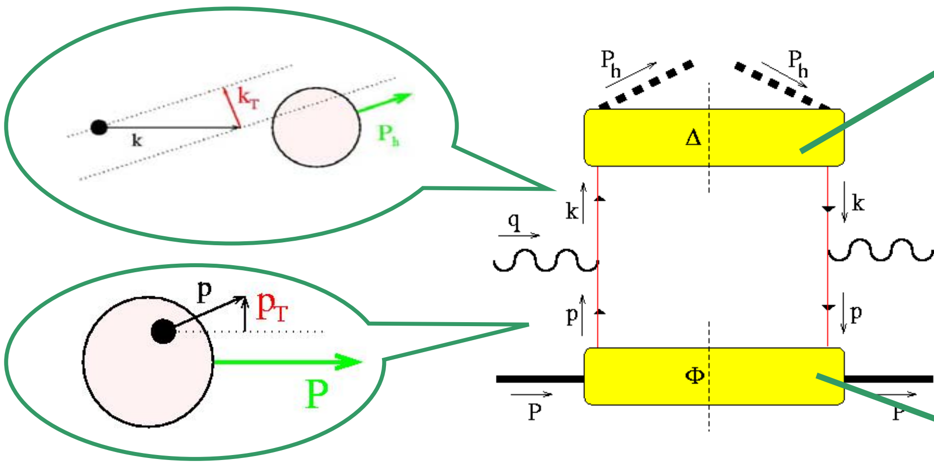
$f_1 =$ 

$g_1 =$ 

$h_1 =$ 

Nucleon Spin Structure & TMD parton densities

Semi Inclusive Deep Inelastic Scattering: $e\,p^{\uparrow} \to e'\,\pi^{\pm}\,X$



Fragmentation Correlator → FFs

The 3 momenta {P,q,P_h} CANNOT be all collinear ; in T-frame, keeping the cross section differential in dq_T: sensitivity to the parton transverse momenta in the hard vertex ⇔ TMD parton densities !

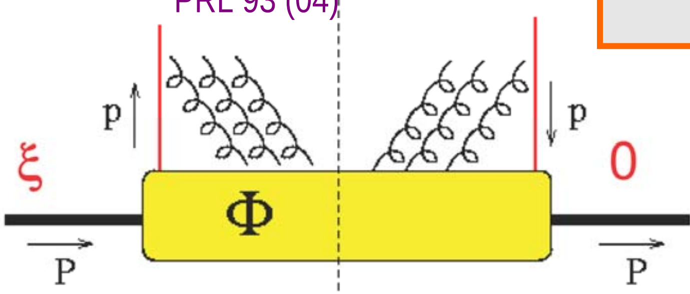
Quark-Quark Correlator → PDFs

Hadronic tensor in the Parton Model (tree level, leading twist):

$$2MW^{\mu\nu}(q,P,S,P_h) = \int d^2\mathbf{p}_T\, d^2\mathbf{k}_T\, \delta^{(2)}(\mathbf{p}_T + \mathbf{q}_T - \mathbf{k}_T)\, \mathrm{Tr}\left[\Phi(x,\mathbf{p}_T,S)\,\gamma^\mu\,\Delta(z_h,\mathbf{k}_T)\,\gamma^\nu\right]$$

$$\Phi(x,\mathbf{p}_T,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ip\cdot\xi}\,\langle P,S\,|\,\bar{\psi}(0)\,\mathcal{U}_{[0,\xi]}\,\psi(\xi)\,|\,P,S\,\rangle$$

TMD hard/soft factorization: Ji, Ma, Yuan, PRD 71 (04); Collins, Metz, PRL 93 (04)



$$p \approx (0, xP^+, \mathbf{p}_T) \Rightarrow \xi = (\xi^-, 0, \boldsymbol{\xi}_T)$$

Diagonal matrix elements of bilocal operators, built with quark fields, on hadronic states

Nucleon Spin Structure & TMD parton densities (2)

Projecting over various Dirac structures, all leading twist TMD parton distribution functions can be extracted, with probabilistic interpretation

$$\Phi^{[\Gamma]}(x, \mathbf{p}_T, S) = \mathrm{Tr}\left[\Phi(x, \mathbf{p}_T, S)\,\Gamma\right]$$

TMD	T-even U	T-even L	T-even T	T-odd U	T-odd L	T-odd T
u	f_1					$f_{1T}^{\perp}$
l		g_{1L}	g_{1T}			
t		$h_{1L}^{\perp}$	h_{1T}, $h_{1T}^{\perp}$	$h_1^{\perp}$		

chiral-odd

h_1

Known x-parametrization, poorly known p_T one (gaussian and with no flavour dependence; other possible functional forms! Connection with orbital L!)

Hautmann, arXiv:0805.1049 [hep-ph]

"Hadronic final states containing multiple jet events…. will play a central role in the LHC physics program…. Owing to the complex kinematics involving multiple hard scales and the large phase space opening up at very high energies, multi-jet events are potentially sensitive to QCD initial-state radiation that depend on the finite transverse- momentum tail of partonic matrix elements and distributions…"

It is of great importance to devise models showing the ability to predict a non-trivial p_T-dependence for TMD densities!

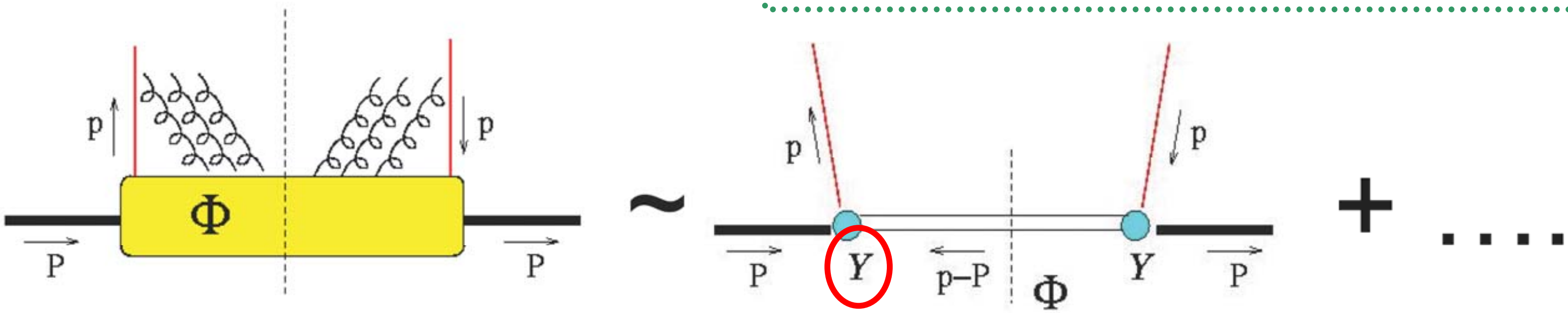
The Spectator Diquark model

The Φ correlator involves matrix elements on bound hadronic states, whose partonic content is neither known nor computable in pQCD (low energy region!) ➔ model calculations required!

SPECTATOR DIQUARK model:

(Jakob, Mulders, Rodrigues, A626 (97) 937,
Bacchetta, Schaefer, Yang, P.L. B578 (04) 109)

- Replace the sum over intermediate states in Φ with a single state of definite mass (on shell) and coloured.
- Its quantum numbers are determined by the action of the quark fields on $|P, S\rangle$, so are those of a diquark!



$$\Phi(x, \mathbf{p}_T, S) = \int \frac{d^4\xi}{(2\pi)^3} e^{ip\cdot\xi} \langle P, S|\bar{\psi}(0)\, U_{[0,\xi]}\, \psi(\xi)|P, S\rangle \approx \frac{1}{(2\pi)^3} \frac{1}{2(1-x)P^+} \overline{\mathcal{M}}^{(0)}(S)\, \mathcal{M}^{(0)}(S) + \ldots$$

$$\mathcal{M}^{(0)}(S) = \langle p - P|\psi(0)|P, S\rangle$$

$$(p-P)^2 = M_D^2 \longrightarrow p^2 = \tau(x, \mathbf{p}_T^2) = -\frac{\mathbf{p}_T^2 + L_D^2(m^2)}{1-x} + m^2$$

$$L_D^2(m^2) = xM_D^2 + (1-x)m^2 - x(1-x)M^2$$

Simple, Covariant model: analytic results, mainly 3 parameters.

The Spectator Diquark model (2)

Nucleon (N)-quark (q)-diquark (Dq) vertex:

Dq Spin = 0 : flavour-singlet [~{ud-du}] $\mathcal{Y}_s = i\, g_s(p^2)\, \mathbf{1}$

Dq Spin = 1 : flavour-triplet [~{dd,ud+du,uu}] $\mathcal{Y}_a^\mu = i \frac{g_a(p^2)}{\sqrt{2}} \gamma^\mu \gamma_5$

⟺ Need of Axial-Vector diquarks in order to describe d in N!

N-q-Dq vertex form factors (non-pointlike nature of N and Dq):

- Pointlike: $g_{s/a}(p^2) = N_{s/a}$
- Dipolar: $g_{s/a}(p^2) = N_{s/a} \frac{p^2 - m^2}{|p^2 - \Lambda^2|^\alpha}$
- Exponential: $g_{s/a}(p^2) = N_{s/a}\, \mathrm{e}^{(p^2 - m^2)/m^2}$

• kill pole $(p^2-m^2)^{-1}$ and log divergences
• suppresses large $\mathbf{p}_T$
• ~ $(1-x)^3$ for $x \to 1$ (Drell-Yan-West)

Virtual S=1 Dq propagator (⟺ real Dq polarization sum): $d^{\mu\nu}(p-P) = \sum_{\lambda_a} \epsilon^{*\,\mu}_{\lambda_a}(p-P)\, \epsilon^{\nu}_{\lambda_a}(p-P)$

$$d^{\mu\nu}(p-P) = \begin{cases} -g^{\mu\nu}\,, \\ -g^{\mu\nu} + \frac{(p-P)^\mu (p-P)^\nu}{M_a^2}\,, \\ -g^{\mu\nu} + \frac{(p-P)^\mu n_-^\nu + (p-P)^\nu n_-^\mu}{(p-P)\cdot n_-} - \frac{M_a^2}{[(p-P)\cdot n_-]^2} n_-^\mu n_-^\nu \end{cases}$$

'Feynman': $\lambda_a = \pm, 0, t$
Bacchetta, Schaefer, Yang, P.L.B578 (04) 109

'Covariant': $\lambda_a = \pm, 0$
Gamberg, Goldstein, Schlegel, arXiv:0708.0324 [hep-ph]

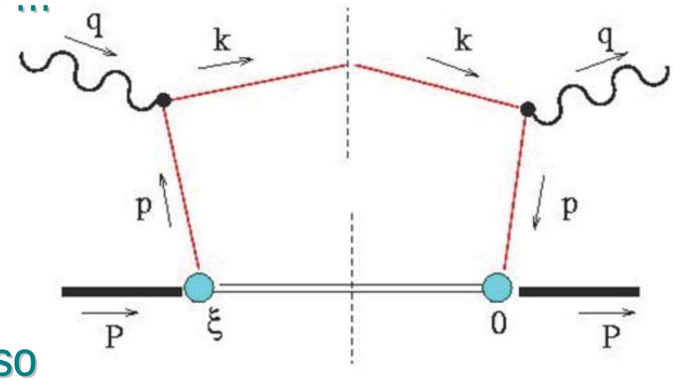
'Light-Cone': $\lambda_a = \pm$
Brodsky, Hwang, Ma, Schmidt, N.P.B593 (01) 311

The Spectator Diquark model (3)

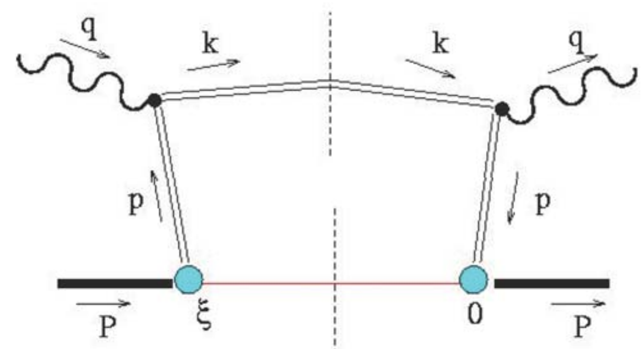
Why should we privilege 'Light-Cone' (LC) gauge?

In DIS process, the exchanged virtual photon can in in principle probe not only the quark, but also the diquark, this latter being a charged boson

Not only ...



... but also



S=0 diquark contributes to F_L only:

~~Callan-Gross~~ $\frac{F_L}{F_T} \overset{Q^2\to\infty}{\not\to} 0$ but

$$2\,F_1(x) = \sum_q e_q^2\,[f_1^q(x) + f_1^{\bar{q}}(x)]$$

Adopting LC gauge, the same holds true for S=1 diquark also, while other gauges give contributions to F_T as well!

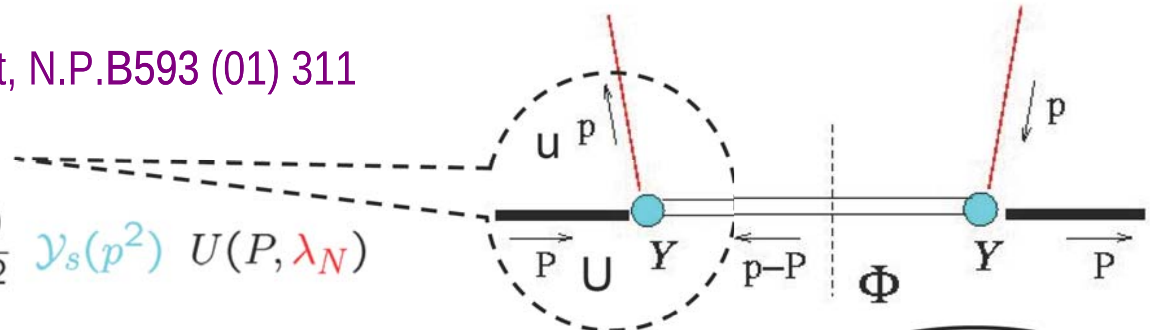
In our model:

- ✓ Systematic calculation of ALL leading twist T-even and T-odd TMD functions (hence of related PDF also)
- ✓ Several functional forms for N-q-Dq vertex form factors and S=1 Dq propagator
- ✓ Moreover, Overlap Representation of all TMD functions in terms of LCWFs!

Overlap representation for T-even TMD

The light-cone Fock wave-functions (LCWF) are the frame independent interpolating functions between hadron and quark/gluon degrees of freedom

following Brodksy, Hwang, Ma, Schmidt, N.P.B593 (01) 311



spin=0 Dq

$$\psi^{\lambda_N}_{\lambda_q}(x,\mathbf{p}_T) = \sqrt{\frac{p^+}{(P-p)^+}}\,\frac{\bar{u}(p,\lambda_q)}{p^2-m^2}\,\mathcal{Y}_s(p^2)\;U(P,\lambda_N)$$

$$\psi^+_\pm \qquad \psi^-_+ = -[\psi^+_-]^*,\;\psi^-_- = \psi^+_+$$

spin=1 Dq

$$\psi^{\lambda_N}_{\lambda_q,\lambda_a}(x,\mathbf{p}_T) = \sqrt{\frac{p^+}{(P-p)^+}}\,\frac{\bar{u}(p,\lambda_q)}{p^2-m^2}\,\epsilon^*_\mu(p-P,\lambda_a)\;\mathcal{Y}^\mu_a(p^2)\;U(P,\lambda_N)$$

$L_{q\text{-}Dq} = 1, 0$: $\psi^+_{+,\pm},\ \psi^+_{-,+}$

$L_{q\text{-}Dq} = 2$: $\psi^+_{-,-} = \psi^-_{+,+} = 0$

$$\psi^-_{+,-} = -\psi^+_{-,+}$$
$$\psi^-_{-,+} = [\psi^+_{+,-}]^*$$
$$\psi^-_{-,-} = [\psi^+_{+,+}]^*$$

Angular momentum conservation: $\lambda_q\;(+\lambda_a) + L_{z,\{qD\}} = \lambda_N$

E.g. :

$$\psi^+_+(x,\mathbf{p}_T) = (m + xM)\,\phi_s/x \qquad L=0$$
$$\psi^+_-(x,\mathbf{p}_T) = -(p_x + ip_y)\,\phi_s/x \qquad L=1$$

⬄ L=1 component relativistically enhanced w.r.t. L=0 one! ⬄ Spin Crisis as a relativistic effect ?!

➡ Non-zero relative orbital angular momentum between q and Dq:
the g.s. of q in N is NOT $J^P=1/2^+$; NO SU(4) spin-isospin symmetry for N wave-function!

Overlap representation for T-even TMD

Besides the Feynman diagram approach, Time-Even TMD densities can be also calculated in terms of overlaps of our spectator diquark model LCWFs

For the Unpolarized TMD parton distribution function, e.g. (using LC gauge for axial vector diquark):

$$f_1^s(x,\mathbf{p}_T^2) = \frac{1}{4}\frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\,\mathrm{Tr}\left[\left(\bar{\mathcal{M}}_s^{(0)}(S)\mathcal{M}_s^{(0)}(S)+\bar{\mathcal{M}}_s^{(0)}(-S)\mathcal{M}_s^{(0)}(-S)\right)\gamma^+\right]+\mathrm{h.c.}$$
$$= \frac{N_s^2}{(2\pi)^3}\frac{[\mathbf{p}_T^2+(m+xM)^2](1-x)^3}{2\left(\mathbf{p}_T^2+L_s^2(\Lambda_s^2)\right)^4} = \frac{1}{16\pi^3}\frac{1}{2}\sum_{\lambda_N=\pm}\sum_{\lambda_q=\pm}\left|\psi_{\lambda_q}^{\lambda_N}\right|^2$$

NO x- pT factorization & NON-gaussian pT dependence !

$$f_1^a(x,\mathbf{p}_T^2) = \frac{1}{4}\frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\,\mathrm{Tr}\left[\left(\bar{\mathcal{M}}_a^{(0)}(S)\mathcal{M}_a^{(0)}(S)+\bar{\mathcal{M}}_a^{(0)}(-S)\mathcal{M}_a^{(0)}(-S)\right)\gamma^+\right]+\mathrm{h.c.}$$
$$= \frac{N_a^2}{(2\pi)^3}\frac{[\mathbf{p}_T^2(1+x^2)+(m+xM)^2(1-x)^2](1-x)}{2\left(\mathbf{p}_T^2+L_a^2(\Lambda_a^2)\right)^4} = \frac{1}{16\pi^3}\frac{1}{2}\sum_{\lambda_N=\pm}\sum_{\lambda_q=\pm}\sum_{\lambda_a=\pm}\left|\psi_{\lambda_q,\lambda_a}^{\lambda_N}\right|^2$$

Parameters Fixing

Jakob, Mulders, Rodrigues, N.P. A626 (97) 937

SU(4) for |p>:

$$f_1^u = \frac{3}{2} f_1^s + \frac{1}{2} f_1^a$$
$$f_1^d = f_1^a$$

(s: S=0, I=0;
a: S=1, I=0)

~~SU(4)~~ for |p>:

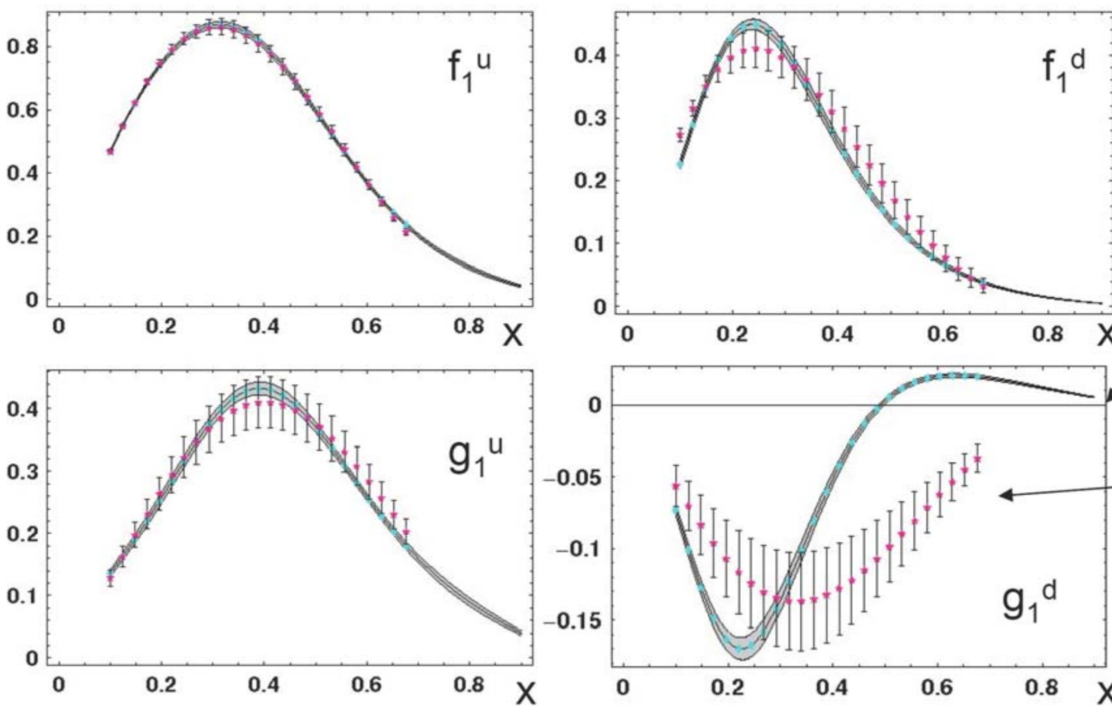
(a': S=1, I=1)

model parameters!

$$f_1^u = c_s^2 \, f_1^s + c_a^2 \, f_1^a$$
$$f_1^d = c_{a'}^2 \, f_1^{a'}$$

Parameters: m=M/3, $N_{s/a/a'}$ (fixed from $||f_1^{s/a/a'}||=1$), $M_{s/a/a'}$, $\Lambda_{s/a/a'}$, $c_{s/a/a'}$ (from a joint fit to data on u & d unpolarized and polarized PDF: ZEUS for f_1 @ Q^2=0.3 GeV^2, GRSV01 at LO for g_1 @ Q^2=0.26 GeV^2)

Hadronic scale of the model: $Q_0^2 \sim 0.3$ GeV^2



χ^2 / d.o.f. = 3.88

Chekanov *et al.* (ZEUS), P.R.D**67** (03) 012007

our fit; error band from MINUIT covariant error matrix

phenomenological parametrizations

GRSV01 at LO

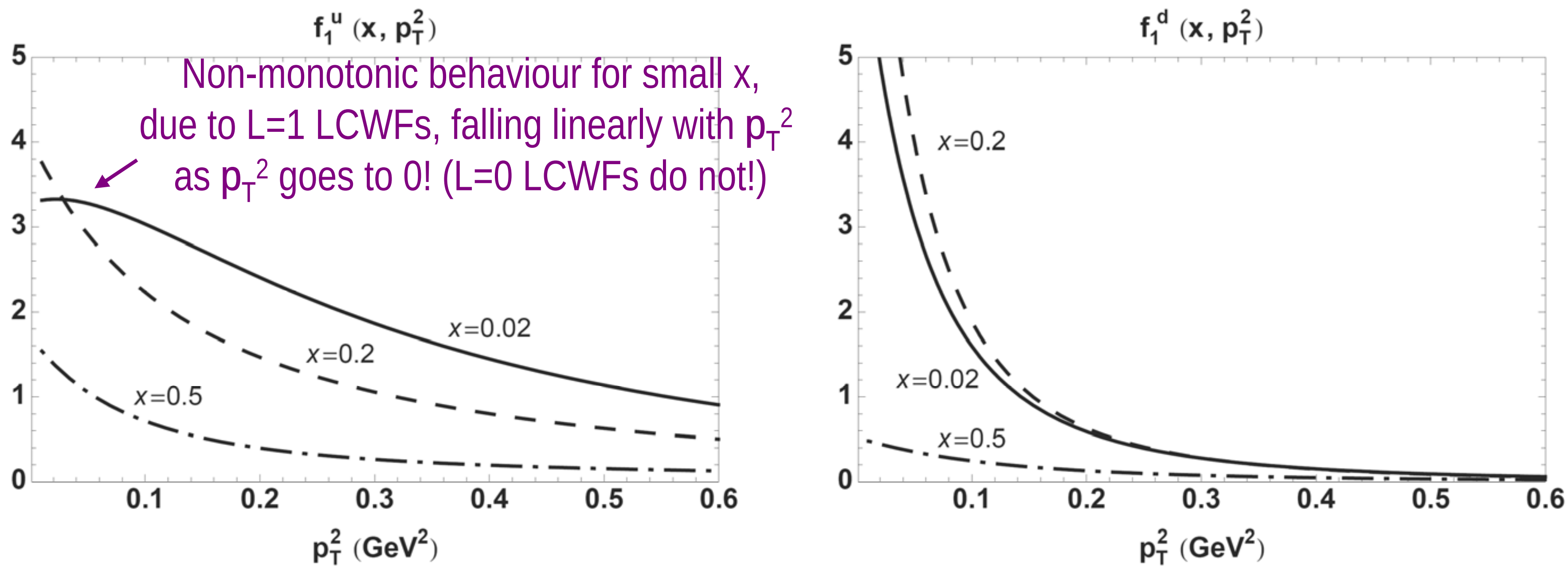
$$P_q \equiv \int_0^1 dx\, x\, (f_1^u(x) + f_1^d(x)) = 0.584 \pm 0.010$$
$$\text{ZEUS} = 0.55$$

$$g_A \equiv \int_0^1 dx\, (g_1^u(x) - g_1^d(x)) = 0.966 \pm 0.038$$
$$\text{GRSV01} = 0.969 \pm 0.096$$

p_T- model dependence

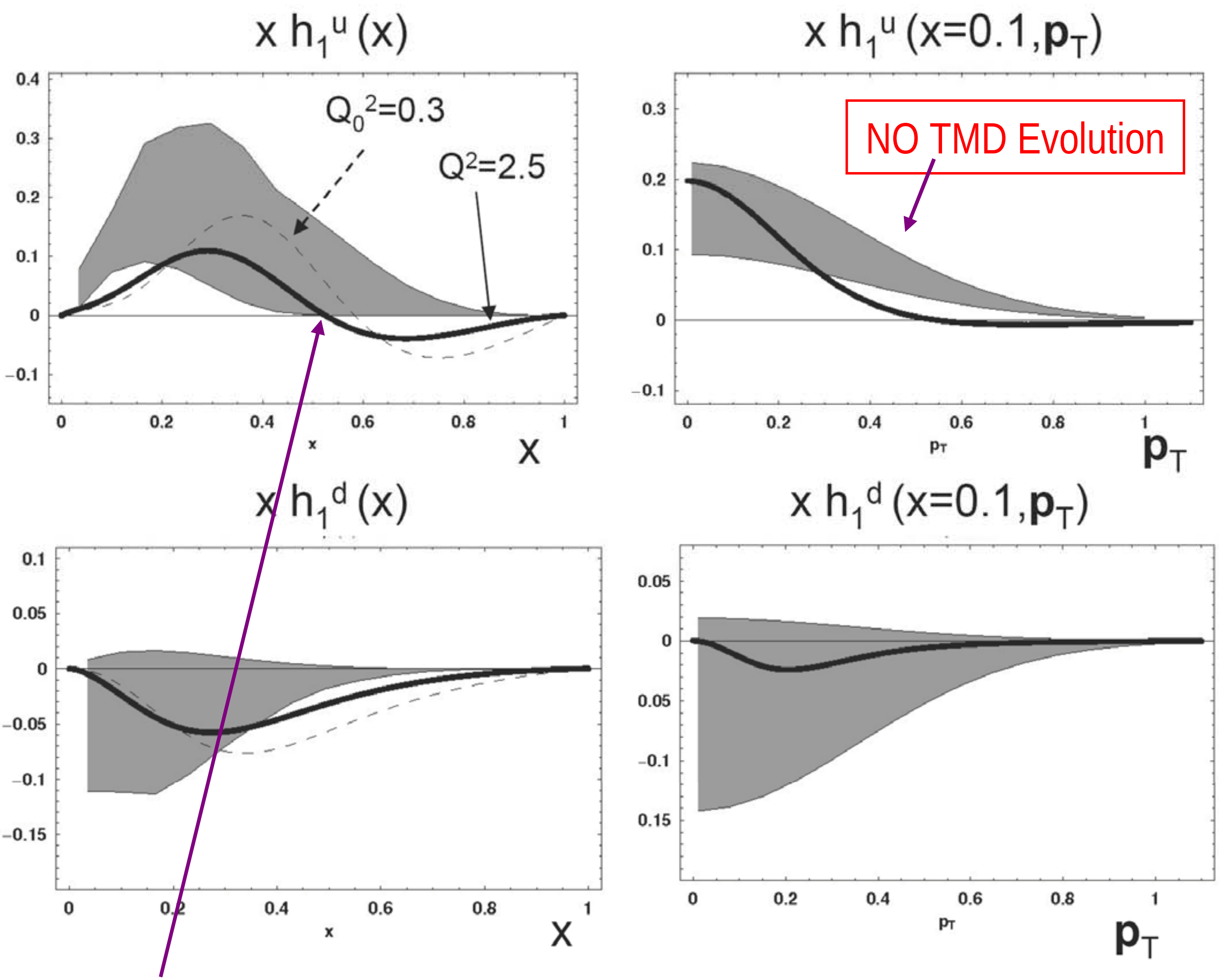
General behaviour: in our diquark model the average quark transverse momentum decreases as x increases, and down quarks on average carry less transverse momentum than up quarks

→ x- and flavour-dependence !

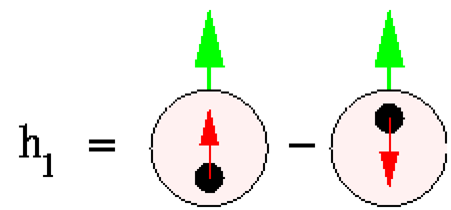


The study of p_T-dependence shed light on the spin/orbital angular momentum structure of the Nucleon!

Transversity



DGLAP Evolution @ LO using code from Hirai, Kumano, Miyama, C.P.C.111 (98) 150



Transverse Spin distribution

Change of sign at x=0.5, due to the negative S=1 Dq contributions, which become dominant at high x

Parametrization: p_T- dependence ~ exp[- p_T^2 / $\langle p_T^2 \rangle$]

Anselmino *et al.* x- dependence ~ $x^\alpha(1-x)^\beta$ ...

P.R.D75 (07) 054032 **no change of sign allowed!**

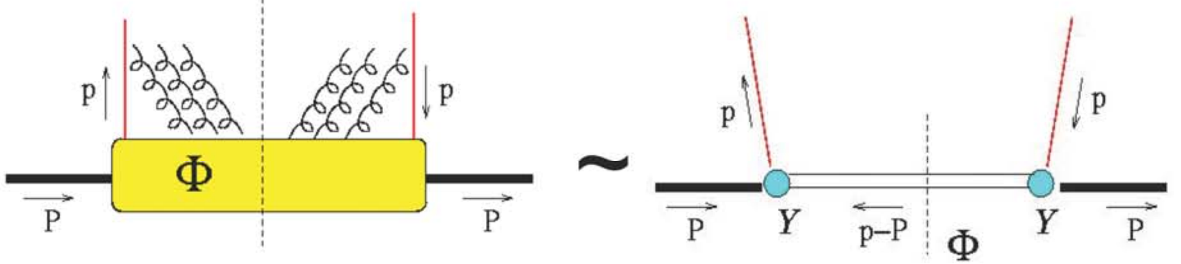
Time-Odd TMD distributions

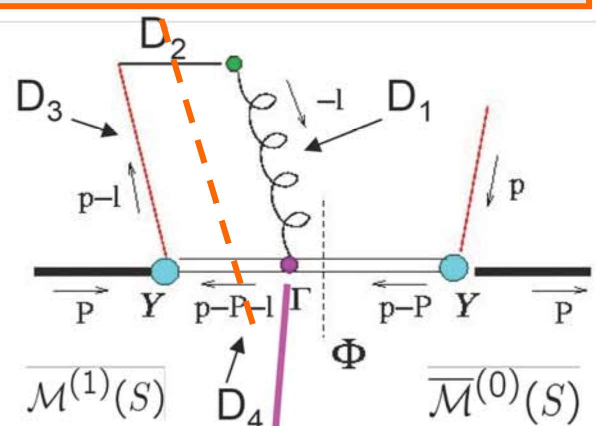
T-odd distributions: crucial to explain the evidences of SSA! Their existence is bound to the Gauge Link operator (⇔ QCD gauge invariance), producing the necessary non-trivial T-odd phases!

$$\Phi(x,\mathbf{p}_T,S) = \int \frac{d^4\xi}{(2\pi)^3}\, e^{ip\cdot\xi}\, \langle P,S|\bar{\psi}(0)\, U_{[0,\xi]}\, \psi(\xi)|P,S\rangle \;\approx\; \frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\,\overline{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(0)}(S) +$$

1 gluon-loop contribution: first order approximation of the Gauge link! ⇔

$$\frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\left[\overline{\mathcal{M}}^{(0)}(S)\,\mathcal{M}^{(1)}(S) + \overline{\mathcal{M}}^{(1)}(S)\,\mathcal{M}^{(0)}(S)\right]$$





$$\mathcal{M}^{(1)}(S) = \int \frac{d^4 l}{(2\pi)^4}\, \frac{ie\, n^\rho_-\,(\not p - \not l + m)}{D_1 D_2 D_3 D_4} \begin{cases} \Gamma_{s\,\rho}\; \Upsilon_s\, U(P,S) \\ \epsilon^*_\sigma(p-P,\lambda_a)\; \Gamma^{\nu\sigma}_{a\,\rho}\; d_{\mu\nu}(p-l-P)\; \Upsilon^\mu_a\, U(P,S) \end{cases}$$

$$\Gamma_{s\,\rho} = ie\,(2P-2p+l)_\rho$$

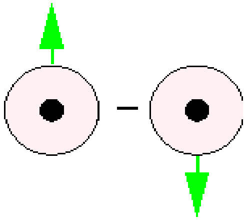
$$\Gamma^{\nu\sigma}_{a\,\rho} = -ie\left[(2P-2p+l)_\rho\, g^{\nu\sigma} - (P-p+(1+v)\,l)^\sigma\, g^\nu_\rho - (P-p-vl)^\nu\, g^\sigma_\rho\right]$$

v: an. mag. mom. of S=1 Dq.
v=1 ⇔ γWW vertex!

Imaginary part: Cutkoski cutting rules! Put on-shell D2 and D4. Analytic results!

Time-Odd TMD distributions (2)

Sivers function appears in the TMD distribution of an unpolarized quark, and describes the possibility for the latter to be distorted due to the parent Proton transverse polarization:



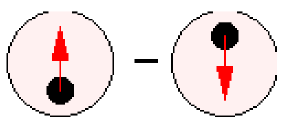
$$\frac{\varepsilon_T^{ij}\, p_{Ti}\, \hat{S}_{Tj}}{M}\, f_{1T}^{\perp\, s/a}(x, \mathbf{p}_T^2) = -\frac{1}{4}\frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\,\mathrm{Tr}\left[\left(\bar{\mathcal{M}}^{(1)}_{s/a}(S)\mathcal{M}^{(0)}_{s/a}(S) - \bar{\mathcal{M}}^{(1)}_{s/a}(-S)\mathcal{M}^{(0)}_{s/a}(-S)\right)\gamma^+\right] + \mathrm{h.c.}$$

$$f_{1T}^{\perp\, s}(x, \mathbf{p}_T^2) = -\frac{N_s^2}{(2\pi)^4}\frac{Me^2(m+xM)\,(1-x)^3}{4L_s^2(\Lambda_s^2)\left(\mathbf{p}_T^2 + L_s^2(\Lambda_s^2)\right)^3}$$

$$f_{1T}^{\perp\, a}(x, \mathbf{p}_T^2) = \frac{N_a^2}{(2\pi)^4}\frac{Me^2(m+xM)\,x\,(1-x)^2}{4L_a^2(\Lambda_a^2)\left(\mathbf{p}_T^2 + L_a^2(\Lambda_a^2)\right)^3}$$

Both provide crucial information on partons Orbital Angluar Momentum contributions to the Proton spin!

Boer-Mulders function describes the transverse spin distribution of a quark in an unpolarized Proton:



$$\frac{\varepsilon_T^{ij}\, p_{Tj}}{M}\, h_1^{\perp\, s/a}(x, \mathbf{p}_T^2) = \frac{1}{4}\frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\,\mathrm{Tr}\left[\left(\bar{\mathcal{M}}^{(1)}_{s/a}(S)\mathcal{M}^{(0)}_{s/a}(S) + \bar{\mathcal{M}}^{(1)}_{s/a}(-S)\mathcal{M}^{(0)}_{s/a}(-S)\right) i\sigma^{i+}\gamma_5\right] + \mathrm{h.c.}$$

$$h_1^{\perp\, s}(x, \mathbf{p}_T^2) = f_{1T}^{\perp\, s}(x, \mathbf{p}_T^2)$$

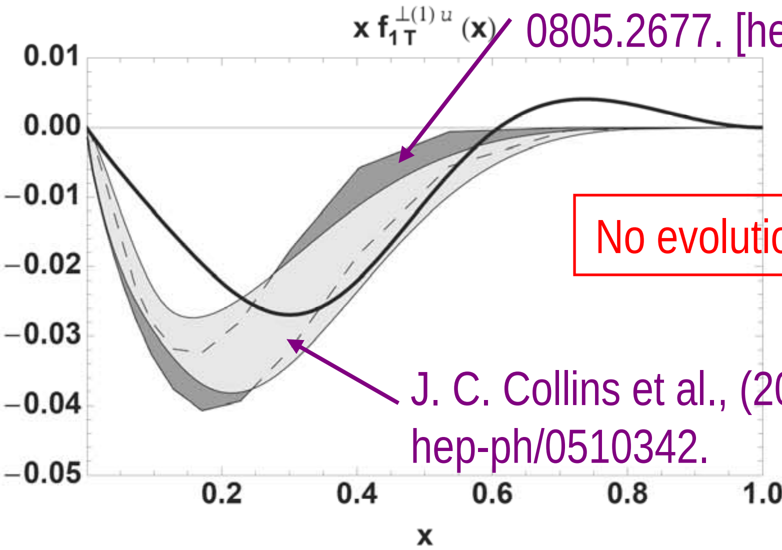
$$h_1^{\perp\, a}(x, \mathbf{p}_T^2) = -\frac{f_{1T}^{\perp\, a}(x, \mathbf{p}_T^2)}{x}$$

Sivers ⇔ Boer-Mulders:
identity for S=0 Dq, simple relation for S=1 Dq (but only using LC gauge!)

Sivers moments

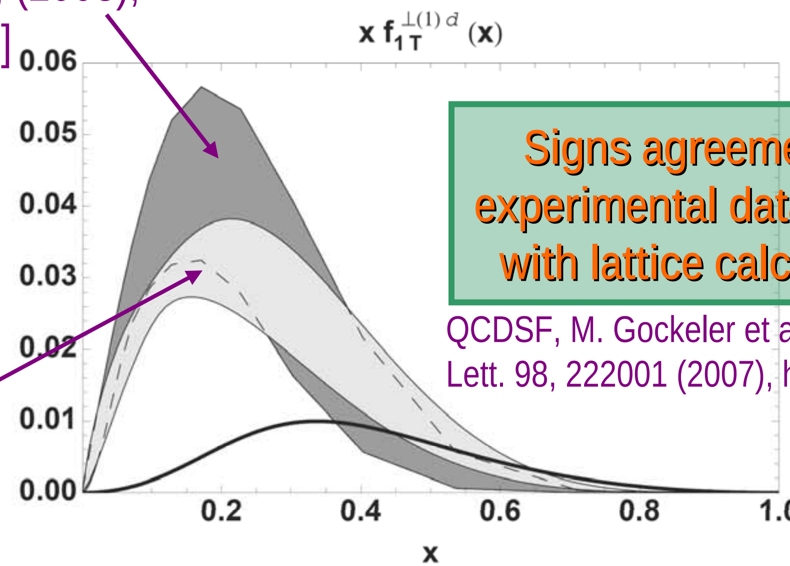
$$f_{1T}^{\perp(1)}(x) \equiv \int d^2\mathbf{p}_T \, \frac{\mathbf{p}_T^2}{2M^2} \, f_{1T}^{\perp}(x, \mathbf{p}_T^2)$$

M. Anselmino et al., (2008), 0805.2677. [hep-ph]



No evolution!

J. C. Collins et al., (2005), hep-ph/0510342.

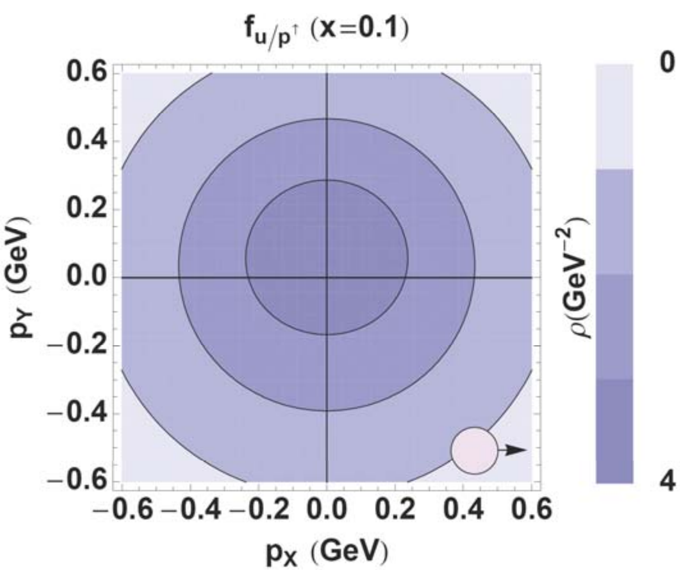


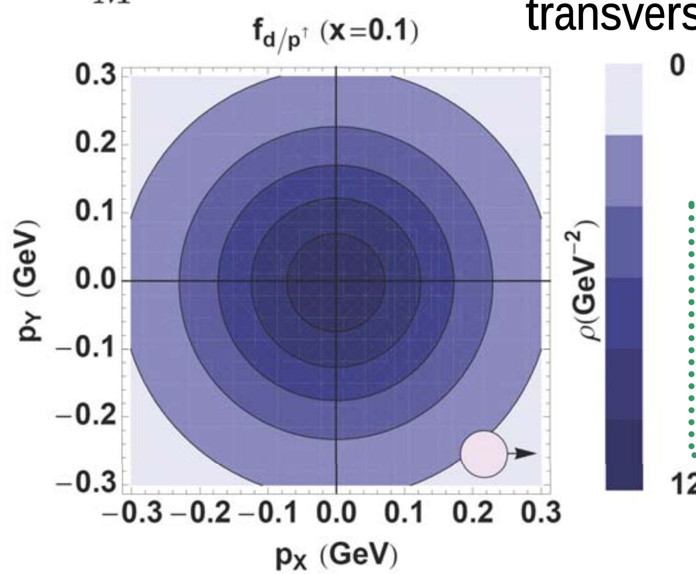
Signs agreement with experimental data and also with lattice calculations!

QCDSF, M. Gockeler et al., Phys. Rev. Lett. 98, 222001 (2007), hep-lat/0612032

$$f_{q/p^\uparrow}(x, \mathbf{p}_T) = f_1^q(x, \mathbf{p}_T^2) - f_{1T}^{\perp q}(x, \mathbf{p}_T^2) \, \frac{(\mathbf{p}_T \times \hat{\mathbf{S}}_T) \cdot \hat{\mathbf{P}}}{M}$$

: Spin density of unpol. q quark in a transversely pol. proton





Trento conventions for SIDIS:

- **P** || z || **q**

if $s_{Tx} \Rightarrow$ deformation Δ

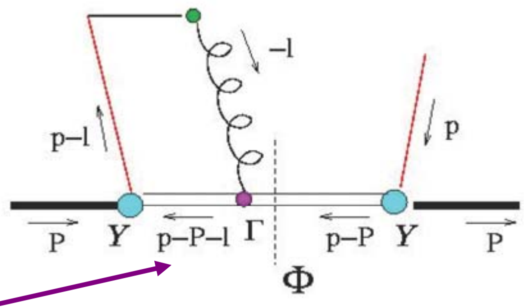
$\Delta \propto -p_{Ty} / M$

$f_{1T}^{\perp u} < 0 \;\Rightarrow \Delta > 0$

$f_{1T}^{\perp d} > 0 \;\Rightarrow \Delta < 0$

Overlap representation for T-odd TMD

Tree level LCWFs are not enough! We need a convolution over transverse momenta, including a Final State Interactions kernel G to model the one-gluon exchange residual interaction!



So far, only results for Sivers function and S=0 diquark

Brodsky, Gardner, P.L.B643 (06) 22

Zu, Schmidt, P.R.D75 (07) 073008

$$\frac{2\,(\mathbf{p}_T\times\hat{\mathbf{S}}_T)\cdot\hat{\boldsymbol{z}}}{M}\, f_{1T}^{\perp s}(x,\mathbf{p}_T^2) = \int\frac{d^2\mathbf{p}'_T}{16\pi^3}\, G(x,\mathbf{p}_T,\mathbf{p}'_T)\sum_{\lambda_q}\left[\psi_{\lambda_q}^{\hat{\mathbf{S}}_T *}(x,\mathbf{p}_T)\,\psi_{\lambda_q}^{\hat{\mathbf{S}}_T}(x,\mathbf{p}'_T) - \psi_{\lambda_q}^{-\hat{\mathbf{S}}_T *}(x,\mathbf{p}_T)\,\psi_{\lambda_q}^{-\hat{\mathbf{S}}_T}(x,\mathbf{p}'_T)\right]\left(+\text{ h.c.}\right)$$

$$\frac{2\,(\mathbf{p}_T\times\hat{\mathbf{S}}_T)\cdot\hat{\boldsymbol{z}}}{M}\, f_{1T}^{\perp a}(x,\mathbf{p}_T^2) = \int\frac{d^2\mathbf{p}'_T}{16\pi^3}\, G(x,\mathbf{p}_T,\mathbf{p}'_T)\sum_{\lambda_q,\lambda_a}\left[\psi_{\lambda_q,\lambda_a}^{\hat{\mathbf{S}}_T *}(x,\mathbf{p}_T)\,\psi_{\lambda_q,\lambda_a}^{\hat{\mathbf{S}}_T}(x,\mathbf{p}'_T) - \psi_{\lambda_q,\lambda_a}^{-\hat{\mathbf{S}}_T *}(x,\mathbf{p}_T)\,\psi_{\lambda_q,\lambda_a}^{-\hat{\mathbf{S}}_T}(x,\mathbf{p}'_T)\right]$$

$$\frac{(\mathbf{p}_T\times\hat{\mathbf{s}}_{qT})\cdot\hat{\boldsymbol{z}}}{M}\, h_1^{\perp s}(x,\mathbf{p}_T^2) = \int\frac{d^2\mathbf{p}'_T}{16\pi^3}\, G(x,\mathbf{p}_T,\mathbf{p}'_T)\sum_{\lambda_N}\left[\psi_{\hat{\mathbf{s}}_{qT}}^{\lambda_N *}(x,\mathbf{p}_T)\,\psi_{\hat{\mathbf{s}}_{qT}}^{\lambda_N}(x,\mathbf{p}'_T) - \psi_{-\hat{\mathbf{s}}_{qT}}^{\lambda_N *}(x,\mathbf{p}_T)\,\psi_{-\hat{\mathbf{s}}_{qT}}^{\lambda_N}(x,\mathbf{p}'_T)\right]$$

$$\frac{(\mathbf{p}_T\times\hat{\mathbf{s}}_{qT})\cdot\hat{\boldsymbol{z}}}{M}\, h_1^{\perp s}(x,\mathbf{p}_T^2) = \int\frac{d^2\mathbf{p}'_T}{16\pi^3}\, G(x,\mathbf{p}_T,\mathbf{p}'_T)\sum_{\lambda_N,\lambda_a}\left[\psi_{\hat{\mathbf{s}}_{qT},\lambda_a}^{\lambda_N *}(x,\mathbf{p}_T)\,\psi_{\hat{\mathbf{s}}_{qT},\lambda_a}^{\lambda_N}(x,\mathbf{p}'_T) - \psi_{-\hat{\mathbf{s}}_{qT},\lambda_a}^{\lambda_N *}(x,\mathbf{p}_T)\,\psi_{-\hat{\mathbf{s}}_{qT},\lambda_a}^{\lambda_N}(x,\mathbf{p}'_T)\right]$$

$$\text{Im}\, G(x,\mathbf{p}_T,\mathbf{p}'_T) = -\frac{e^2}{2\,(2\pi)^2}\,\frac{1}{(\mathbf{p}_T-\mathbf{p}'_T)^2}$$

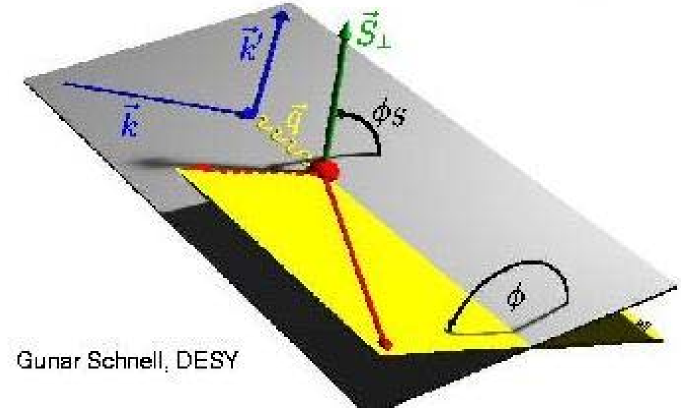
Universal FSI operator G !
(using LC gauge for S=1 Dq)

SIDIS weighted SSAs

$$e\, p^{\uparrow} \to e'\, \pi^{\pm}\, X$$

Single Spin Asymmetry :

$$A_{UT}^{w} \equiv \frac{\int d\phi_S\, d^2\boldsymbol{P}_{h\perp}\; w\; (d^6\sigma_{UT}^{\uparrow} - d^6\sigma_{UT}^{\downarrow})}{\int d\phi_S\, d^2\boldsymbol{P}_{h\perp}\; (d^6\sigma_{UT}^{\uparrow} + d^6\sigma_{UT}^{\downarrow})}$$



$|\boldsymbol{P}_{h\perp}|/M$ weighted asymmetries allow for an analytic deconvolution of the integrals upon transverse momenta, while unweighted ones can only be decoupled through (gaussian) assumptions on TMDs.

Collins Asymmetry:

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{M}\sin(\phi+\phi_S)}(x) \approx \frac{\int_{cuts} dy\, dz\;\; 2B(y)\sum_q e_q^2\, h_1^q(x)\, H_1^{\perp(1),q}(z)}{\int_{cuts} dy\, dz\;\; A(y)\sum_q e_q^2\, f_1^q(x)\, D_1^q(z)}$$

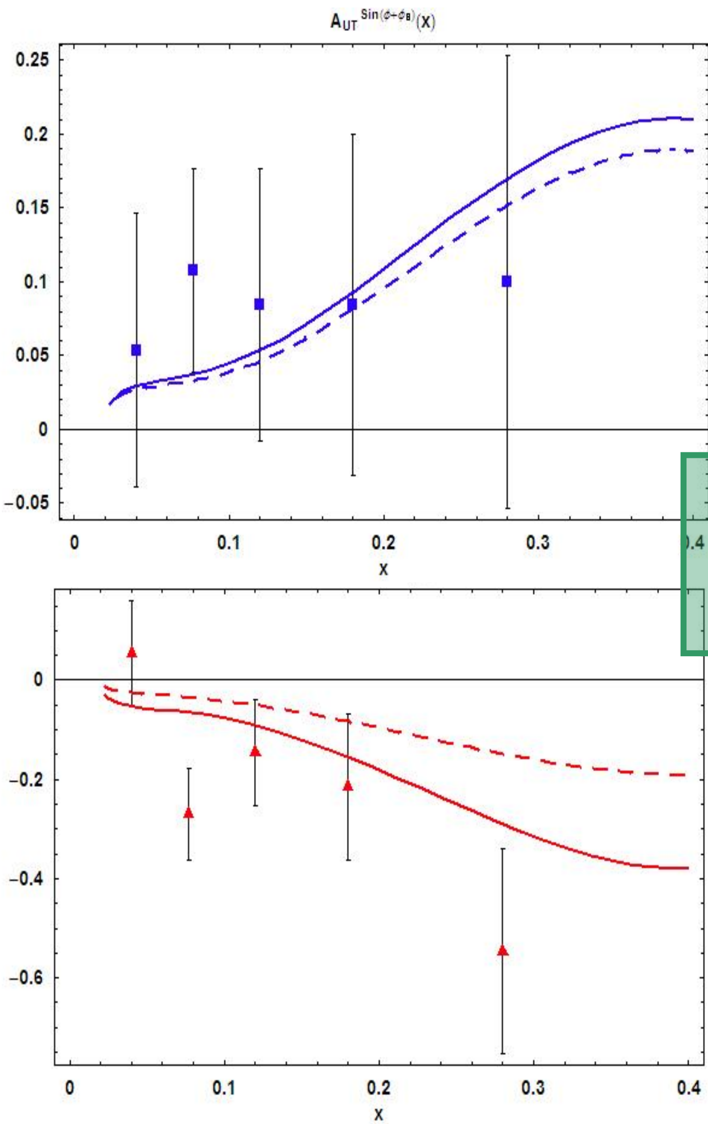
Sivers Asymmetry:

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{M}\sin(\phi-\phi_S)}(x) \approx -\frac{\int_{cuts} dy\, dz\;\; 2A(y)\sum_q e_q^2\, f_{1T}^{\perp(1),q}(x)\, D_1^q(z)}{\int_{cuts} dy\, dz\;\; A(y)\sum_q e_q^2\, f_1^q(x)\, D_1^q(z)}$$

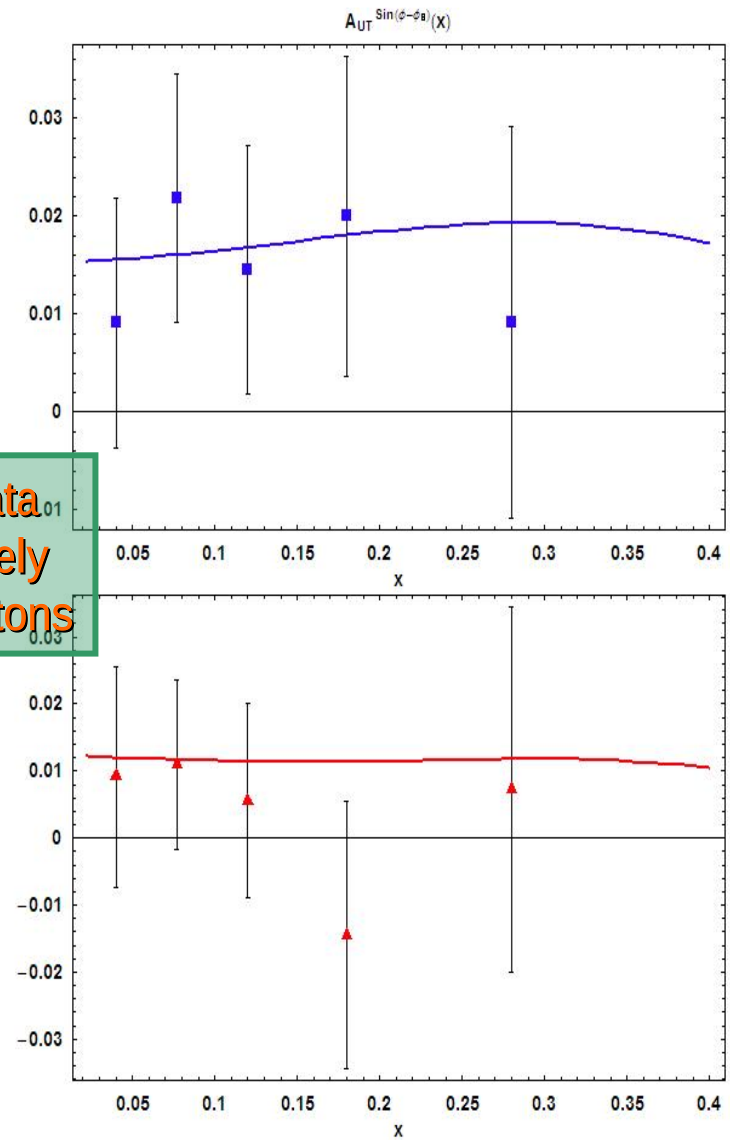
SIDIS weighted SSAs

D_1 from De Florian-Sassot-Stratmann, hep-ph/0703242 (2007).
$H_1^{\perp}$ from diquark model,

Collins Asymmetry :



Sivers Asymmetry :



π^+

HERMES data on transversely polarized protons

π^-

Conclusions & perspectives

- ✓ Why another model for TMD? We actually don't know much about them!

- ✓ Why a spectator diquark model? It's simple, always analytic results! Able to reproduce T-odd effects!
  Why including axial-vector diquarks? Needed for down quarks!

- ✓ What's new in our work? Systematic calculation of ALL leading twist T-even and T-odd TMDs
  Several forms of the N-q-Dq vertex FF and of the S=1 diquark propagator
  9 free parameters fixed by fitting available parametrization for f_1 and g_1
  T-even overlap representation: LCWFs with non-zero L, breaking of SU(4)
  T-odd overlap representation: universal FSI operator

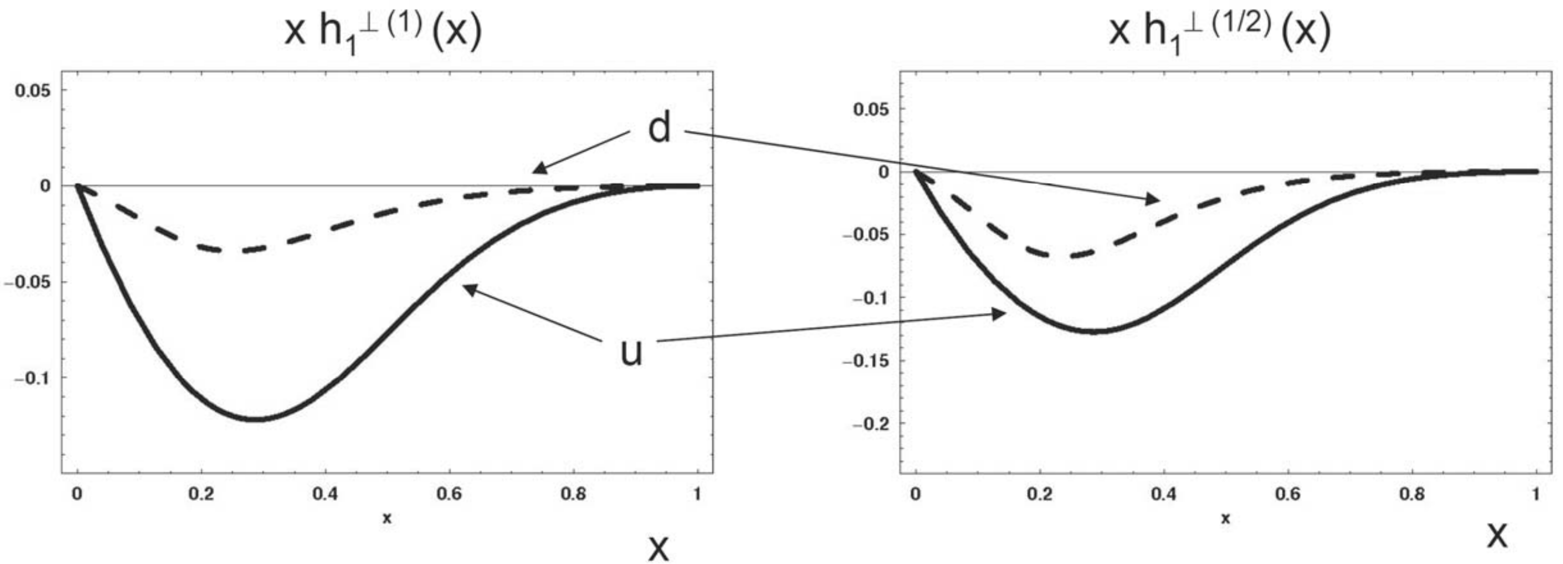
  A.Bacchetta, F.C., M.Radici; arXiv:0807.0321 [hep-ph]

- ✓ Which are the main results? Interesting p_T dependence
  Satisfactory agreement with u & d transversity parametrizations
  Agreement with lattice on T-odd functions signs for all flavours
  Satisfactory agreement with u Sivers moments parametrizations, but understimation for d quark.

Future: calculate observables (SSA) and exploit model LCWFs to compute other fundamental objects, such as nucleon e.m. form factors and GPDs

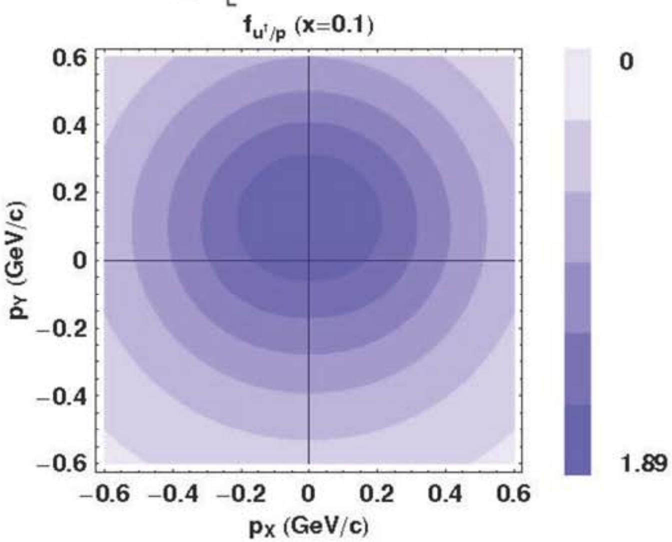
Support Slides

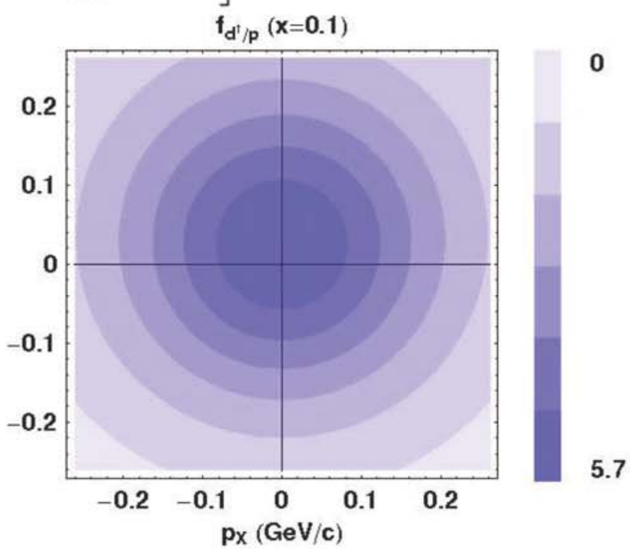
Boer-Mulders function



$$f_{q^\uparrow/p} = \frac{1}{2}\left[f_1^q(x,\mathbf{p}_T^2) - h_1^{\perp\, q}(x,\mathbf{p}_T^2)\,\frac{(\mathbf{p}_T\times\hat{\mathbf{s}}_{qT})\cdot\hat{\mathbf{P}}}{M}\right]$$

density of pol. quarks $q^\uparrow$ in unpol. p





if $s_{qTx} \Rightarrow$ deformation Δ

$\Delta \propto -p_{Ty}/M$

$h_1^{\perp u} < 0 \;\Rightarrow \Delta > 0$

$h_1^{\perp d} < 0 \;\Rightarrow \Delta > 0$

T-even TMD: overlap representation

need to define the state with polarization along $\hat{s}_T = (\cos\phi, \sin\phi)$

$$U(P,\uparrow) = \frac{1}{\sqrt{2}}\left(U(P,+) + e^{i\phi}U(P,-)\right)$$
$$U(P,\downarrow) = \frac{1}{\sqrt{2}}\left(U(P,+) + e^{i(\phi+\pi)}U(P,-)\right)$$

$$\psi^{\lambda_N}_{\lambda_q} = \sqrt{\frac{p^+}{(P-p)^+}}\,\frac{\bar{u}(p,\lambda_q)}{p^2-m^2}\,\mathcal{Y}_s(p^2)\,U(P,\lambda_N)$$

agree with Barone, Ratcliffe, *Transverse Spin Physics* (World Scientific, USA, 2003)

D = s $\lambda_s = \varnothing$
D = a $\lambda_a = \pm$

$$g_{1L}^D(x,\mathbf{p}_T^2) = \frac{1}{16\pi^3}\sum_{\lambda_D}\left[|\psi^+_{+,\lambda_D}|^2 - |\psi^+_{-,\lambda_D}|^2\right]$$

$$\frac{\mathbf{p}_T\cdot\hat{\mathbf{s}}_T}{M}\,g_{1T}^D(x,\mathbf{p}_T^2) = \frac{1}{16\pi^3}\sum_{\lambda_D}\left[|\psi^\uparrow_{+,\lambda_D}|^2 - |\psi^\uparrow_{-,\lambda_D}|^2\right]$$

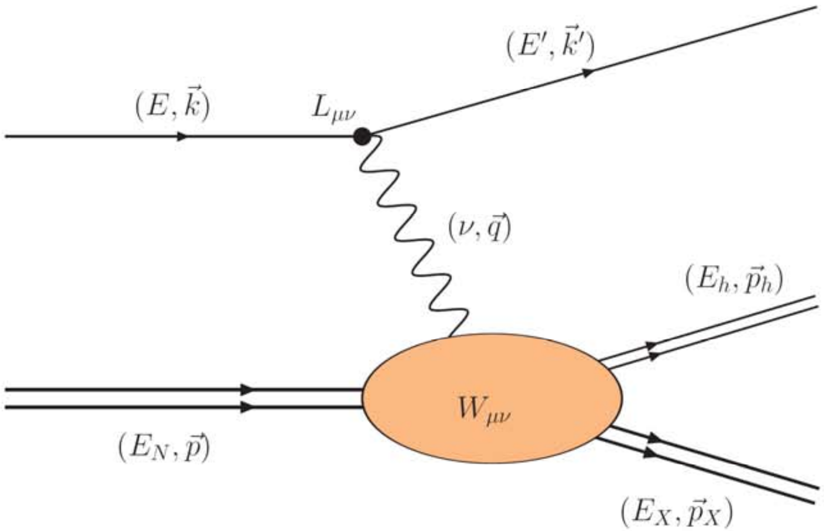
$$\frac{\mathbf{p}_T\cdot\hat{\mathbf{s}}_{qT}}{M}\,h_{1L}^{\perp D}(x,\mathbf{p}_T^2) = \frac{1}{16\pi^3}\sum_{\lambda_D}\left[|\psi^+_{\uparrow,\lambda_D}|^2 - |\psi^+_{\downarrow,\lambda_D}|^2\right]$$

$$\frac{\hat{\mathbf{s}}_T\cdot\hat{\mathbf{s}}_{qT}}{M}\,h_{1T}^D(x,\mathbf{p}_T^2) + \frac{\mathbf{p}_T\cdot\hat{\mathbf{s}}_T}{M}\,\frac{\mathbf{p}_T\cdot\hat{\mathbf{s}}_{qT}}{M}\,h_{1T}^{\perp D}(x,\mathbf{p}_T^2) = \frac{1}{16\pi^3}\sum_{\lambda_D}\left[|\psi^\uparrow_{\uparrow,\lambda_D}|^2 - |\psi^\uparrow_{\downarrow,\lambda_D}|^2\right]$$

Structure of the Nucleon

Deep Inelastic Scattering: $ep \to e'X$

usefulness of an expansion in powers of 1/Q, besides that in powers of α_s (pQCD)



DIS regime:

$$Q^2 = -(\nu, \mathbf{q})^2 \gg M^2$$
$$\nu = E - E' \gg M^2$$
$$x_B = Q^2/2M\nu \qquad \text{fixed}$$

$$\sigma \sim L_{\mu\nu} W^{\mu\nu}$$

Hadronic tensor: information on the hadron internal dynamics (low energy => non-pert. QCD), encoded in terms of *structure functions*, with SCALING properies (Q-independence) in DIS regime

Leptonic tensor: known at any order in pQED

PARTON MODEL:

- almost free (on shell) pointlike partons
- hard/soft factorization theorems: convolution between *hard* elementary cross section and *soft* (non pert.) and universal parton distribution functions => PDF

Asymptotic Freedom / Confinement

Hadronic tensor:

$$2MW^{\mu\nu}(q,P,S) = \frac{1}{2\pi}\sum_X \int \frac{d^3\boldsymbol{P}_X}{(2\pi)^3 2P_X^0}(2\pi)^4\,\delta^{(4)}(q+P-P_X) \times \langle P,S|J^\mu(0)|P_X\rangle\,\langle P_X|J^\nu(0)|P,S\rangle,$$

Fourier transforming the Dirac delta:

DIS kinematical dominance: $\xi^2 \simeq 0$

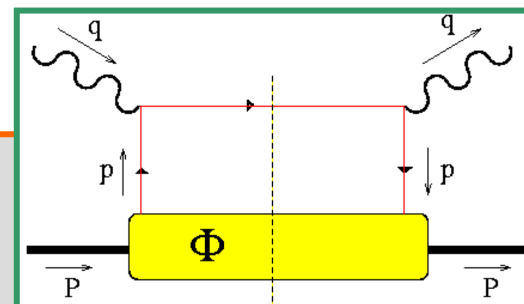
$$2MW^{\mu\nu}(q,P,S) = \frac{1}{2\pi}\int d^4\xi\, e^{iq\cdot\xi}\,\langle P,S|J^\mu(\xi)\,J^\nu(0)|P,S\rangle$$

Light-Cone quantization!

In the PARTON MODEL, at tree level and LEADING TWIST (leading order in 1/Q): incoherent sum of interactions with single quarks

$$2MW^{\mu\nu}(q,P,S) = \sum_f e_f^2 \int d^4p\,\delta\left((p+q)^2 - m^2\right)\theta(p^0+q^0-m) \times \mathrm{Tr}\left[\Phi^f(p,P,S)\gamma^\mu(\not p+\not q+m)\gamma^\nu\right]$$

QUARK-QUARK CORRELATOR: probability of extracting a quark f (with momentum p) in 0 and reintroducing it at ξ



$$\Phi^f_{ji}(p,P,S) = \int \frac{d^4\xi}{(2\pi)^4}\,e^{-ip\cdot\xi}\,\langle P,S|\bar\psi_i^f(\xi)\psi_j^f(0)|P,S\rangle$$

$$= \sum_X \int \frac{d\boldsymbol{P}_X}{(2\pi)^3 2P_X^0}\langle P,S|\bar\psi_i^f(0)|P_X\rangle\,\langle P_X|\psi_j^f(0)|P,S\rangle\,\delta^{(4)}(P-p-P_X)$$

Diagonal matrix elements of bilocal operators, built with quark fields, on hadronic states

Parton Distribution Functions

PDF extractable through projections of the Φ over particular Dirac structures, integrating over the LC direction — (kinematically suppressed) and over the parton transverse momentum

$$\Phi_{ji}(x, S) = \frac{1}{2}\int dp^- d\boldsymbol{p}_T\, \Phi(p, P, S)\Big|_{p^+ = xP^+} = \int \frac{d\xi^-}{4\pi} e^{-ip\cdot\xi} \langle P, S|\bar{\psi}_i(\xi)\, \psi_j(0)|P, S\rangle\Big|_{\xi^+ = \xi_\perp = 0}$$

$$\Phi^{[\Gamma]}(x, S) = \mathrm{Tr}[\Phi(x, S)\,\Gamma]$$

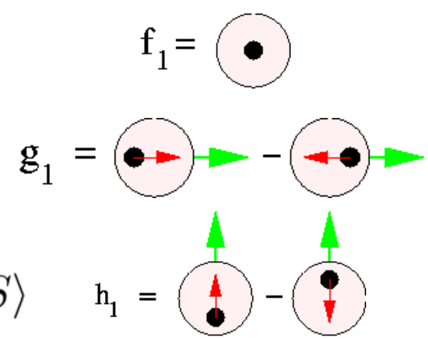
(Non località ristretta alla direzione LC —)

3 LEADING TWIST projections, with probabilistic interpretation as numerical quark densities:

$$f_1(x) = \Phi^{[\gamma^+]} = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P, S|\bar{\psi}(\xi^-)\gamma^+\psi(0)|P, S\rangle$$

$$\lambda\, g_1(x) = \Phi^{[\gamma^+\gamma_5]} = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P, S|\bar{\psi}(\xi^-)\gamma^+\gamma_5\psi(0)|P, S\rangle$$

$$S_T^i\, h_1(x) = \Phi^{[i\sigma^{i+}\gamma_5]} = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P, S|\bar{\psi}(\xi^-)i\sigma^{i+}\gamma_5\psi(0)|P, S\rangle$$



f_1 Momentum distribution (unpolarized PDF)

g_1 Helicity distribution (chirality basis)

h_1 Transverse Spin distribution (transverse spin basis)

Chiral odd, and QCD conserves chirality at tree level => NOT involved in Inclusive DIS !

Open problems: Proton Spin

Experimental information about the Proton Structure Functions:

$$F_1(x) = \frac{1}{2}\sum_q e_q^2\, f_1^q(x)$$

→ measured in unpolarized inlusive DIS; systematic analysis @ HERA, included Q^2-dependence from radiative corrections (scaling violations)

$$G_1(x) = \frac{1}{2}\sum_q e_q^2\, g_1^q(x)$$

→ measured in DIS with longitudinally polarized beam & target, through the (helicity) Double Spin Asymmetry:

$$G_1(x) \sim A^{\parallel} = \frac{\sigma(l^{\uparrow}N^{\uparrow}) - \sigma(l^{\uparrow}N^{\downarrow})}{\sigma(l^{\uparrow}N^{\uparrow}) + \sigma(l^{\uparrow}N^{\downarrow})}$$

From EMC @ CERN results ('80) (+ Isospin and flavour simmetry, QCD sum rules):

$$\Delta\Sigma = \sum_q \int_0^1 dx\, g_1^q(x) \sim 0,2$$

→ small fraction of the Proton Spin determined by the quark spin !

Spin sum rule for a longitudinal Proton: J_z^N

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Quark/Gluon Spin Fraction

Orbital Angular Momentum contributions:

- not directly accessible
- link with GPD…complex extraction!
- Is there any link with PDF?

Open problems: Single Spin Asymmetries

Experimental evidence of large Asymmetries in *the azymuthal distribution* (with respect to the normal at the production plane) of the reaction products in processes involving ONE transversely polarized hadron

$$\mathrm{SSA} = \frac{d\sigma(H^{\uparrow}) - d\sigma(H^{\downarrow})}{d\sigma(H^{\uparrow}) + d\sigma(H^{\downarrow})}$$

$$p\,N \to \Lambda^{\uparrow}\,X$$

$$P_{\Lambda}(x_F)$$
$$x_F = x_1 - x_2 \simeq 2(p_{\Lambda})_L/\sqrt{s}$$

Ex.: Heller *et al.*, P.R.L. 41 ('78) 607

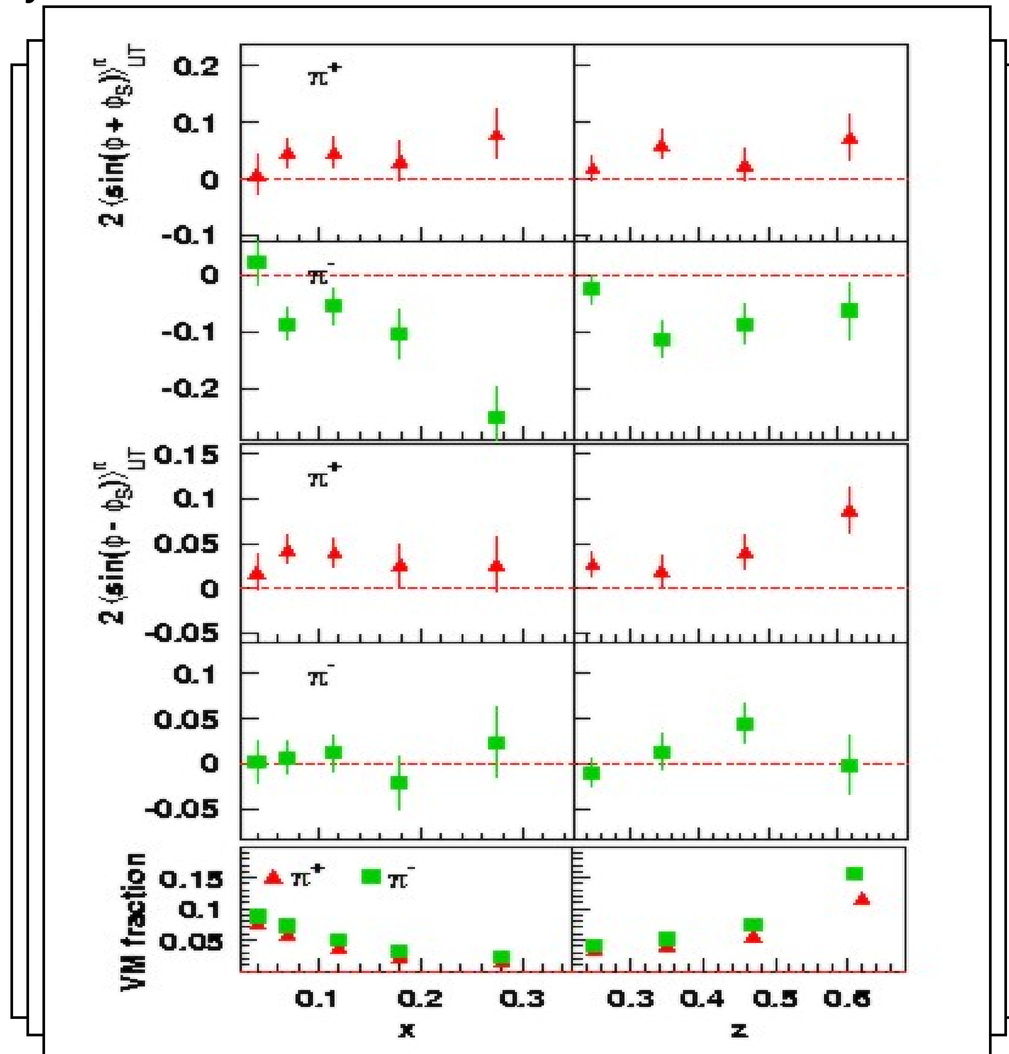
$$p\,p^{\uparrow} \to \pi^0\,X$$

$$A_N(x_F) \quad x_F = 2(p_{\pi})_L/\sqrt{s}$$

Ex.: Adams *et al.*, STAR P.R.L. 92 ('04) 171801

$$e\,p^{\uparrow} \to e'\,\pi^{\pm}\,X$$ (SIDIS)

Ex.:A. Airapetian *et al.* (HERMES) Phys. Lett. B562 182 ('05)



(Quite) simple experiments, but difficult interpretation!

At the parton level:

transverse spin

$$\textbf{SSA} = \frac{d\sigma(q^{\uparrow}) - d\sigma(q^{\downarrow})}{d\sigma(q^{\uparrow}) + d\sigma(q^{\downarrow})} \sim \frac{\langle\uparrow\,|\,\uparrow\rangle - \langle\downarrow\,|\,\downarrow\rangle}{\langle\uparrow\,|\,\uparrow\rangle + \langle\downarrow\,|\,\downarrow\rangle} \;\Rightarrow\; \sim \frac{2\,\mathrm{Im}\langle+|-\rangle}{\langle+|+\rangle + \langle-|-\rangle}$$

$$|\uparrow/\downarrow\rangle = (|+\rangle \pm i|-\rangle)/\sqrt{2}$$

helicity/chirality

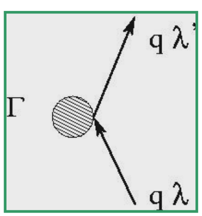
2 conditions for non-zero SSA:

1) Existence of two amplitudes $\mathcal{M}[\gamma^* p(J_z^p) \to F]$, with $J_z^p = \pm\frac{1}{2}$, coupled to the <u>same final state</u> F
2) Different complex phases for the two amplitude: the correlation is linked to the <u>imaginary part</u> of the interference $\mathrm{Im}[\mathcal{M}^*(1/2)\mathcal{M}(-1/2)]$

But :

(p$_T$–dependence integrated away)

1) QCD, in the massless limit (λ=±1) and in *collinear factorization*, conserves chirality
=> helicity flip amplitudes suppressed !



$$\begin{aligned} M &\sim \bar{u}_{\lambda'}\,\Gamma\,u_\lambda \\ &\sim \bar{u}_{\lambda'}\,(1-\lambda'\gamma_5)\,(1-\lambda\gamma_5)\,\Gamma\,u_\lambda \\ &\sim \delta_{\lambda\lambda'}\,\bar{u}_{\lambda'}\,\Gamma\,u_\lambda + o\left(\frac{m_q}{E_q}\right) \end{aligned}$$

$$\left(\begin{aligned} \frac{1+\lambda\gamma_5}{2}\,u_\lambda &= u_\lambda \\ \bar{u}_\lambda\,\frac{1-\lambda\gamma_5}{2} &= \bar{u}_\lambda \end{aligned}\right)$$

2) Born amplitudes are real !

⇒ Need to include transverse momenta and processes beyond tree level !

Origin of T- odd structures

T-odd PDF initially believed to be zero (Collins) due to Time-Reversal invariance of strong interactions. In 2002, however, computation of a non-zero Sivers function in a simple model.

(J. C. Collins, Nucl. Phys. B396, S. J. Brodsky, D. S. Hwang and I. Schmidt, Phys. Lett. B530)

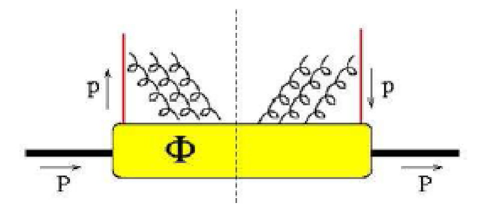
What produces the required complex phases NOT invariant under (naive) Time-Reversal?

The Φ correlator involves bilocal operators and QCD is based on the invariance under Gauge (i.e. local) transformations of colour SU(3)

$$\bar{\psi}(\xi)\psi(0) \longrightarrow \bar{\psi}(\xi) U_{[0,\xi^-]} \left[\equiv \mathcal{P} e^{-ig\int_0^{\xi^-} dw \cdot A(w)} \right] \psi(0)$$

correct Gauge Invariant definition

Gauge Link



Every link can be series expanded at the wanted (n) order, and can be interpretated as the exchange of n soft gluons on the light-cone

$$U_{[0,\xi^-]} = \sum_{n=0}^{\infty} (-ig)^n \int_0^{\xi^-} dw_1^- \, A^+(w_1) \ldots \int_{w_{n-1}}^{\xi^-} dw_n^- \, A^+(w_n) \Big|_{w_i^+=0, \mathbf{w}_T=\mathbf{0}_T}$$

Gauge Link = Residual active quark-spectators Interactions, NOT invarianti under Time Reversal!

If there is also p_T-dependence, twist analysis reveals that the leading order contributions come from both A^+ and A_T (at ∞^-) => NON-trivial link structure, not reducible to identity with A+ =0 gauge

But is there a real Time Reversal invariance violation?

Altough QCD Lagrangian contains terms that would allow it, experimentally there is no evidence of CP- (and hence T-) violation in the strong sector (no neutron e.d.m.)

Sivers and Boer-Mulders functions are associated with coefficients involving **3** (pseudo)vectors, thus changing sign under time axis orientation reversal. This operation alone is defined Naive Time Reversal. In this sense we speak about Naive T-*odd* distributions!

Nevertheless, Time Reversal operation in general also requires an exchange between initial and final states!

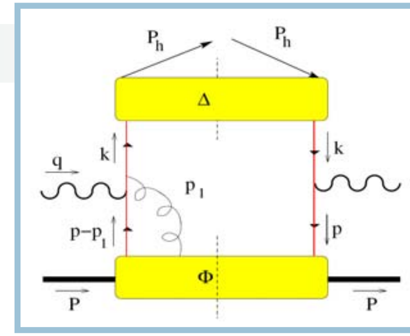
If such an operation turns out not to be trivial, due to the presence of complex phases in the S matrix elements, there could be Naive T-odd spin effects($|T_{if}|^2 \neq |\widetilde{T}_{if}|^2$) in a theory which in general shows CP- and hence T-invariance ($|T_{if}|^2 = |\widetilde{T}_{fi}|^2$)!

$$\langle f|S|i\rangle = \langle i|S|f\rangle^*$$

Naive T-odd Fragmentation Functions (e. g. Collins function) are easier to justify, because the required relative phases can be generated by Final State Interactions (FSI) between leading hadron and jets, dinamically distinguishing initial and final states.

... and for PDF?   **Gauge Link !!!**

Hadronic tensor for SIDIS, at leading twist and at first order in the strong coupling *g*:



$$\Phi^{+}(p,l,P,S)=\int\frac{d^4\xi}{(2\pi)^4}\int\frac{d^4\eta}{(2\pi)^4}\,e^{+ip\cdot\xi}\,e^{+i\,l\cdot(\eta-\xi)}\,\langle P,S|\bar\psi(0)\,g\,A^{+}(\eta)\,\psi(\xi)|P,S\rangle$$

EIKONAL Approximation :

It can be shown that within this approximation the one-gluon loop contribution represents the $\mathcal{O}(g)$ term in a series expansion of the Gauge Link operator!

$$\gamma^{-}\frac{\not k-\not l+m}{(k-l)^2-m^2}A^{+}(\eta)\simeq\gamma^{-}\frac{\not k-\not l+m}{-2k^{-}l^{+}+i\varepsilon}A^{+}(\eta)$$
$$\simeq\left[\frac{\gamma^{-}\not l}{2k^{-}l^{+}-i\varepsilon}-\frac{2k^{-}}{2k^{-}l^{+}-i\varepsilon}+\frac{(\not k-m)}{2k^{-}l^{+}-i\varepsilon}\gamma^{-}\right]A^{+}(\eta)$$
$$\simeq(-)\frac{1}{l^{+}-i\varepsilon}A^{+}(\eta)$$

(A. V. Belitsky, X. Ji, F. Yuan, Nucl. Phys. B656)

the relevant Correlator is now (integration over gluon momentum):

$$\Phi^{+}(p,P,S)\;=\;\int d^4l\,\frac{1}{l^{+}-i\varepsilon}\,\Phi^{+}(p,l,P,S)$$

For the Drell-Yan process (hadronic collisions), the sign of the k momentum is instead reversed and the analogous eikonal approximation now gives: $(l^{+}-i\varepsilon)\;\longrightarrow\;(l^{+}+i\varepsilon)$

Sivers function depends on the imaginary part of an interference between different amplitudes:

$$\rightarrow\quad f_{1T}^{\perp}|_{SIDIS}=-f_{1T}^{\perp}|_{DY}$$ (NON-Universality!)