Ab-initio Approach to Nuclear Matter Dynamics

Andreas Fedoseew • Walter Heupel • Anika Obermann • Urnaa Badarch • Horst Lenske

Universität Gießen, Germany

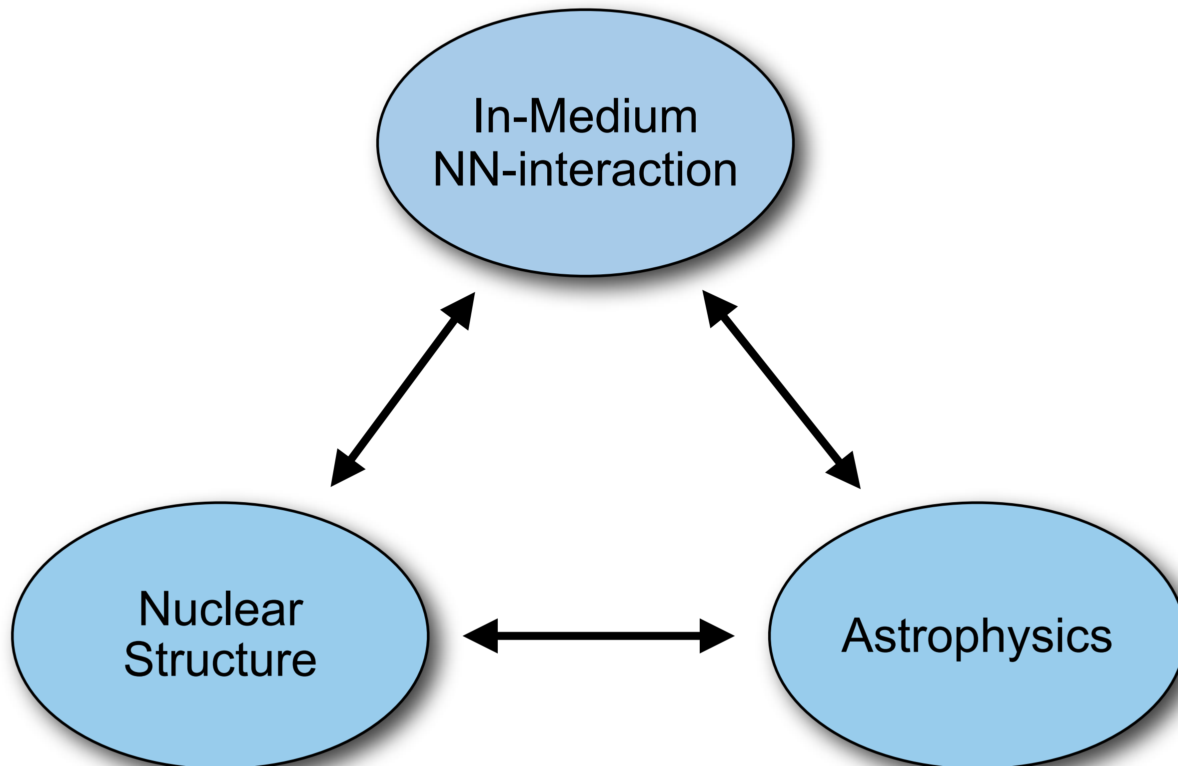
Deutsche Forschungsgemeinschaft
DFG

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

European Graduate School
Complex Systems of Hadrons and Nuclei
Copenhagen - Gießen - Helsinki - Jyväskylä - Torino

Overview

- In-Medium NN-Interactions from Ab-Initio Calculations
- Fermi Liquid Theory and Residual Interaction
- Quasi-Elastic Response Functions
- Neutron Stars

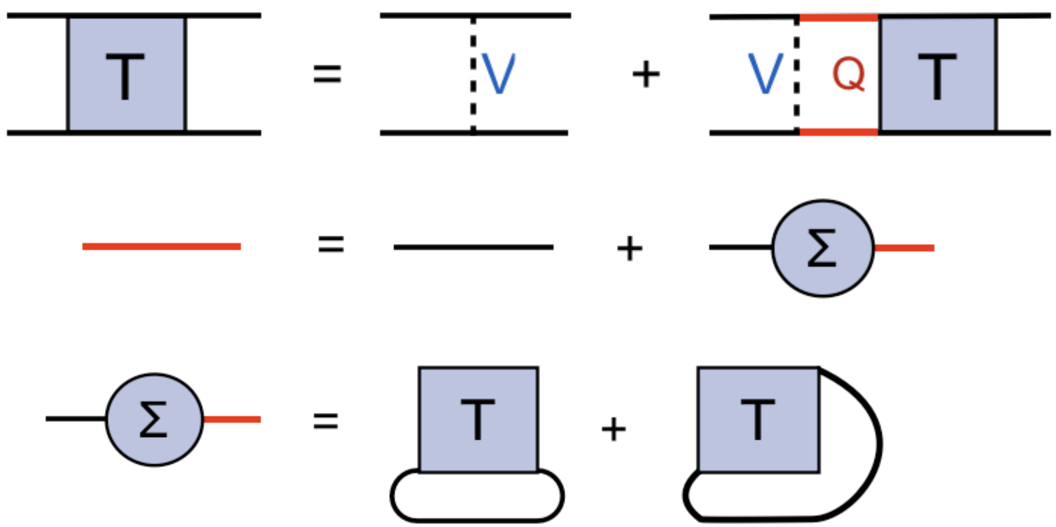


Ab-Initio Approach to Density Functional Theory

- NN Interaction Lagrangian

$$\mathcal{L}_{int} = g_{Nm} \cdot \bar{\Psi}_N \Gamma_\mu \Psi_N \Phi^\mu_m$$

- Self-consistent solution of Bethe-Salpeter equation with Ladder Kernel

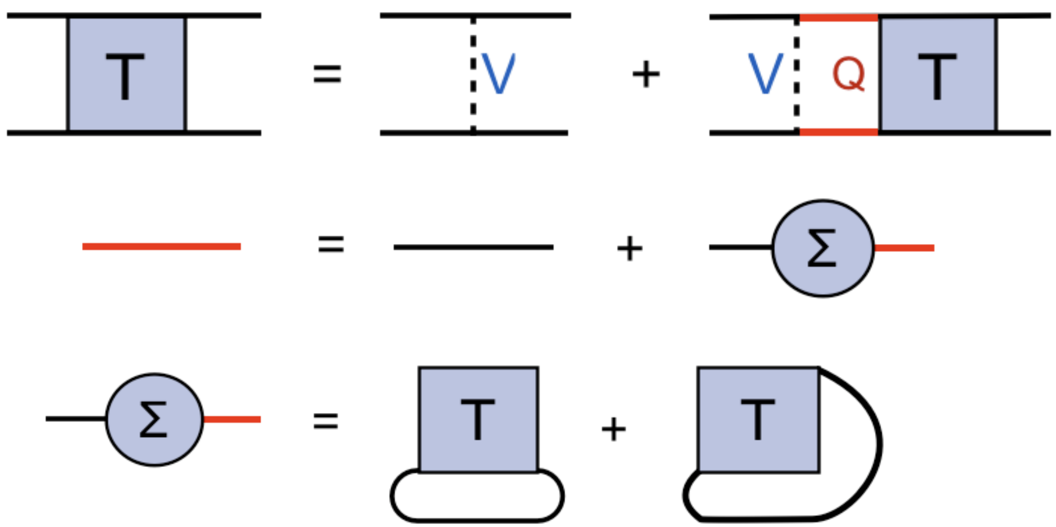


Ab-Initio Approach to Density Functional Theory

- NN Interaction Lagrangian

$$\mathcal{L}_{int} = g_{Nm} \cdot \bar{\Psi}_N \Gamma_\mu \Psi_N \Phi_m^\mu$$

- Self-consistent solution of Bethe-Salpeter equation with Ladder Kernel



- Medium modifications

  Statistical: Pauli Blocking

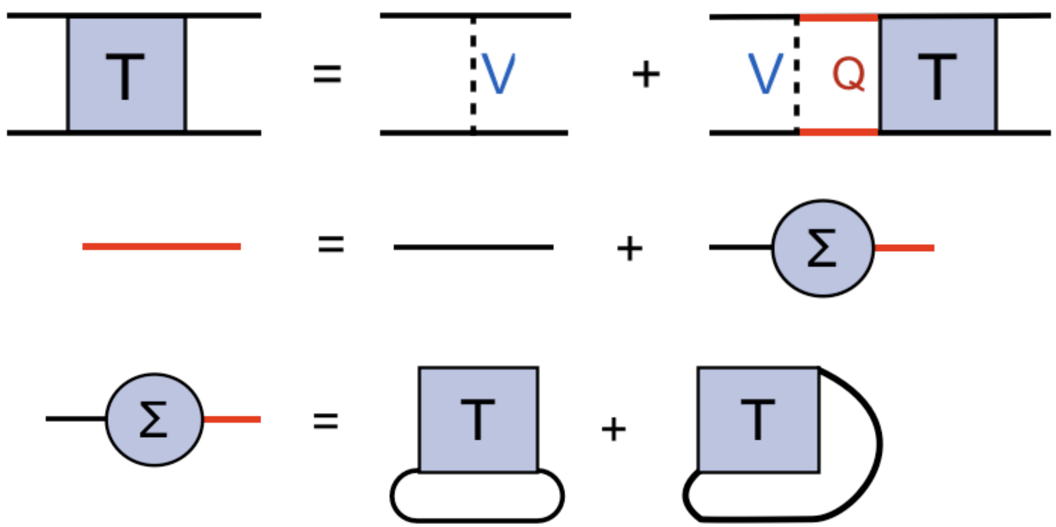
  Dynamical: Self-Energy

Ab-Initio Approach to Density Functional Theory

- NN Interaction Lagrangian

$$\mathcal{L}_{int} = g_{Nm} \cdot \bar{\Psi}_N \Gamma_\mu \Psi_N \Phi_m^\mu$$

- Self-consistent solution of Bethe-Salpeter equation with Ladder Kernel

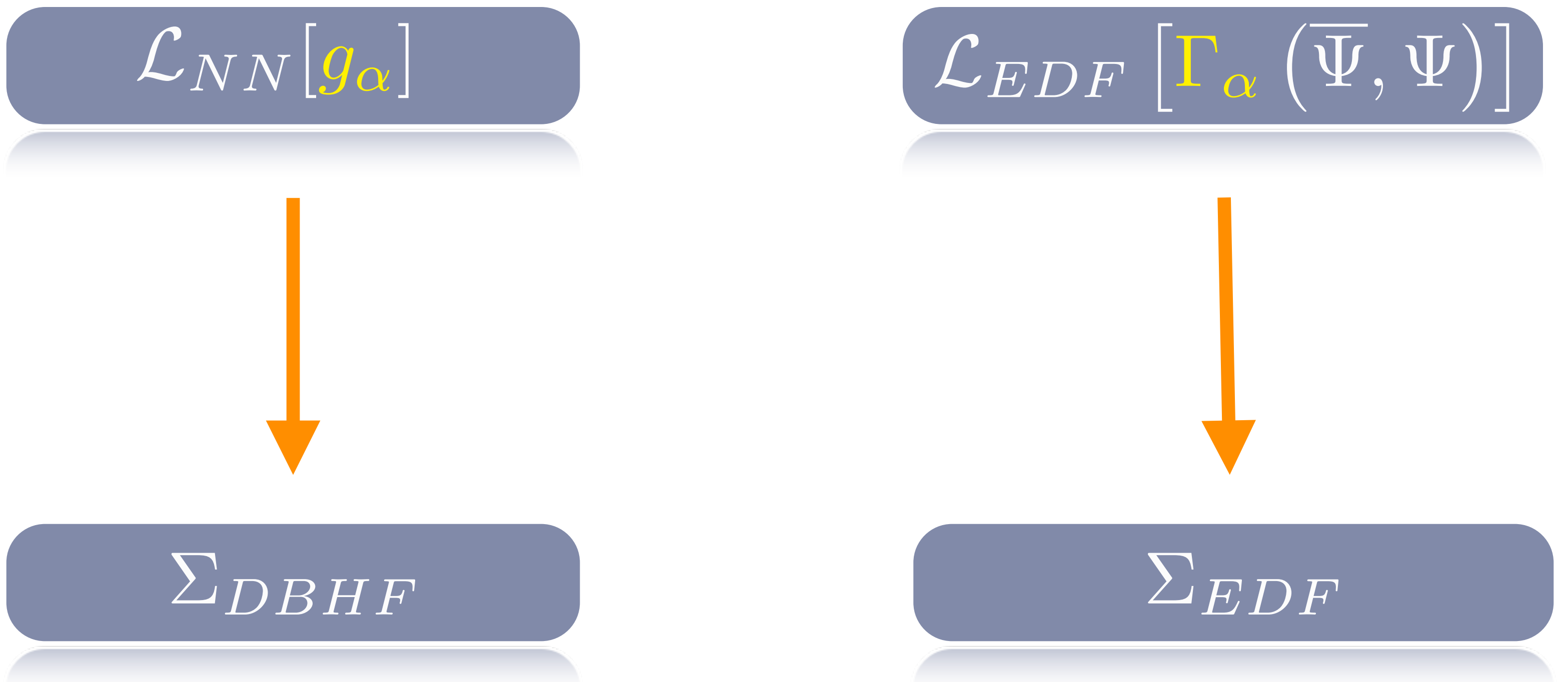


- Medium modifications
  - Statistical: Pauli Blocking
  - Dynamical: Self-Energy
- Density dependent vertex renormalization:

$$T = \sum_m z_m(k_F) V_m \longrightarrow \Gamma_m(k_F) = z_m \cdot g_m$$

Density Dependent Vertices from DB Calculations

- Interaction vertices as Lorentz-scalar **functionals of the field operators**.
- Fully covariant and thermodynamically consistent density functional theory.



Density Dependent Vertices from DB Calculations

- Interaction vertices as Lorentz-scalar **functionals of the field operators**.
- Fully covariant and thermodynamically consistent density functional theory.

$$\mathcal{L}_{NN}[g_\alpha] \qquad\qquad \mathcal{L}_{EDF}\left[\Gamma_\alpha\left(\overline{\Psi},\Psi\right)\right]$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

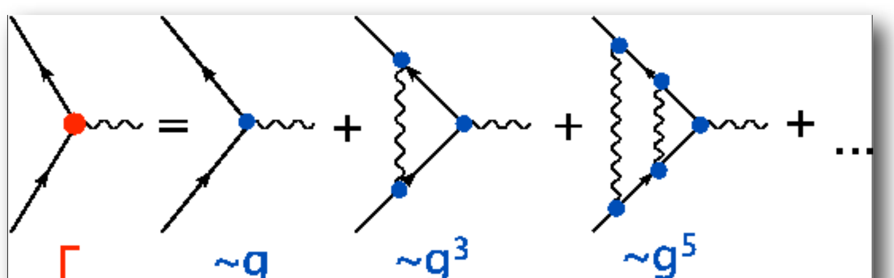
$$\Sigma_{DBHF} \stackrel{!}{=} \Sigma_{EDF}$$

Density Dependent Vertices from DB Calculations

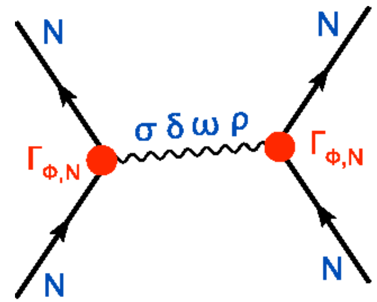
- Interaction vertices as Lorentz-scalar **functionals of the field operators**.
- Fully covariant and thermodynamically consistent density functional theory.

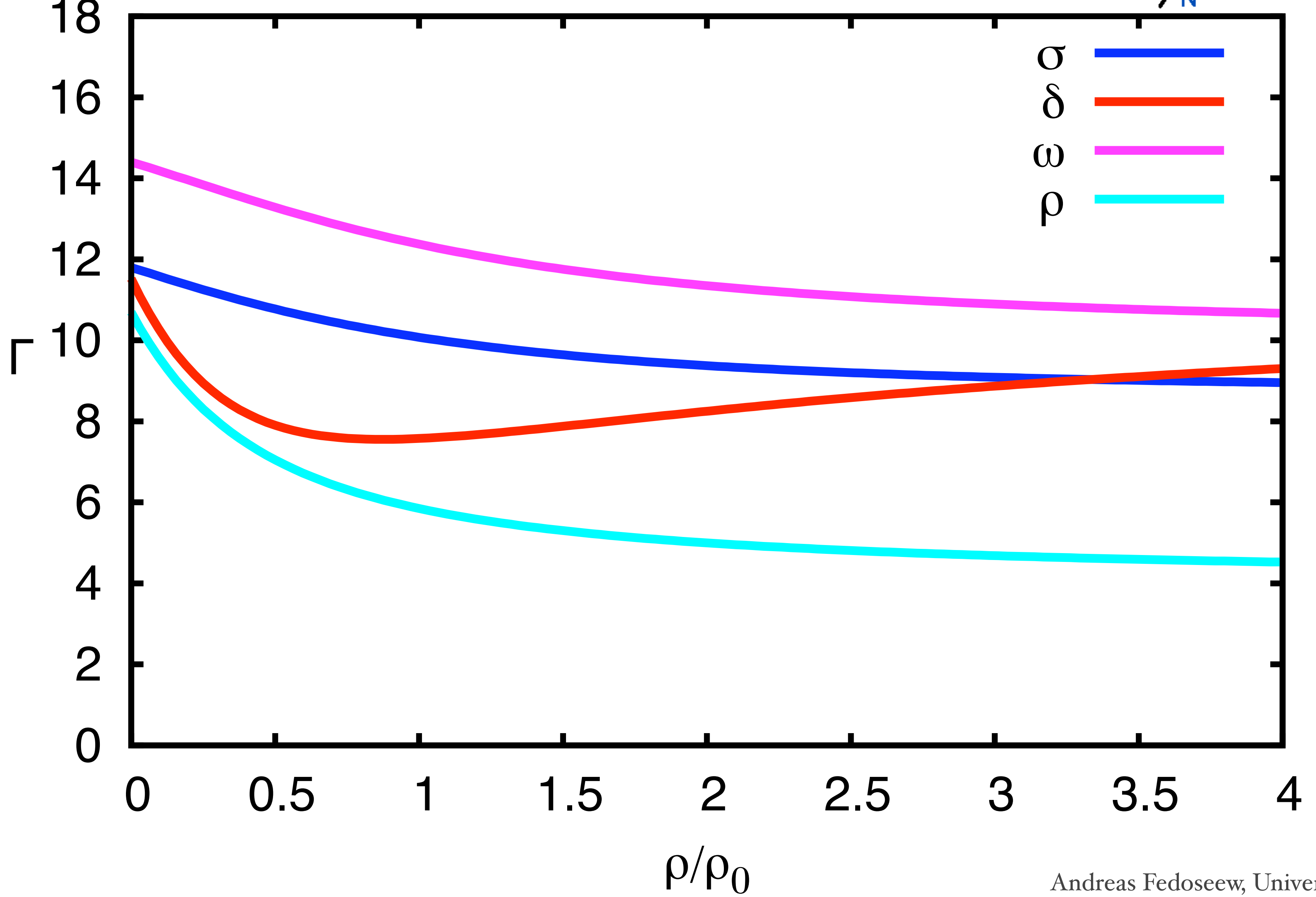
$$\mathcal{L}_{NN}[g_\alpha] \qquad \mathcal{L}_{EDF}\left[\Gamma_\alpha\left(\overline{\Psi},\Psi\right)\right]$$

$$\Sigma_{DBHF} \overset{!}{=} \Sigma_{EDF}$$



$$\mathcal{L}_{int} = \overline{\Psi}\Gamma_\sigma(\hat\rho)\Psi\Phi_\sigma - \overline{\Psi}\Gamma_\omega(\hat\rho)\gamma_\mu\Psi A^\mu_\omega$$
$$\overline{\Psi}\Gamma_\delta(\hat\rho)\tau\Psi\mathbf{\Phi}_\delta - \overline{\Psi}\Gamma_\rho(\hat\rho)\gamma_\mu\tau\Psi\mathbf{A}^\mu_\omega$$





Andreas Fedoseew, Universität Giessen

- The **D**ensity **D**ependent **R**elativistic **H**adron Field Theory

  Rearrangement self-energies

$$\hat{\Sigma}^{\mu} = \hat{\Sigma}^{\mu(0)} + \hat{\Sigma}^{\mu(r)}$$

  Thermodynamical consistency

$$\frac{1}{3}\langle T^{ii}\rangle = \rho^2 \frac{\partial}{\partial\rho}\left(\frac{\epsilon}{\rho}\right)$$

  Inclusion of the isovector-scalar interaction leads to different *effective Masses* between *protons* and *neutrons*.

- **The Density Dependent Relativistic Hadron Field Theory**

Rearrangement self-energies

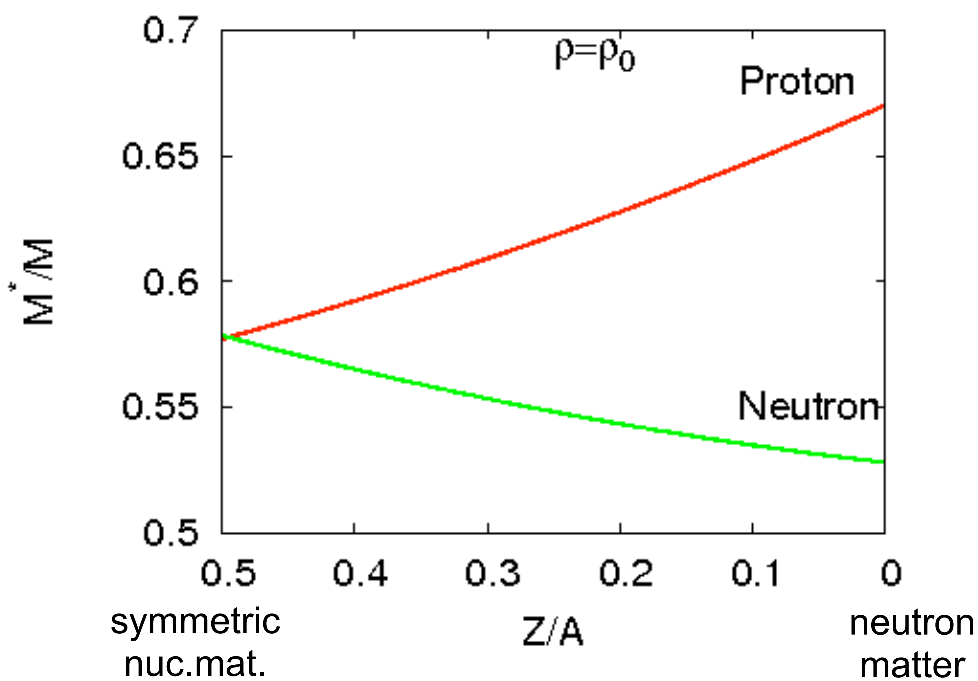
$$\hat{\Sigma}^{\mu} = \hat{\Sigma}^{\mu(0)} + \hat{\Sigma}^{\mu(r)}$$

Thermodynamical consistency

$$\frac{1}{3}\langle T^{ii}\rangle = \rho^2 \frac{\partial}{\partial\rho}\left(\frac{\epsilon}{\rho}\right)$$

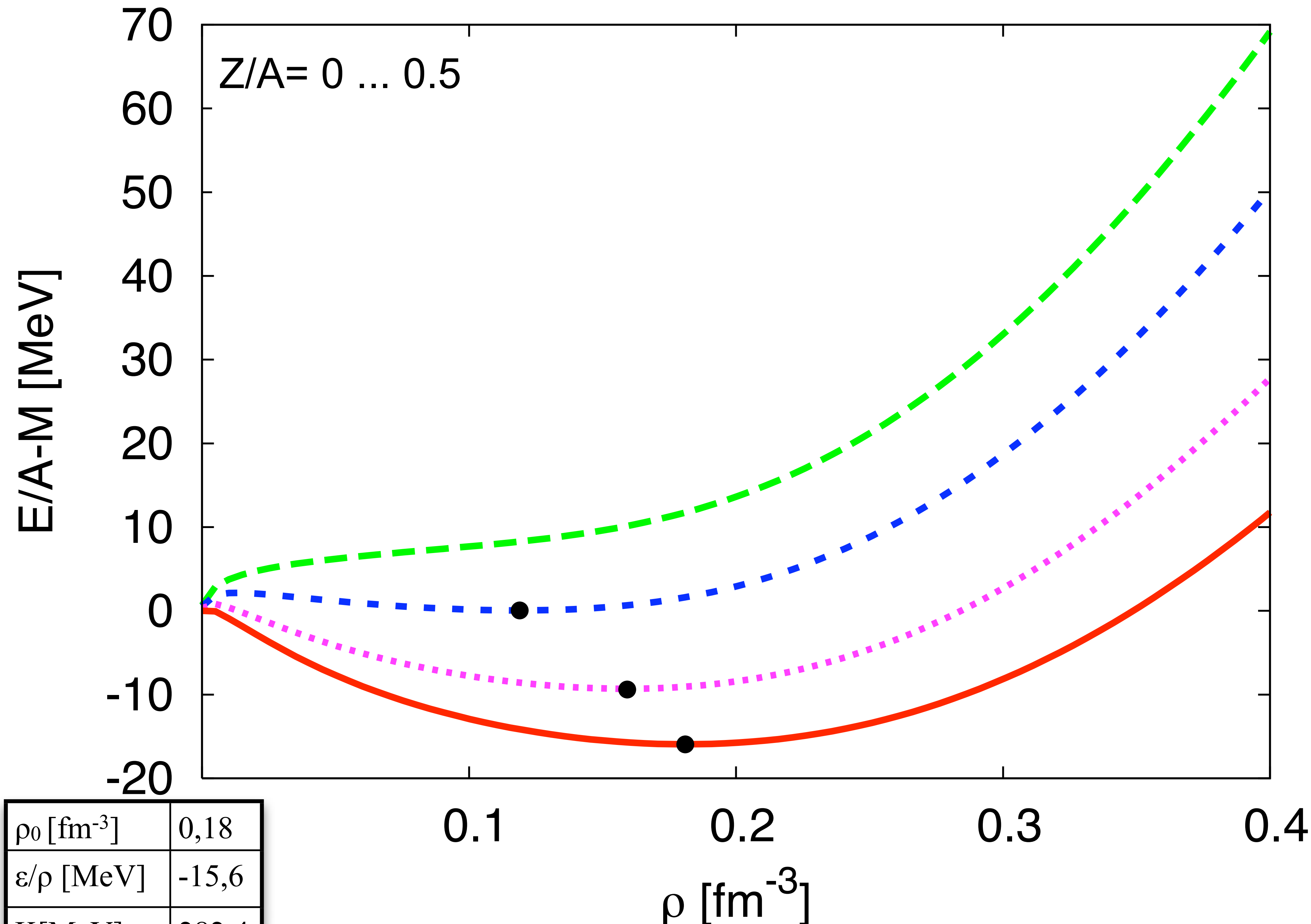
Inclusion of the isovector-scalar interaction leads to different *effective Masses* between *protons* and *neutrons*.

$$M_b^* = M_b - \Gamma_\sigma \Phi_\sigma - \tau_b \Gamma_\delta \Phi_\delta$$



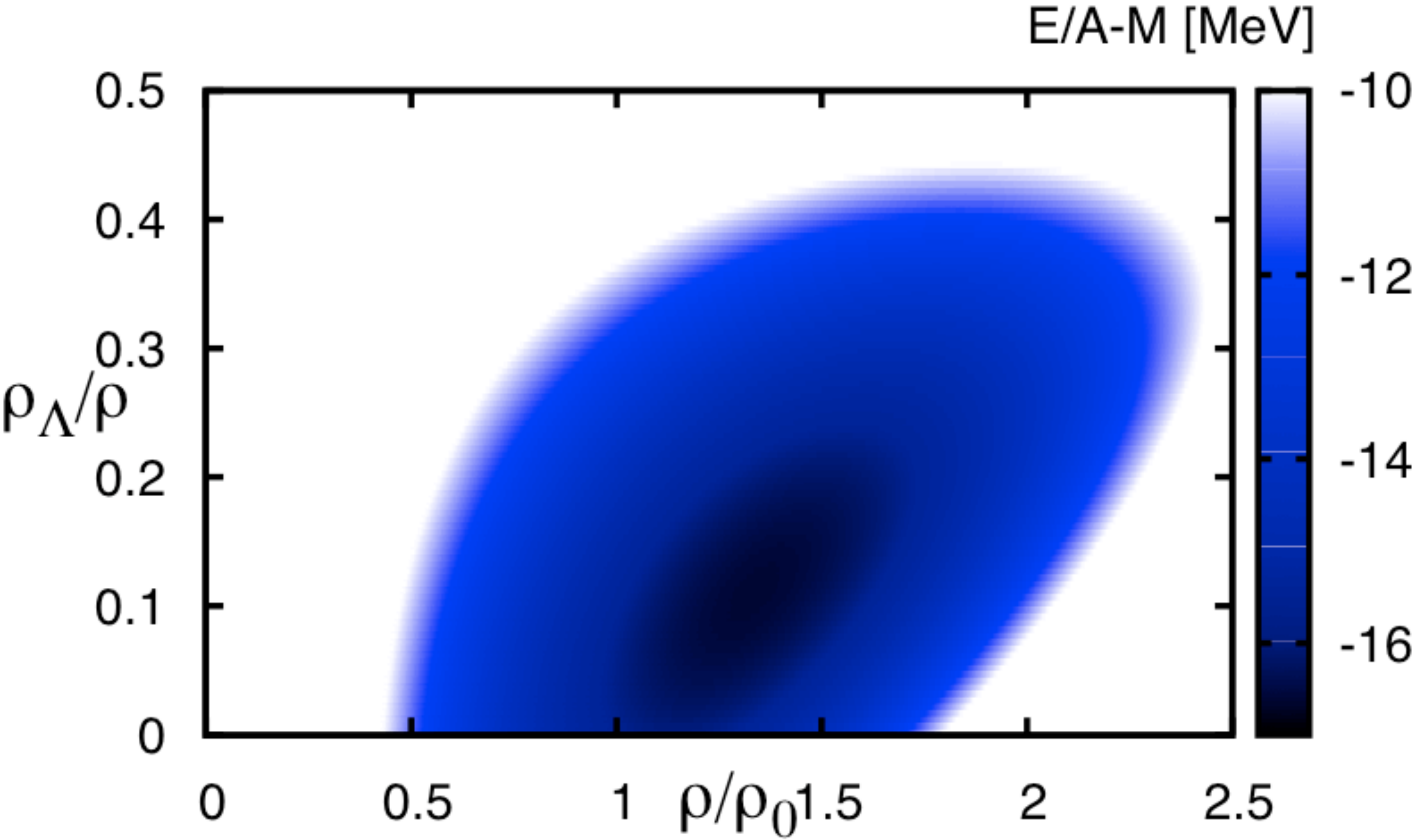
Andreas Fedoseew, Universität Giessen

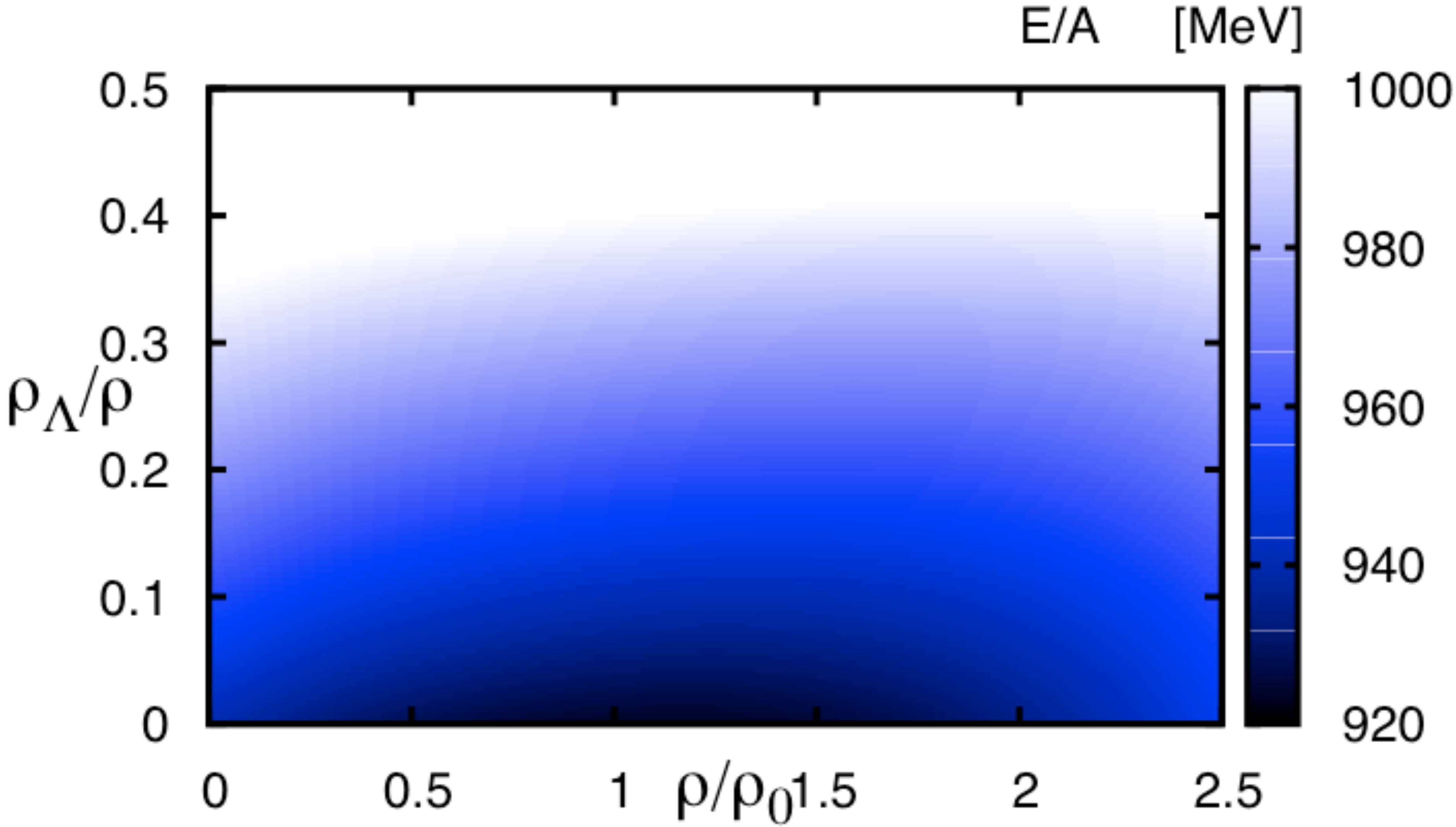
Equation of State



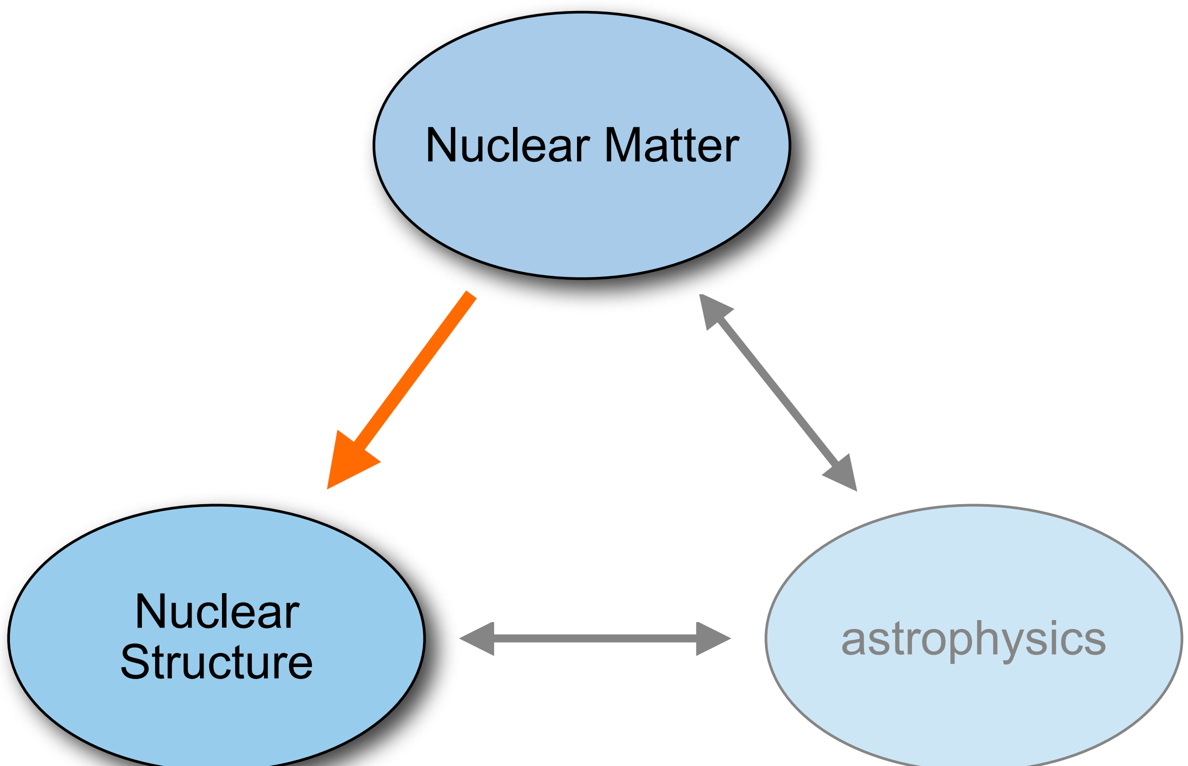
ρ_0 [fm^{-3}]	0,18
ε/ρ [MeV]	-15,6
K[MeV]	282,4

Andreas Fedoseew, Universität Giessen


E/A-M [MeV]
0.5
0.4
0.3
0.2
0.1
0
ρ_Λ/ρ
0
0.5
1
1.5
2
2.5
ρ/ρ_0
-10
-12
-14
-16

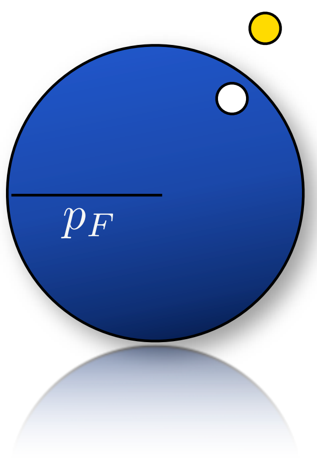

E/A [MeV]
0.5
0.4
0.3
0.2
0.1
0
ρΛ/ρ
0
0.5
1
ρ/ρ0
1.5
2
2.5
1000
980
960
940
920

Dynamical Properties

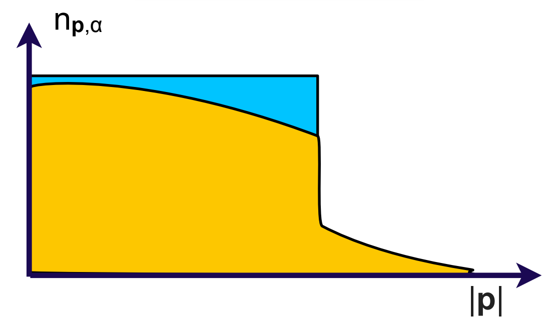


- Achieve a better understanding of dynamical effects in n.m.
  - investigate contributions from density dependent couplings

$$\delta n_{\vec{p},\alpha} = n_{\vec{p},\alpha} - n^0_{\vec{p},\alpha}$$



- Extend the approach of Landau's **Fermi-liquid theory** to relativistic density functionals.
  - Retain the field theoretical structure
  - derive the interaction functional by second variation with respect to the field operators

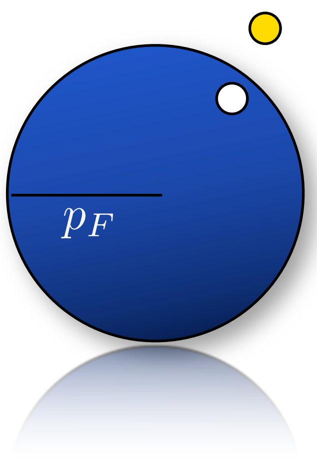


- Quasi-Particle Residual Interaction

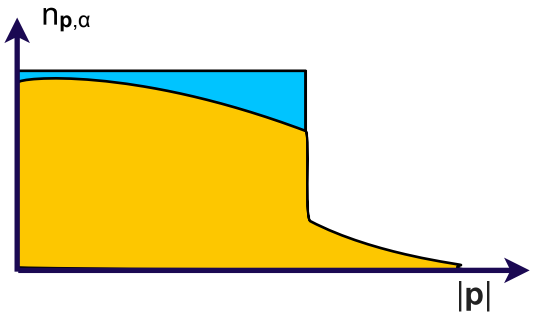
$$\delta E = \sum_k \frac{\delta E}{\delta n_k} \delta n_k + \frac{1}{2} \sum_{k_1,k_2} \frac{\delta^2 E}{\delta n_{k_1} \delta n_{k_2}} \delta n_{k_1} \delta n_{k_2}$$

- Achieve a better understanding of dynamical effects in n.m.
  - investigate contributions from density dependent couplings

$$\delta n_{\vec{p},\alpha} = n_{\vec{p},\alpha} - n^{0}_{\vec{p},\alpha}$$



- Extend the approach of Landau's **Fermi-liquid theory** to relativistic density functionals.
  - Retain the field theoretical structure
  - derive the interaction functional by second variation with respect to the field operators



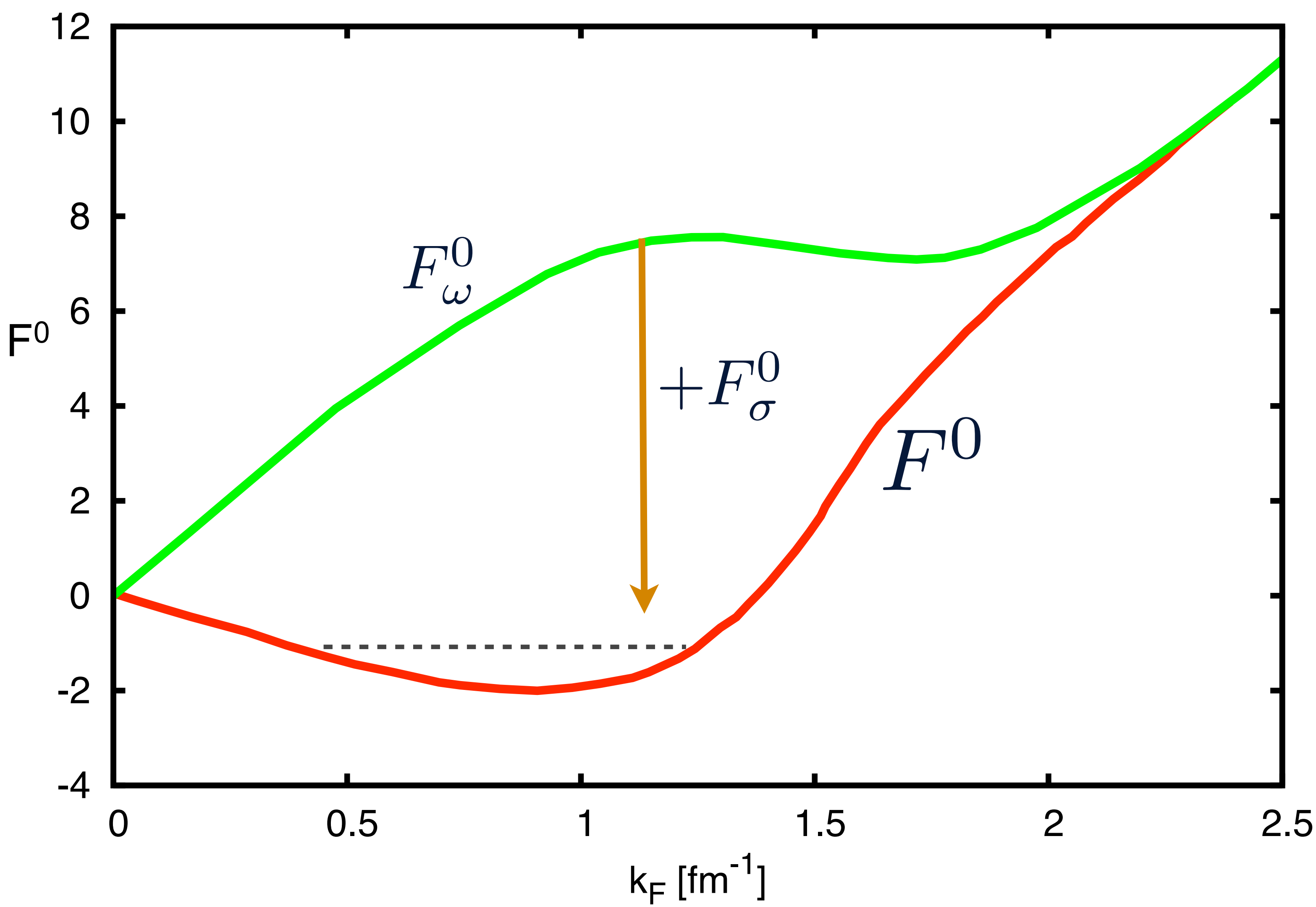
- Quasi-Particle Residual Interaction

$$\delta E = \sum_{k} \frac{\delta E}{\delta n_k} \delta n_k + \frac{1}{2} \sum_{k_1,k_2} \frac{\delta^2 E}{\delta n_{k_1} \delta n_{k_2}} \delta n_{k_1} \delta n_{k_2}$$

$$f_{k_1,k_2} = \frac{\delta^2 E[n_k]}{\delta n_{k_1} \delta n_{k_2}} = \sum_{l} f_l P_l(\cos\theta)$$

Simplest Case - Walecka Model

The contribution of the ω and σ meson to F^0



- Retain the full field theoretical structure until to the very end

$$\hat{\mathcal{F}}^{\alpha\beta} = \frac{\delta^2 E[\Phi, \bar{\Psi}\Psi]}{\delta\hat{\rho}_\alpha \delta\hat{\rho}_\beta} = \hat{\mathcal{F}}_0^{\alpha\beta} + \hat{\mathcal{F}}^{(r)\alpha\beta}$$

Variation with respect to the fields

$$\delta\Phi_m = D_m \hat{\Gamma}_m \delta\hat{\rho} + \hat{\rho} D_m \delta\hat{\Gamma}_m$$

- Retain the full field theoretical structure until to the very end

$$\hat{\mathcal{F}}^{\alpha\beta} = \frac{\delta^2 E[\Phi, \bar{\Psi}\Psi]}{\delta\hat{\rho}_\alpha \delta\hat{\rho}_\beta} = \hat{\mathcal{F}}_0^{\alpha\beta} + \hat{\mathcal{F}}^{(r)\alpha\beta}$$

Variation with respect to the fields

$$\delta\Phi_m = D_m \hat{\Gamma}_m \delta\hat{\rho} + \hat{\rho} D_m \delta\hat{\Gamma}_m$$

- Evaluate expectation values in the last step of the calculations

- Retain the full field theoretical structure until to the very end

$$\hat{\mathcal{F}}^{\alpha\beta} = \frac{\delta^2 E[\Phi, \bar{\Psi}\Psi]}{\delta\hat{\rho}_\alpha \delta\hat{\rho}_\beta} = \hat{\mathcal{F}}_0^{\alpha\beta} + \hat{\mathcal{F}}^{(r)\alpha\beta}$$

Variation with respect to the fields

$$\delta\Phi_m = D_m \hat{\Gamma}_m \delta\hat{\rho} + \hat{\rho} D_m \delta\hat{\Gamma}_m$$

- Evaluate expectation values in the last step of the calculations
- Expand the interaction amplitudes around the ground state expectation value

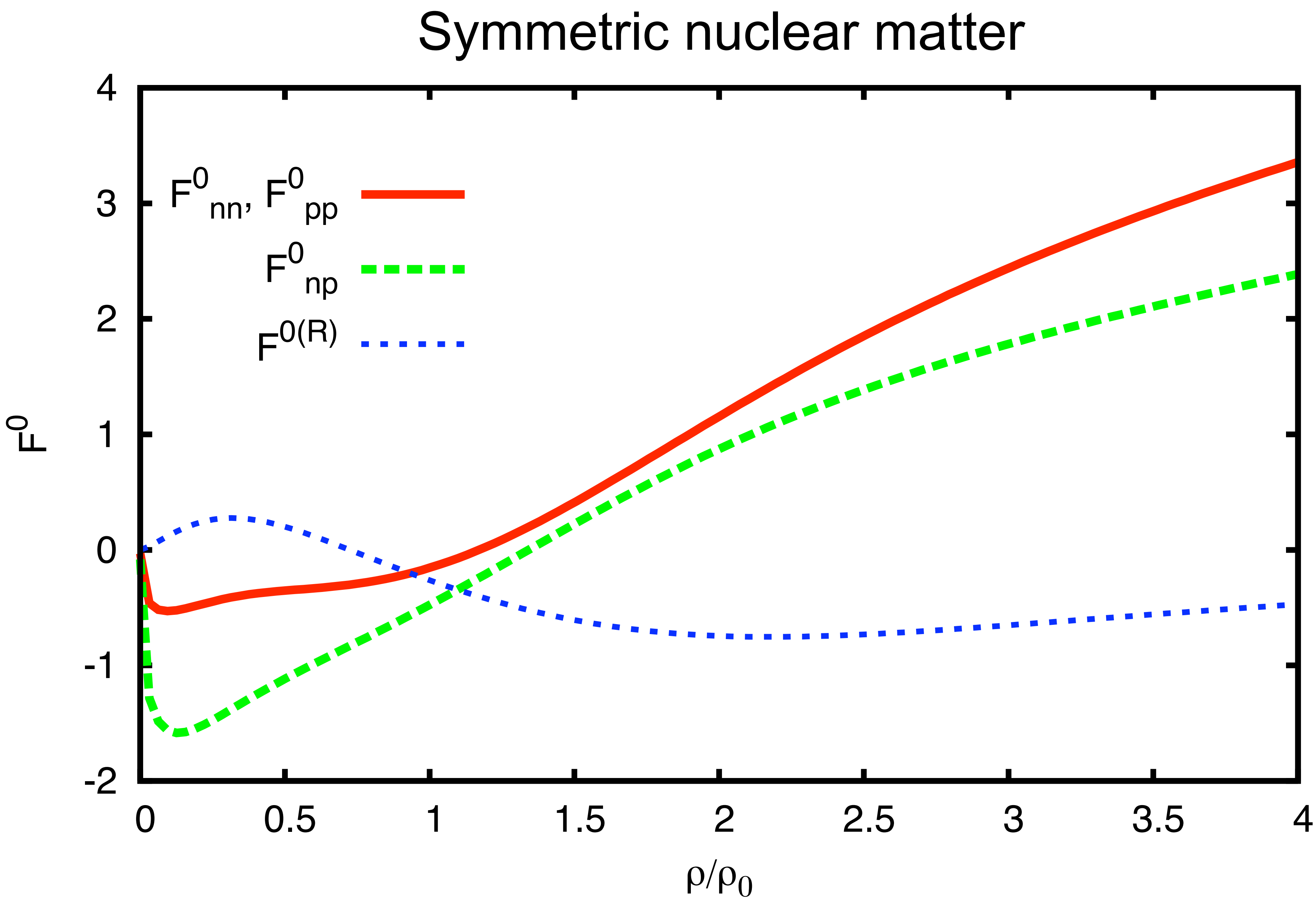
- Retain the full field theoretical structure until to the very end

$$\hat{\mathcal{F}}^{\alpha\beta} = \frac{\delta^2 E[\Phi, \bar{\Psi}\Psi]}{\delta\hat{\rho}_\alpha \delta\hat{\rho}_\beta} = \hat{\mathcal{F}}_0^{\alpha\beta} + \hat{\mathcal{F}}^{(r)\alpha\beta}$$

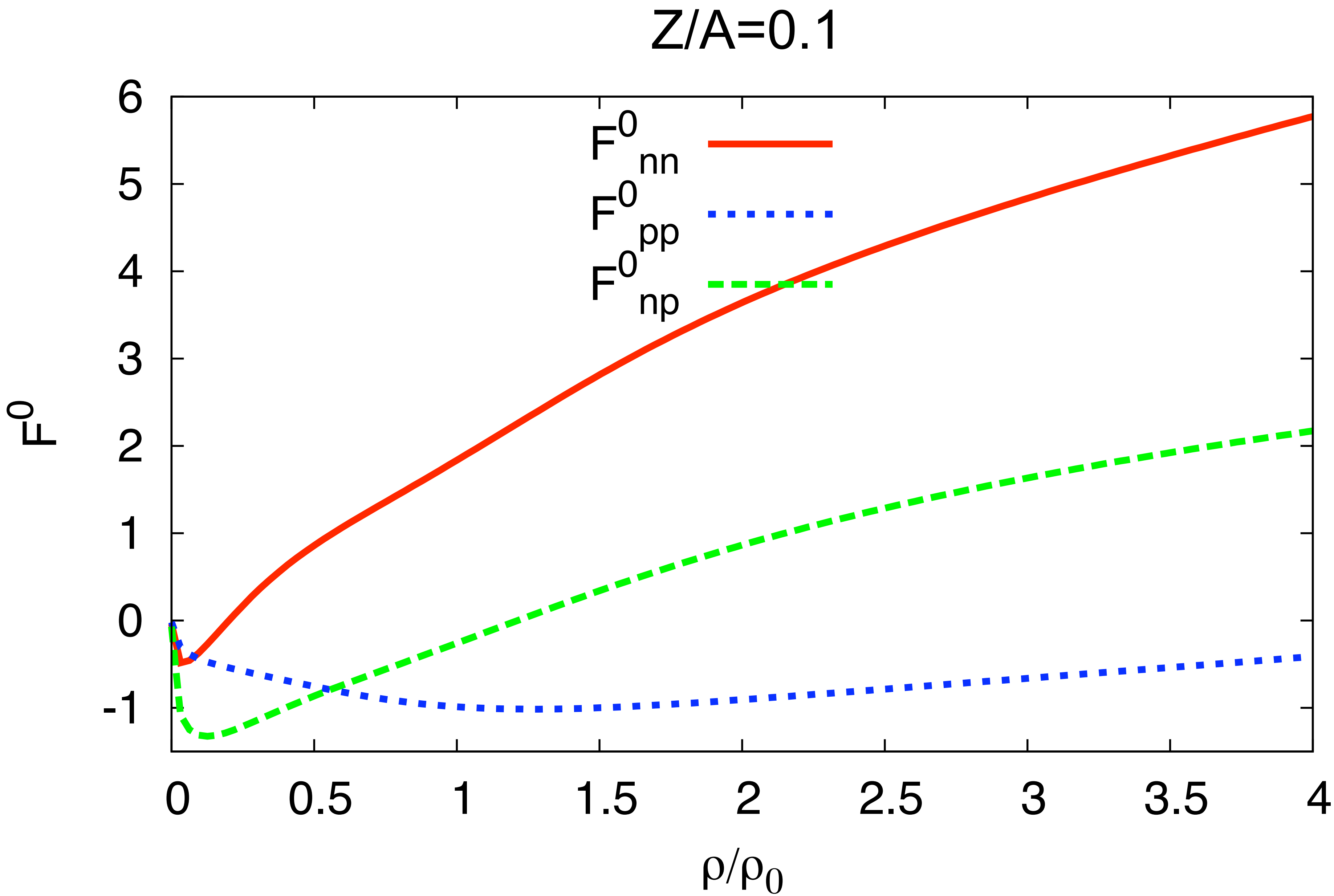
Variation with respect to the fields

$$\delta\Phi_m = D_m \hat{\Gamma}_m \delta\hat{\rho} + \hat{\rho} D_m \delta\hat{\Gamma}_m$$

- Evaluate expectation values in the last step of the calculations
- Expand the interaction amplitudes around the ground state expectation value
- Density dependent vertex functionals

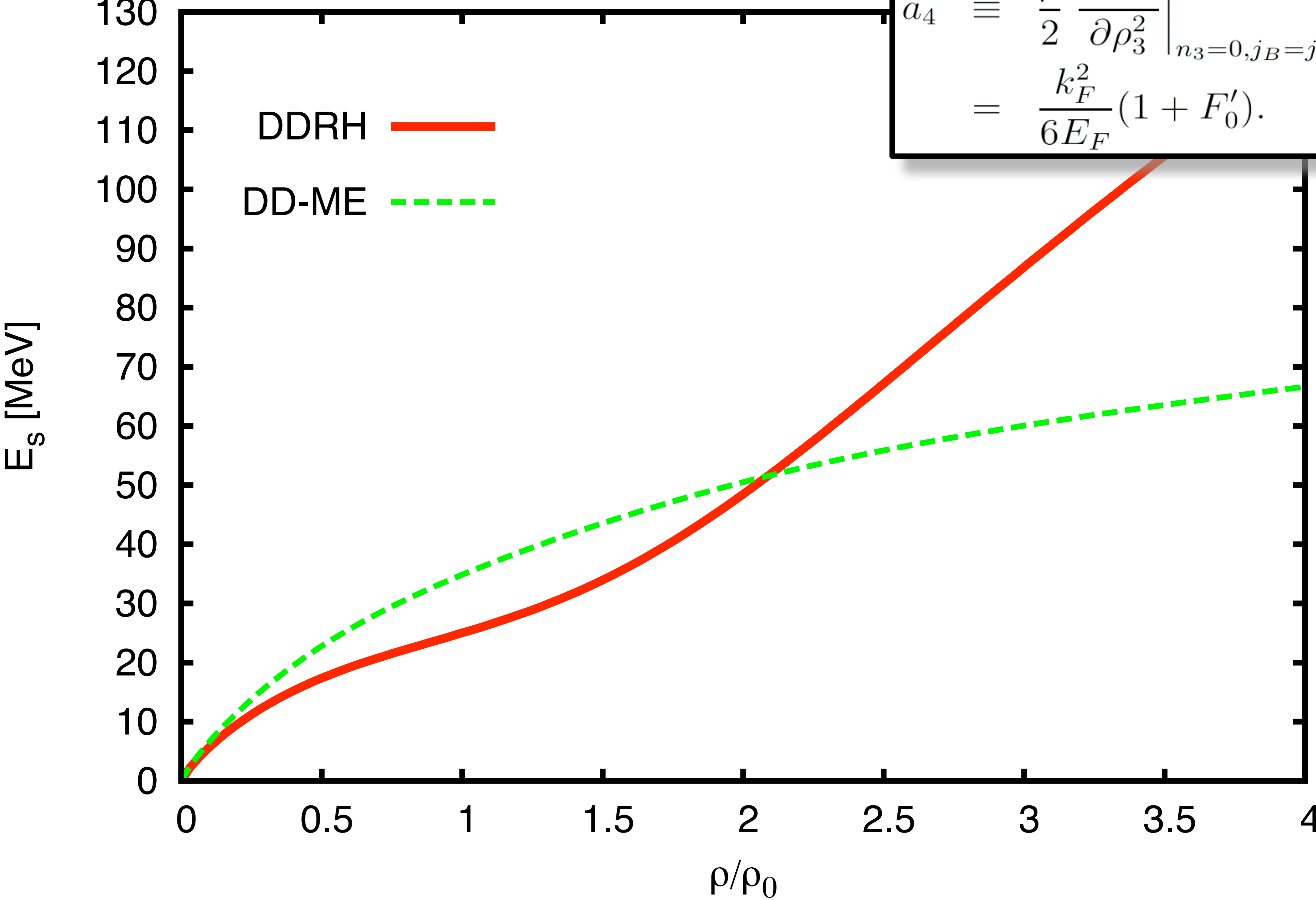

Symmetric nuclear matter
F0nn, F0pp
F0np
F0(R)
F0
ρ/ρ0
4
3
2
1
0
-1
-2
0
0.5
1
1.5
2
2.5
3
3.5
4

Landau Migdal Parameters



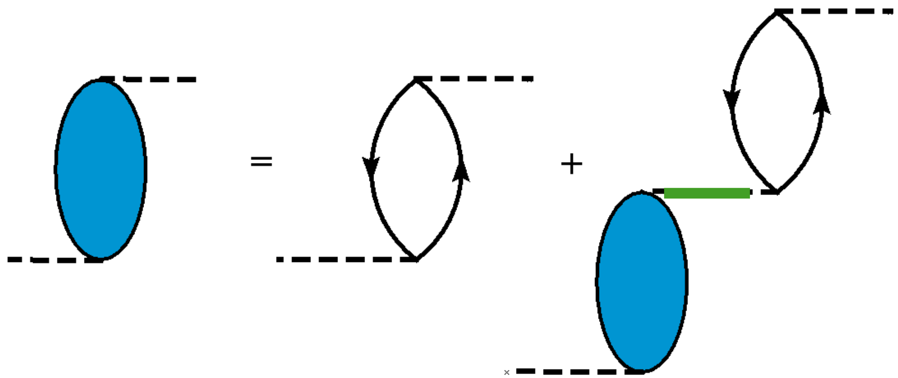
Symmetry Energy

$$a_4 \equiv \frac{\rho}{2} \frac{\partial^2 E'}{\partial \rho_3^2}\Big|_{n_3=0, j_B=j_3=0} = \frac{k_F^2}{6E_F}(1+F_0').$$



Nuclear Response

- Calculation of the response functions using RRPA

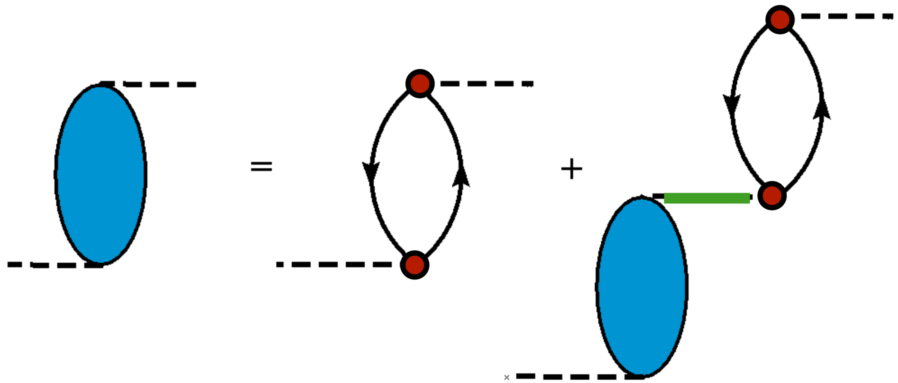


$$\Pi_{AB} = \Pi^0_{AB} + \Pi^0_{AC} V_{CD} \Pi_{DB}$$

$$\Pi^0_{A,B} = \frac{1}{i} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} Tr \left[\Gamma_A G_0(p+q) \Gamma_B G_0(p) \right]$$

- $\Pi = \Pi_D + \Pi_F$
  - Π_D : ph + part of $N\bar{N}$ excitations
  - Π_F : vacuum polarization
- In Medium Correlations
  - QP interaction
  - Meson mixing

- Calculation of the response functions using RRPA



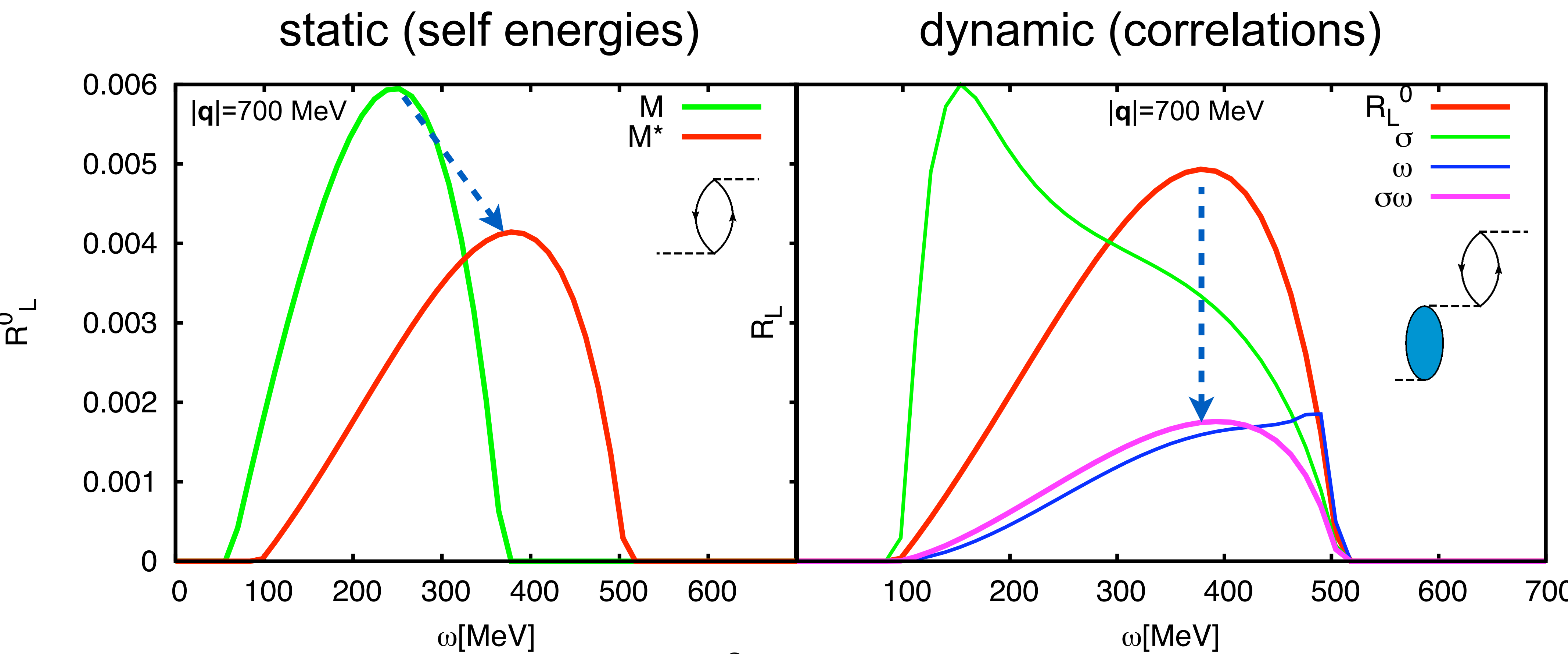
$$\Pi_{AB} = \Pi^0_{AB} + \Pi^0_{AC} V_{CD} \Pi_{DB}$$

$$\Pi^0_{A,B} = \frac{1}{i} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} Tr\left[\Gamma_A G_0(p+q) \Gamma_B G_0(p)\right]$$

- $\Pi = \Pi_D + \Pi_F$
  - Π_D : ph + part of $N\bar{N}$ excitations
  - Π_F : vacuum polarization
- In Medium Correlations
  - QP interaction
  - Meson mixing
- Beyond RPA:
  - Calculation with **dressed vertices** from **ab-initio** calculations

$$R_{L/T} \sim -\Im\Pi_{L/T}$$

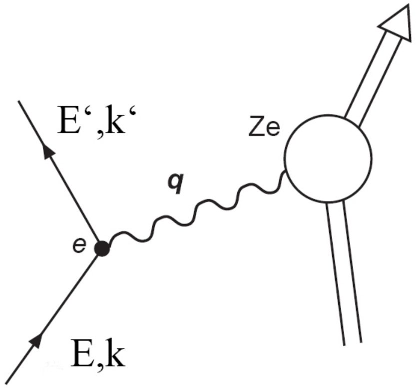
Medium Effects



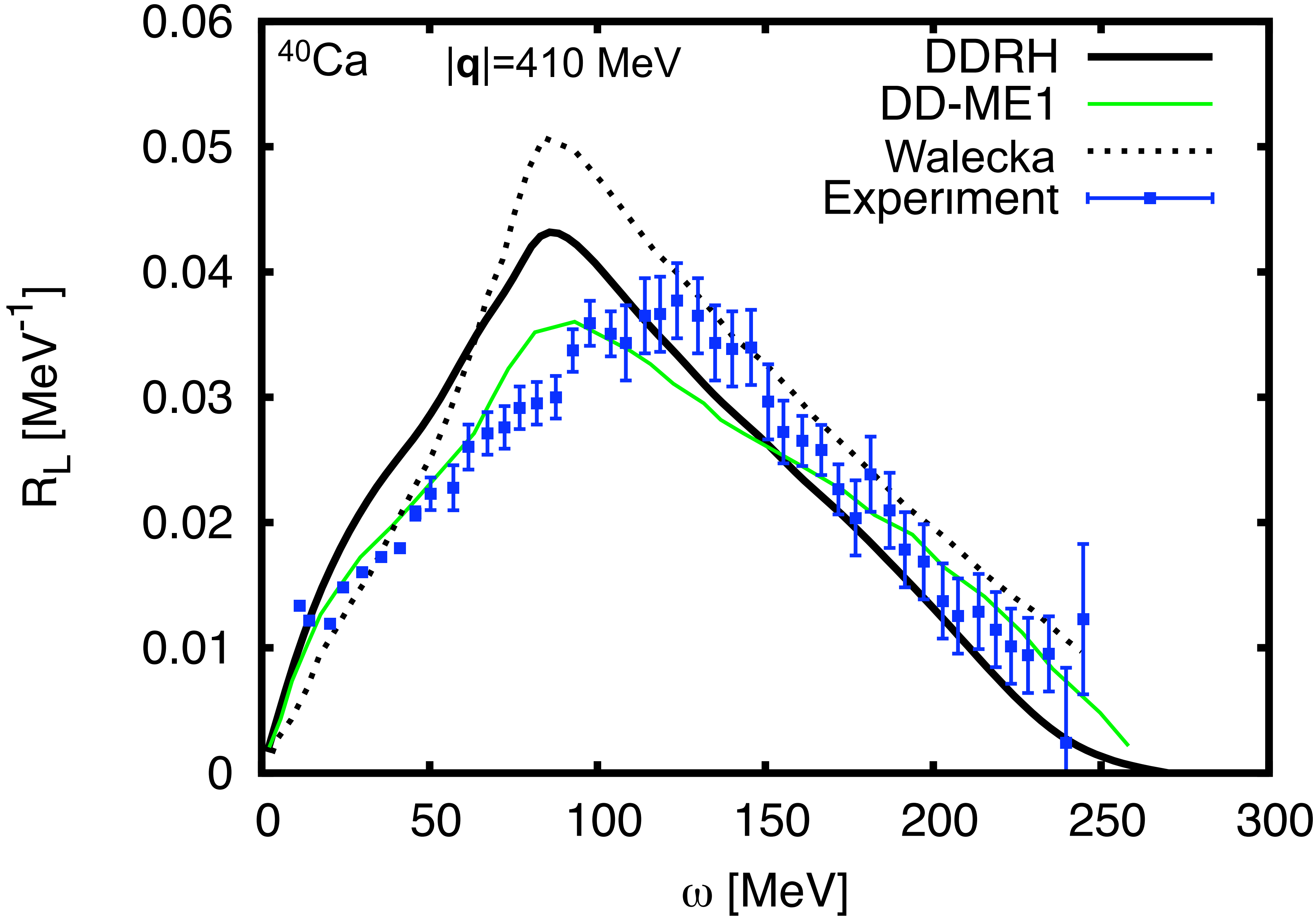
$$\omega_{max} \approx \frac{\mathbf{q}^2}{2M_N^*} \quad \Gamma \approx |\mathbf{q}| k_F / M_N^*$$

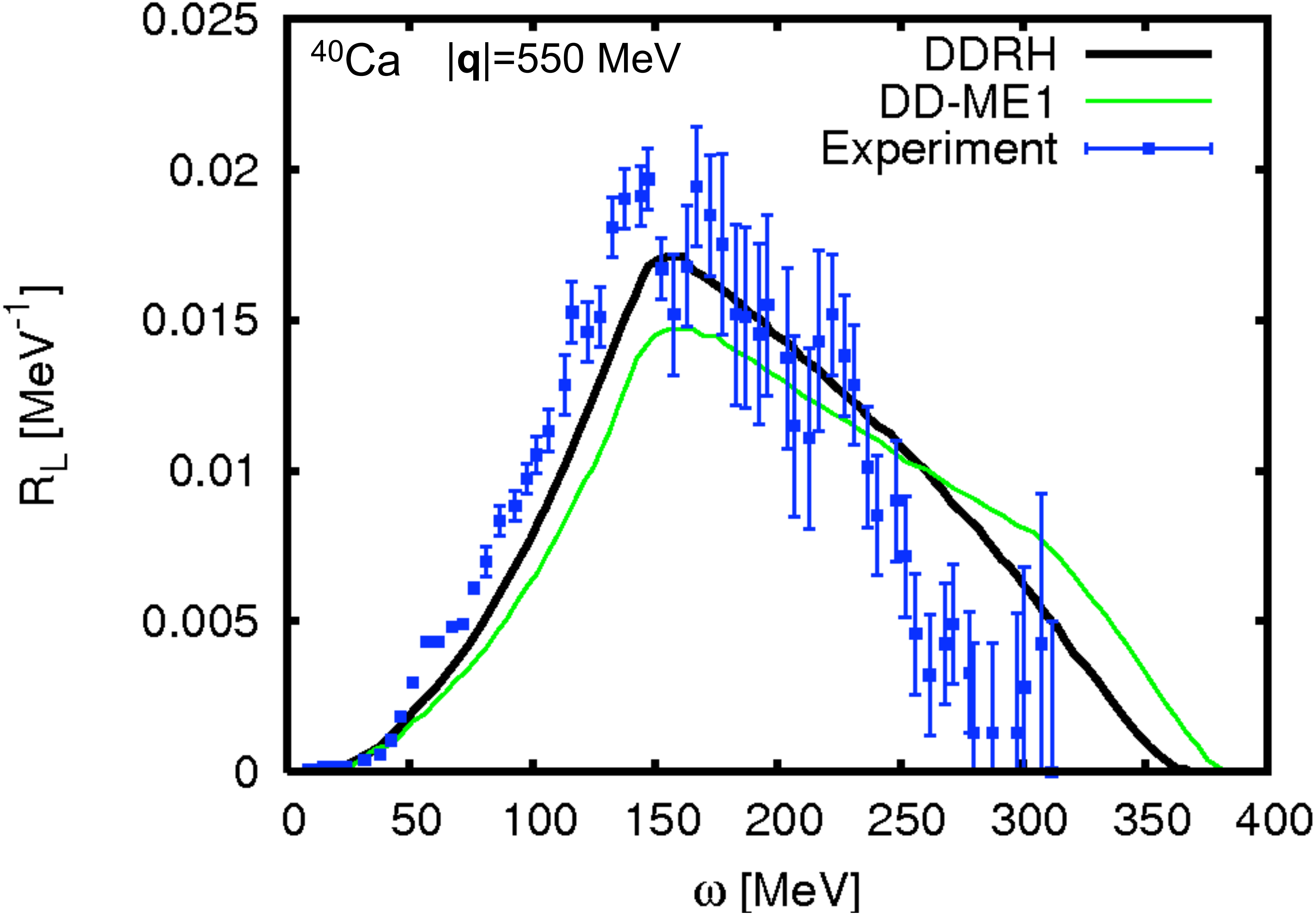
Andreas Fedoseew, Universität Giessen

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_{\mathrm{M}} \left[\left(\frac{Q^2}{q^2} \right) R_L(q,\omega) + \left(\frac{Q^2}{2q^2} + \tan^2 \frac{\theta}{2} \right) R_T(q,\omega) \right]$$

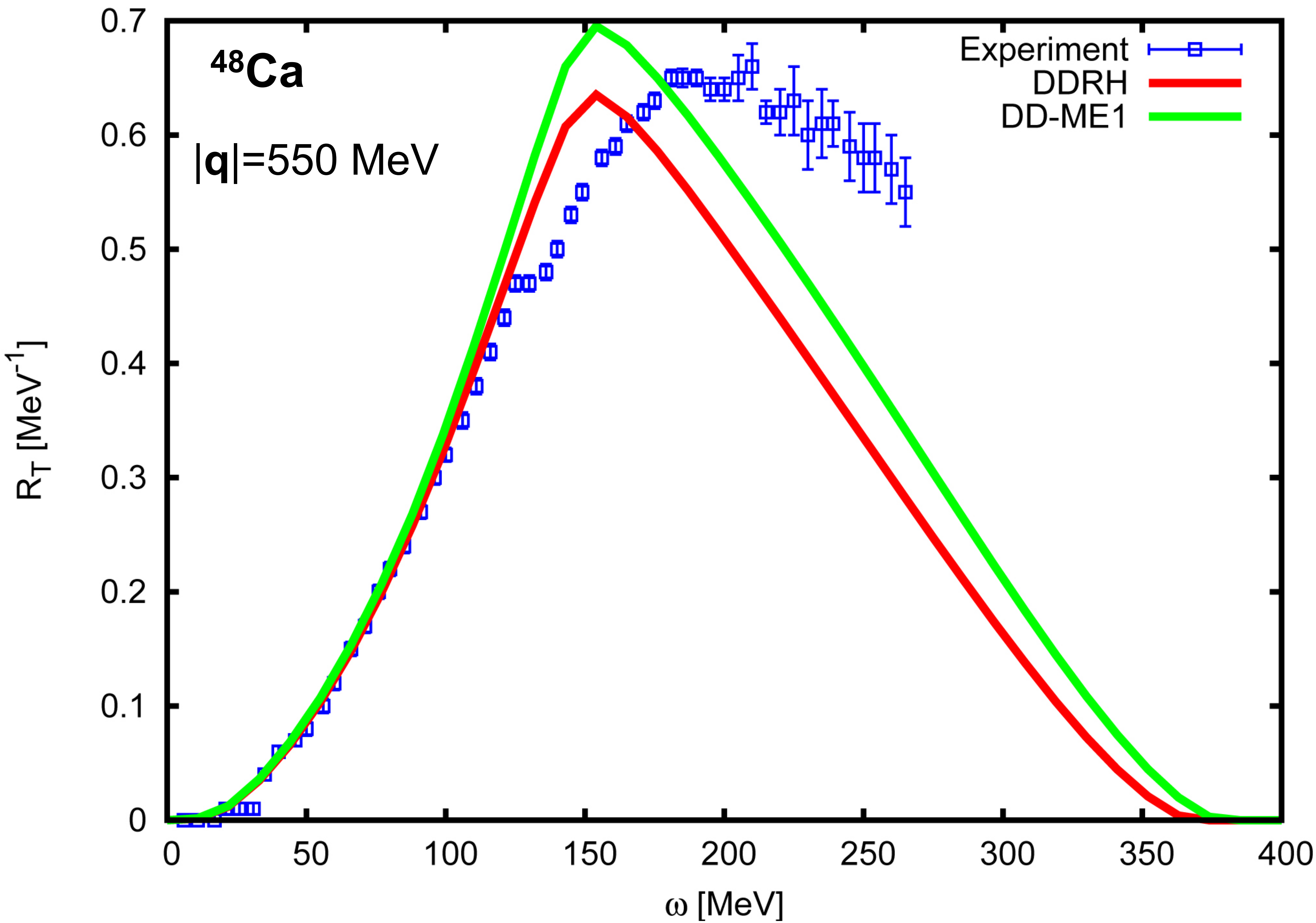


- Local Density Approximation
- Self-consistent relativistic Thomas Fermi calculations used for the density distribution
- Include Nucleon Form Factors


0.06
0.05
0.04
0.03
0.02
0.01
0
R_L [MeV^{-1}]
0
50
100
150
200
250
300
ω [MeV]
^{40}Ca
|**q**|=410 MeV
DDRH
DD-ME1
Walecka
Experiment

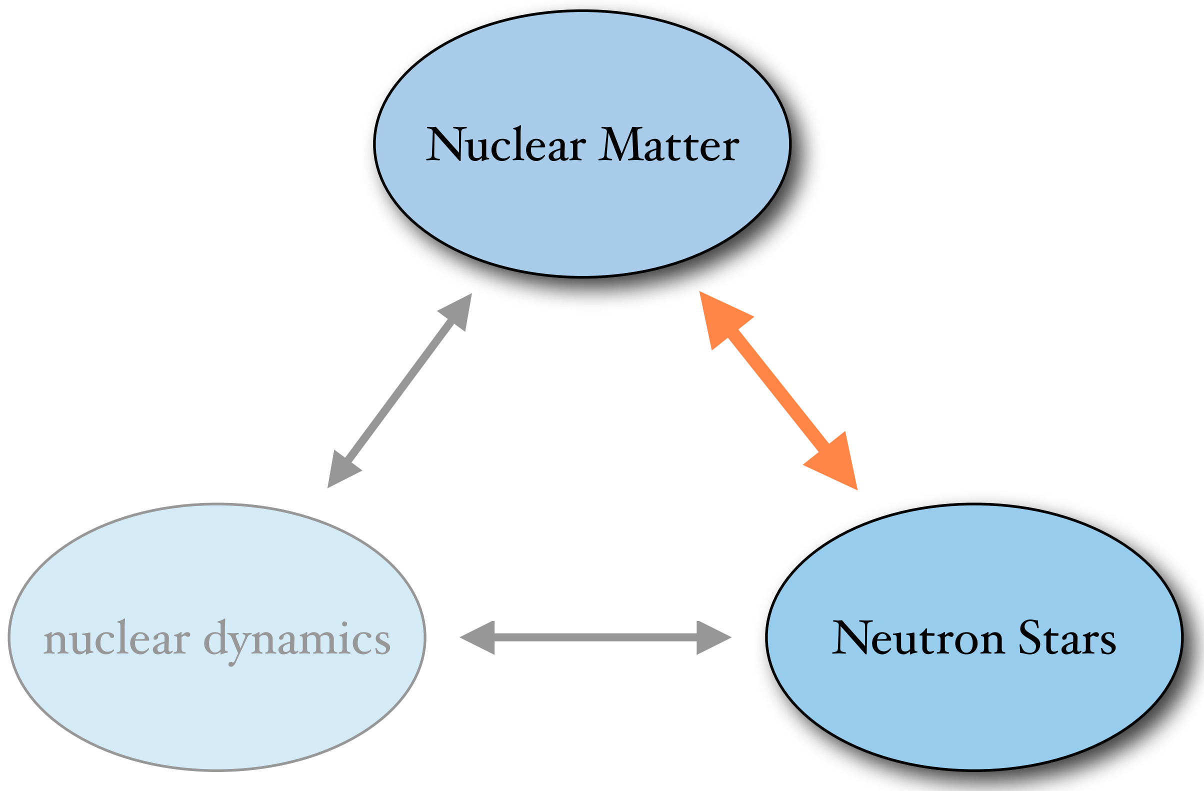

0.025
0.02
0.015
0.01
0.005
0
^{40}Ca |**q**|=550 MeV
DDRH
DD-ME1
Experiment
R_L [MeV^{-1}]
0
50
100
150
200
250
300
350
400
ω [MeV]

Results



^{48}Ca
|q|=550 MeV
Experiment
DDRH
DD-ME1
R_T [MeV^{-1}]
ω [MeV]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
50
100
150
200
250
300
350
400

Strange Nuclear Matter



- Natural astrophysical laboratories to study the behavior of the EoS at high density.
- Long-lived systems in β equilibrium

Equilibrium conditions:

$$n \leftrightarrow p + e^- + \bar{\nu}_e$$

$$N + N \leftrightarrow N + H + K$$

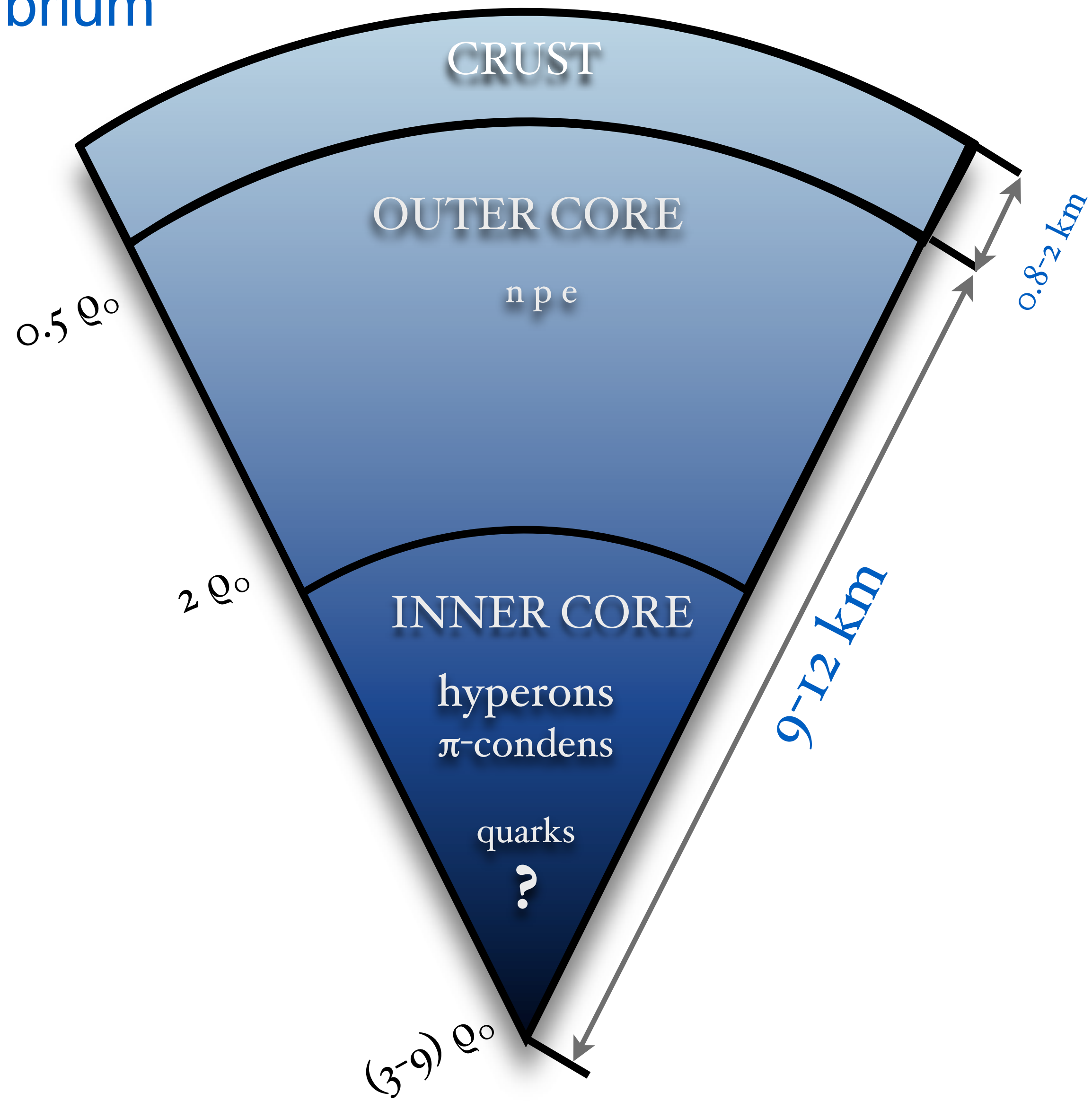
$$\mu = b\mu_N - q\mu_e$$

Baryon number conservation:

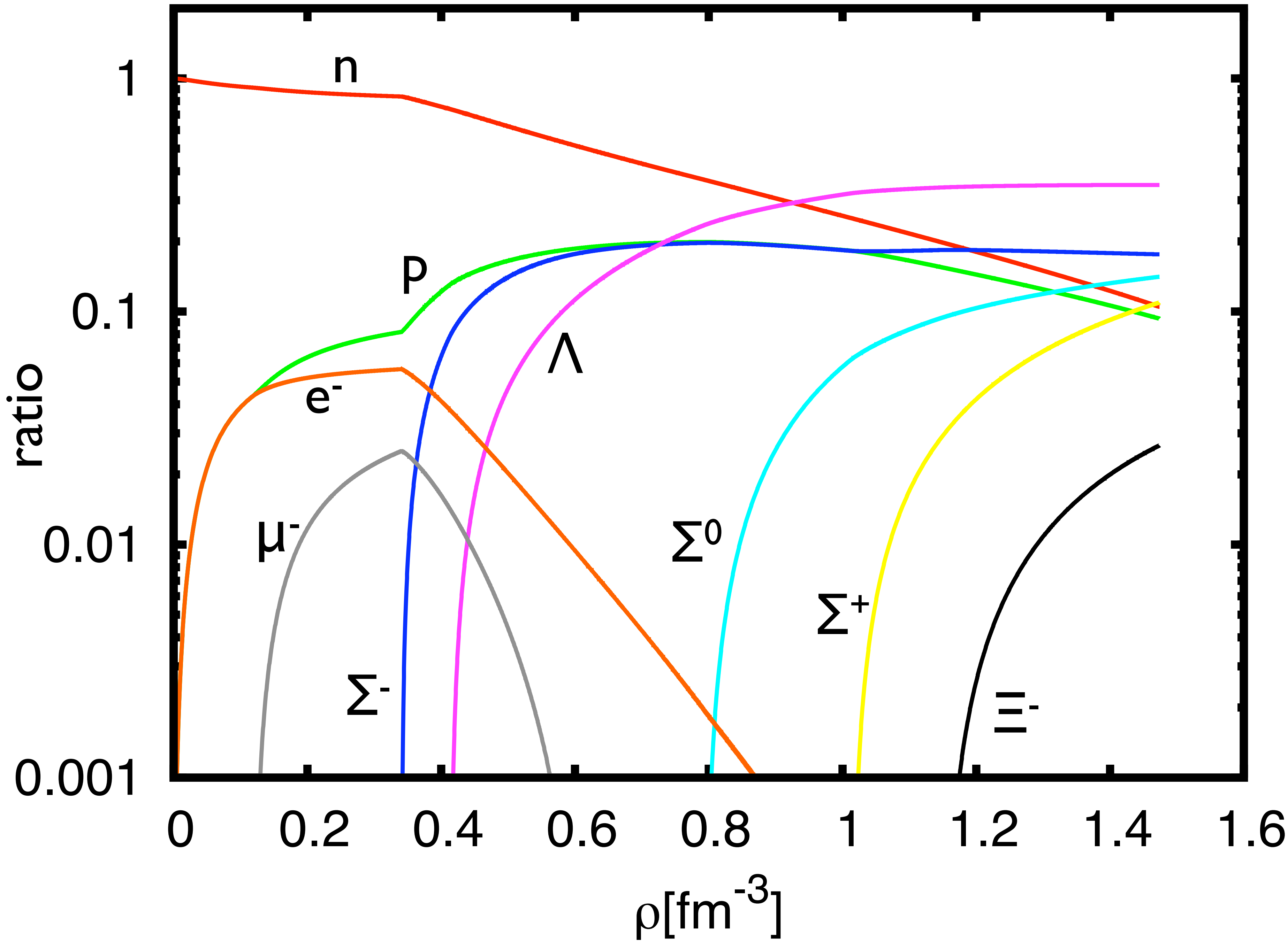
$$\sum_B \rho_B \cdot b_B = const.$$

Charge conservation:

$$\sum_i \rho_i \cdot q_i = 0$$



Particle Ratios

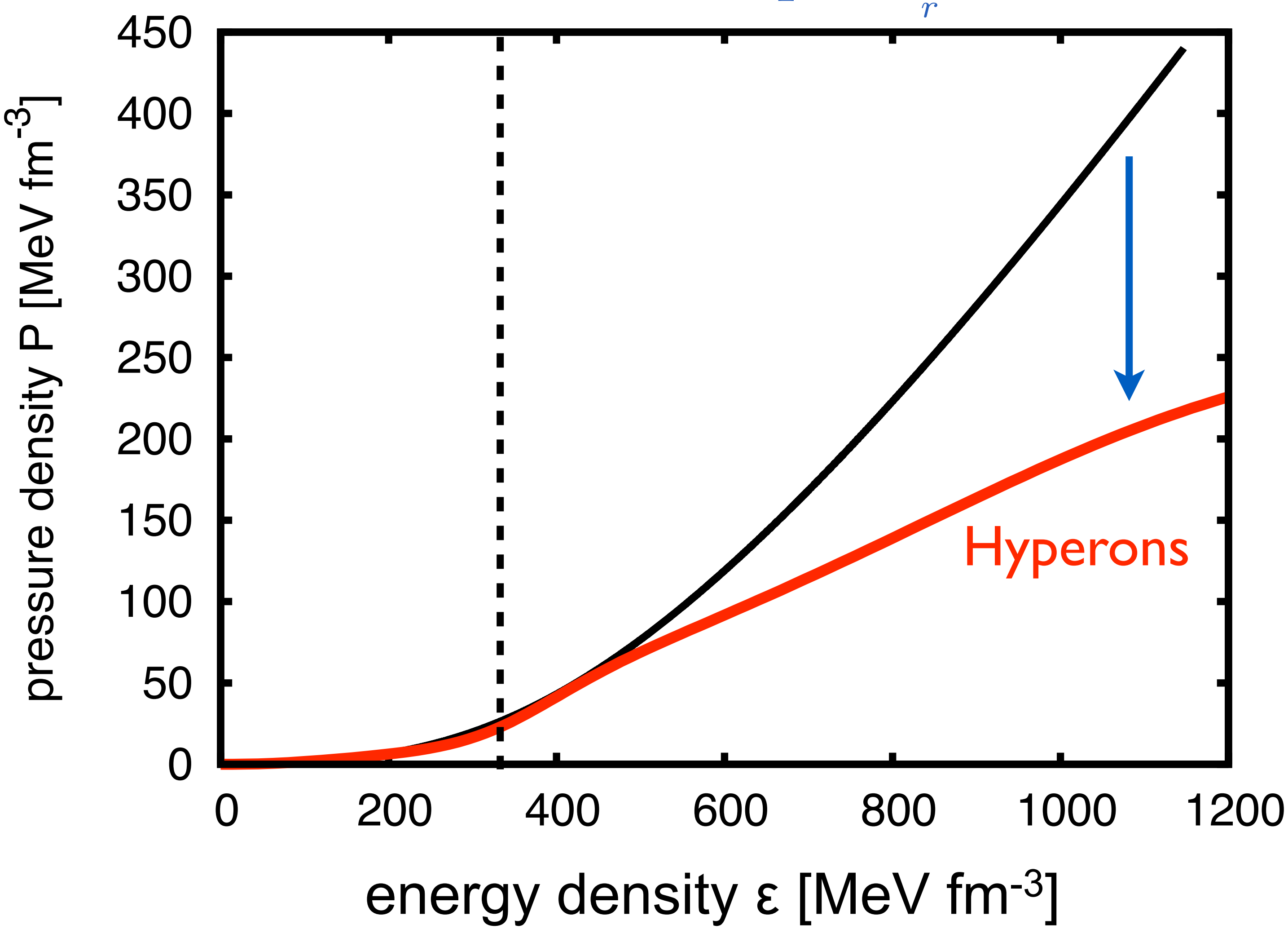


Andreas Fedoseew, Universität Giessen

Tolmann-Oppenheimer-Volkoff Equation

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r)$$

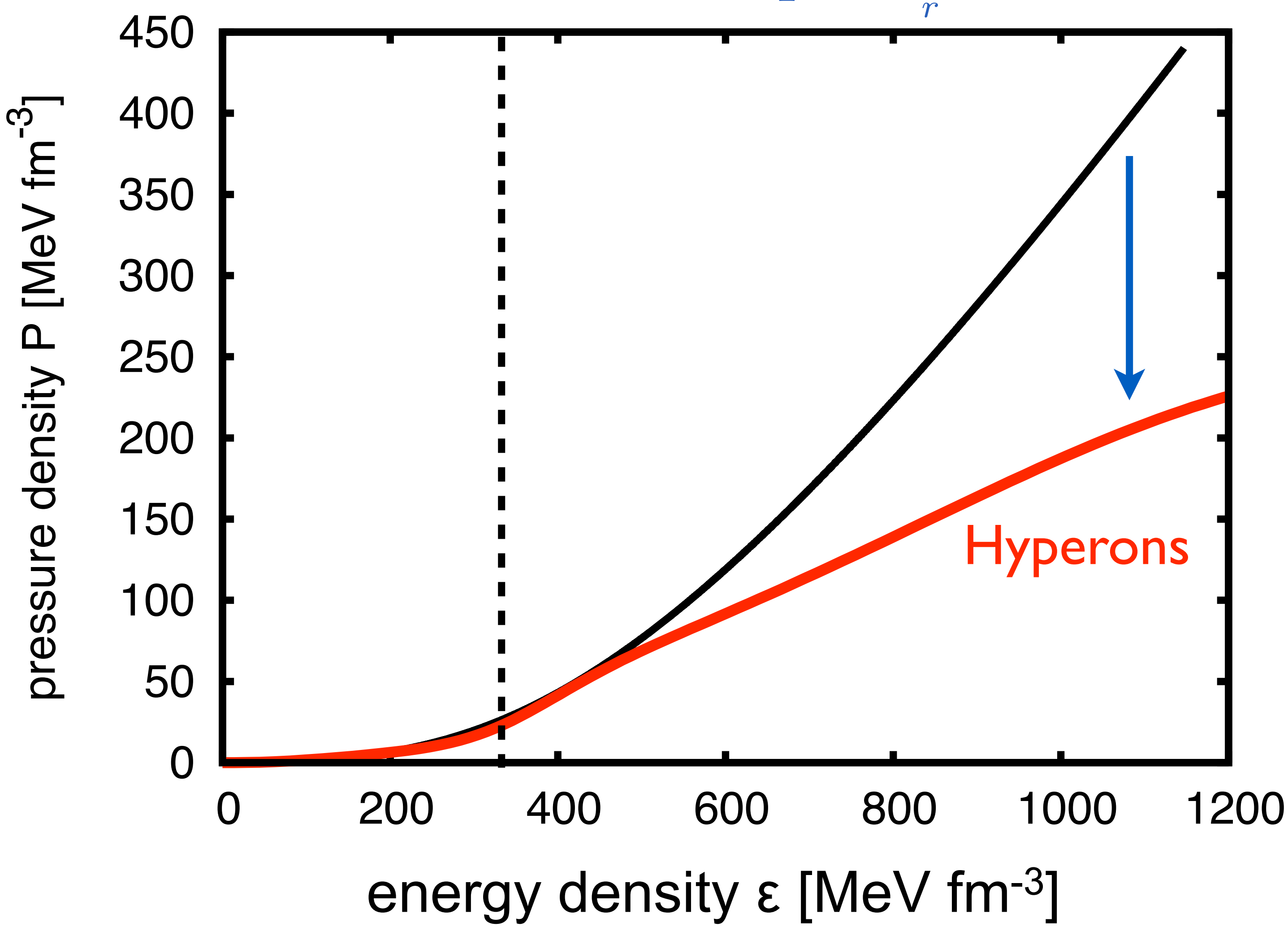
$$\frac{dP(r)}{dr} = -\frac{G}{r^2}\frac{(P(r)+\epsilon(r))(M(r)+4\pi r^3 P(r))}{1-\frac{2GM(r)}{r}}$$



Tolmann-Oppenheimer-Volkoff Equation

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r)$$

$$\frac{dP(r)}{dr} = -\frac{G}{r^2}\frac{(P(r)+\epsilon(r))(M(r)+4\pi r^3 P(r))}{1-\frac{2GM(r)}{r}}$$



Hyperon Couplings

$$\Gamma_{\alpha Y} = R_{\alpha Y} \cdot \Gamma_{\alpha N}$$

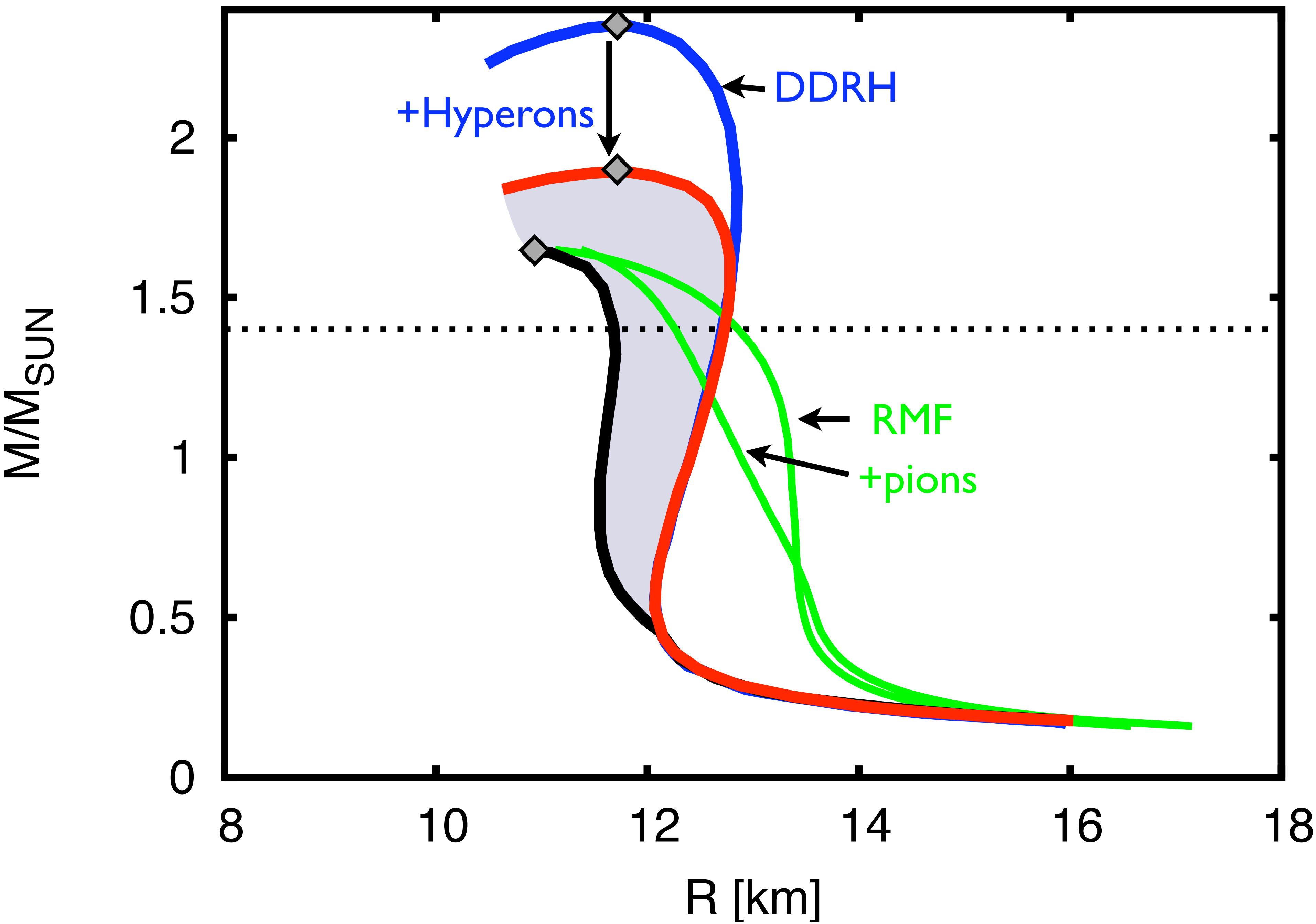
$SU(3)_f$:

$$R_{\sigma\Lambda} = \frac{2}{3}$$

Microscopic Calculations:

$$R_{\sigma\Lambda} = 0.49$$

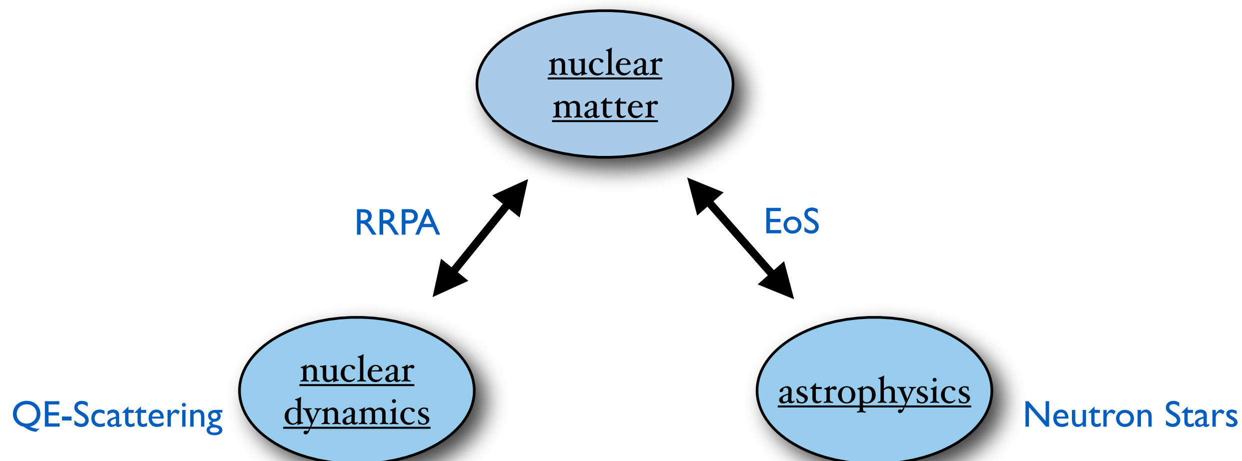
$$R_{\omega\Lambda} = 0.48 \pm 0.3$$



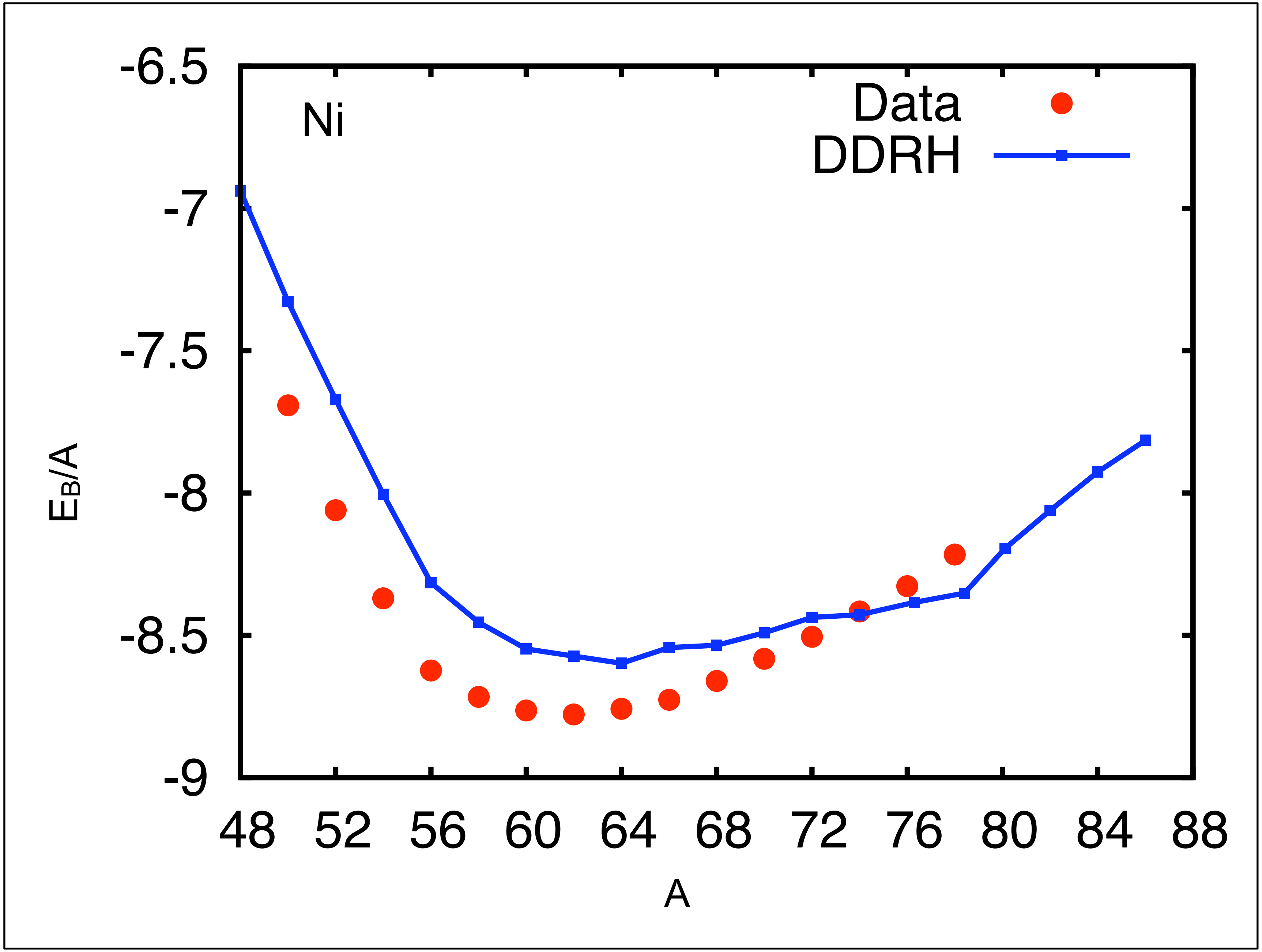
DDRH
+Hyperons
RMF
+pions
2
1.5
1
0.5
0
8
10
12
14
16
18
R [km]
M/M$_{SUN}$

Conclusion

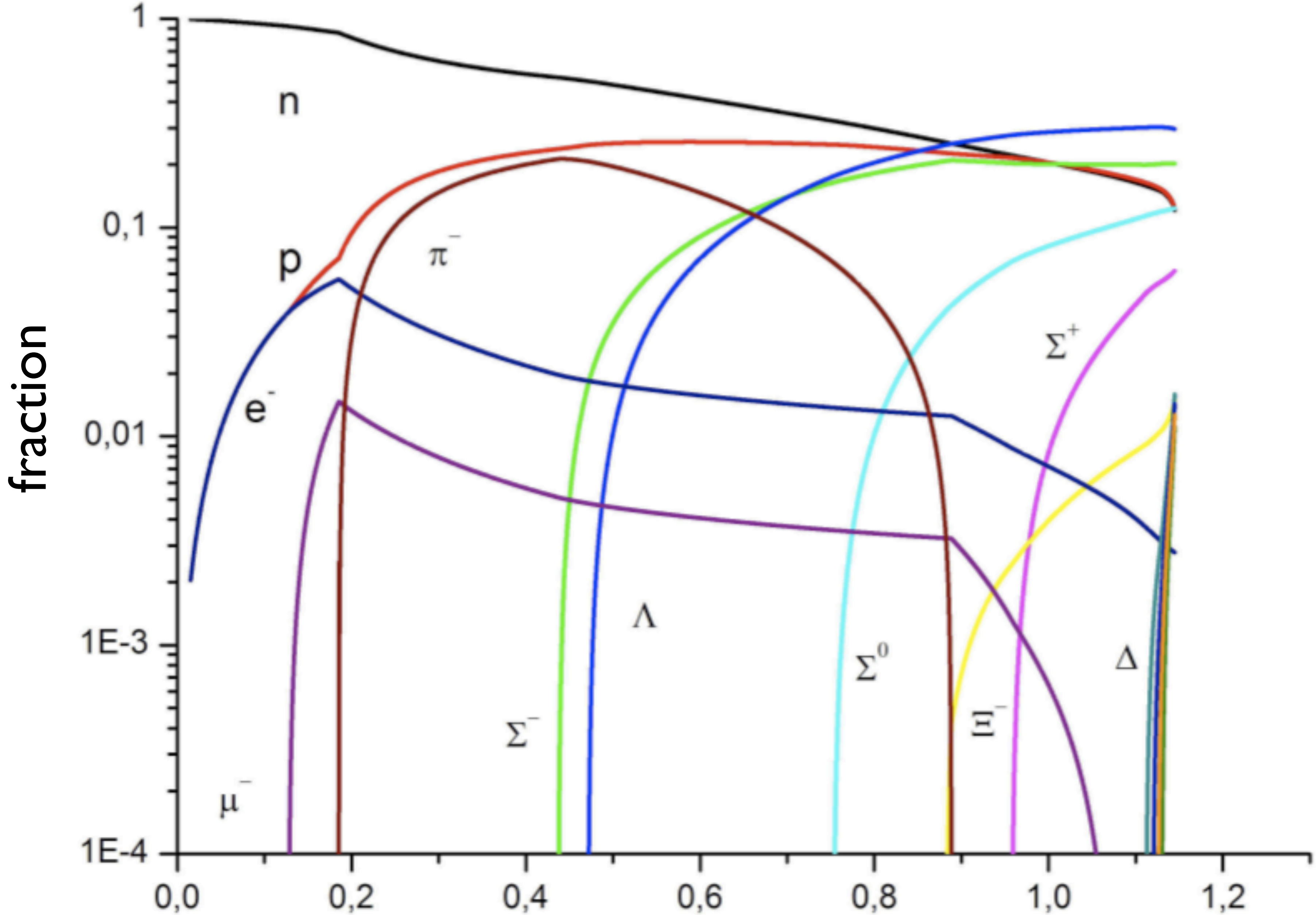
- From Free Space to In-Medium NN-interaction
  - Effective density dependent NN-couplings from ab-initio calculations
  - Rearrangement contributions
- NN Residual Interaction
  - Isospin dependent QP-interaction
  - Consequences for the symmetry energy
- Response Functions
  - RRPA and beyond
  - Quasi-elastic electron scattering on nuclei sheds more light on the in-medium effects of the NN-interaction
  - Differences between microscopic and phenomenological calculation
- Neutron Stars
  - β-equilibrium
  - Calculation of the M/R relation with hyperons

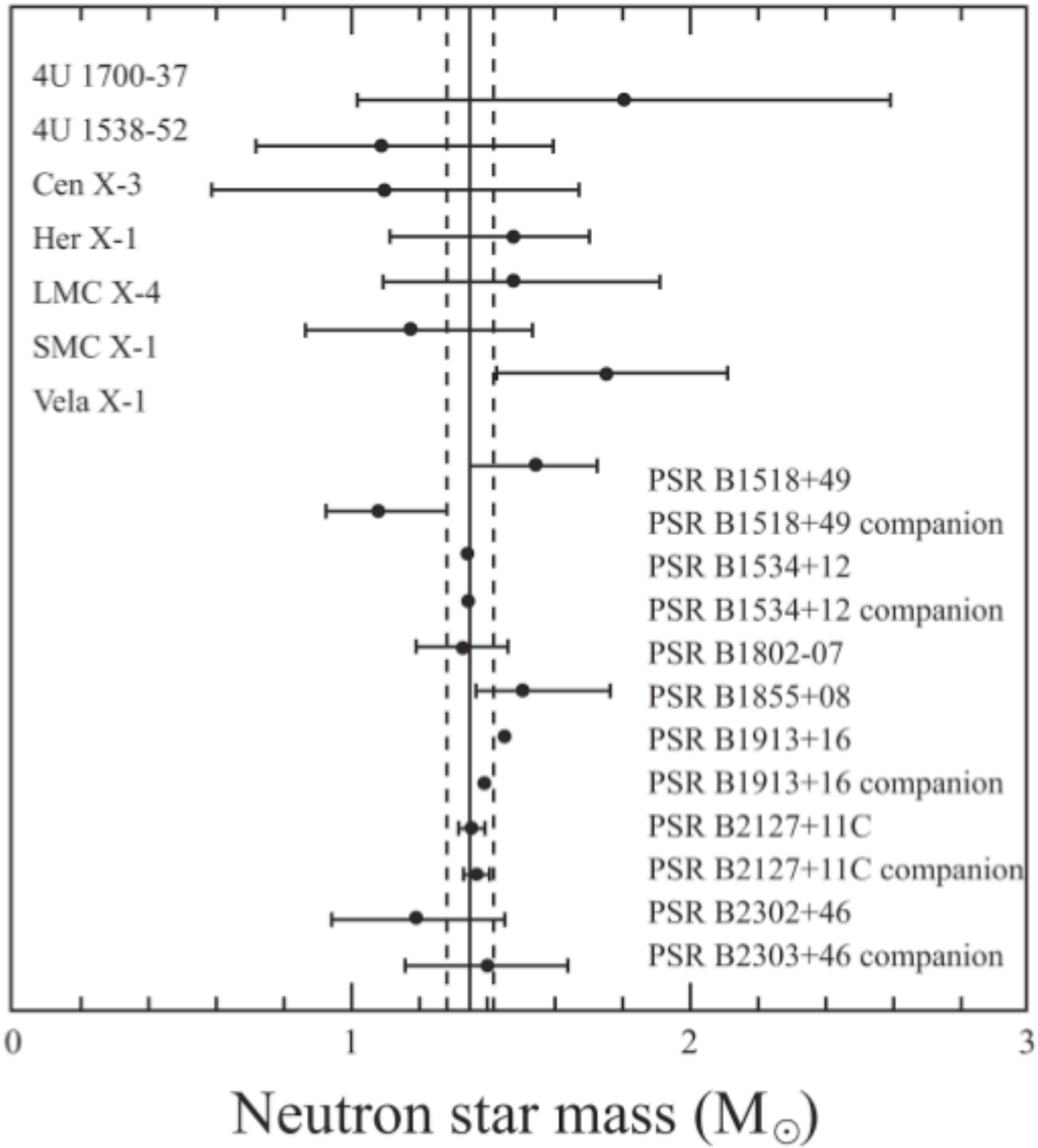


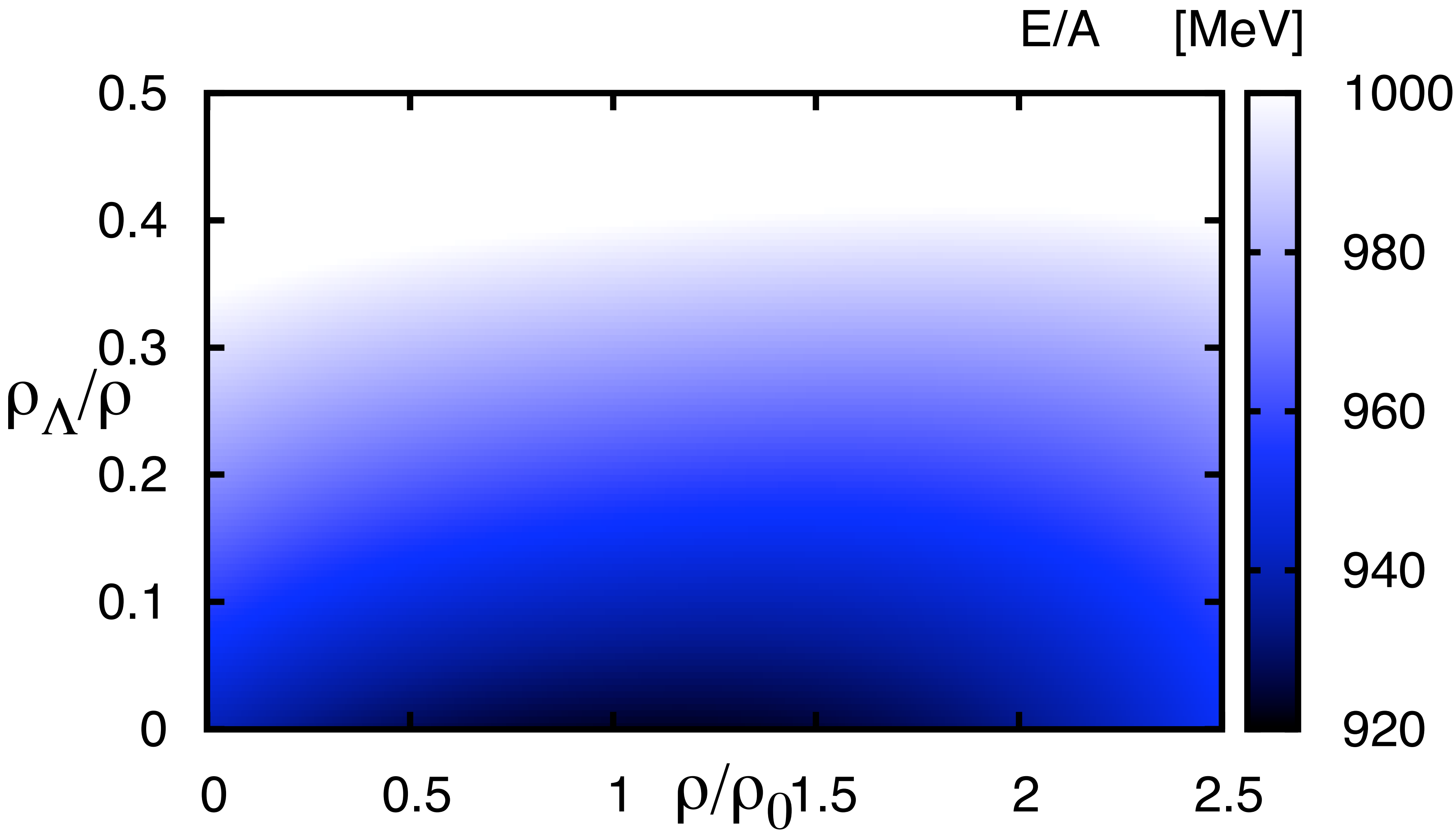
Walter Heupel • Anika Obermann • Christoph Valentin
• Urnaa Badarch • Horst Lenske



Ni
Data
DDRH
-6.5
-7
-7.5
-8
-8.5
-9
48
52
56
60
64
68
72
76
80
84
88
A
E_B/A


fraction
1
0,1
0,01
1E-3
1E-4
0,0
0,2
0,4
0,6
0,8
1,0
1,2
n
p
e⁻
μ⁻
π⁻
Σ⁻
Λ
Σ⁰
Ξ⁻
Σ⁺
Δ


4U 1700-37
4U 1538-52
Cen X-3
Her X-1
LMC X-4
SMC X-1
Vela X-1
PSR B1518+49
PSR B1518+49 companion
PSR B1534+12
PSR B1534+12 companion
PSR B1802-07
PSR B1855+08
PSR B1913+16
PSR B1913+16 companion
PSR B2127+11C
PSR B2127+11C companion
PSR B2302+46
PSR B2303+46 companion
0
1
2
3
Neutron star mass ($M_{\odot}$)



E/A [MeV]
0.5
0.4
0.3
0.2
0.1
0
ρ_Λ/ρ
0
0.5
1
ρ/ρ_0
1.5
2
2.5
1000
980
960
940
920