Generalized parton distributions

C. Weiss (Jefferson Lab), HANUC European Graduate School, Torino, 23-Mar–09

- Unified description of quark/gluon structure of hadrons

  parton distributions — elastic form factors

- Probed in exclusive processes in eN scattering

  $eN \to e' + N + M \quad (M = \gamma, \text{meson})$ — HERA, HERMES, COMPASS, JLab

  QCD factorization at high Q^2 — Future facilities: JLab 12 GeV, EIC

- New tool for exploring nucleon structure in QCD

  transverse quark/gluon imaging

  orbital angular momentum

- Many interesting applications

  small–x physics — RHIC, Tevatron, LHC

  diffractive pp scattering, . . .

Jefferson Lab

Plan

I) Concepts: QCD factorization, GPDs, quark/gluon imaging

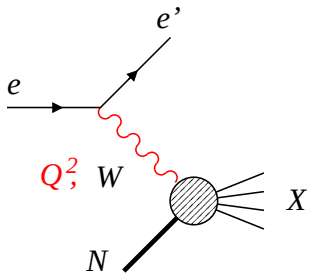
- Inclusive eN scattering: QCD factorization, parton distributions
- Elastic eN scattering: Form factors
- Exclusive processes: GPDs
- GPDs and nucleon structure: Imaging, sum rules

II) Applications: GPDs in high–Q^2 processes

- Tests of reaction mechanism and factorization
- Deeply–virtual Compton scattering
- Exclusive meson production
- GPDs in high–energy scattering ("small–x physics")
- GPDs pp scattering

Overview: Basic eN scattering experiments

high–Q^2 inclusive scattering

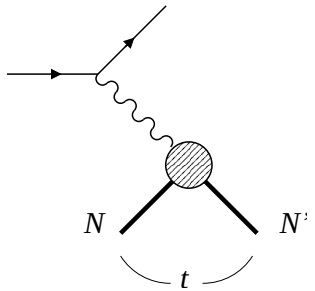


parton distributions

longitudinal momentum distribution of quarks/gluons in fast–moving nucleon

$Q^2 \leftrightarrow$ resolution scale

elastic scattering

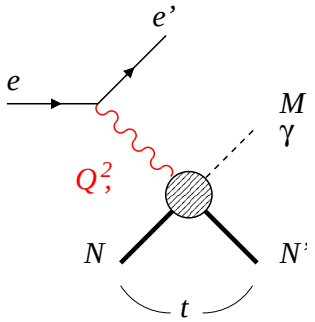


form factors

spatial distribution of charge/magnetization

$t \leftrightarrow$ size of object

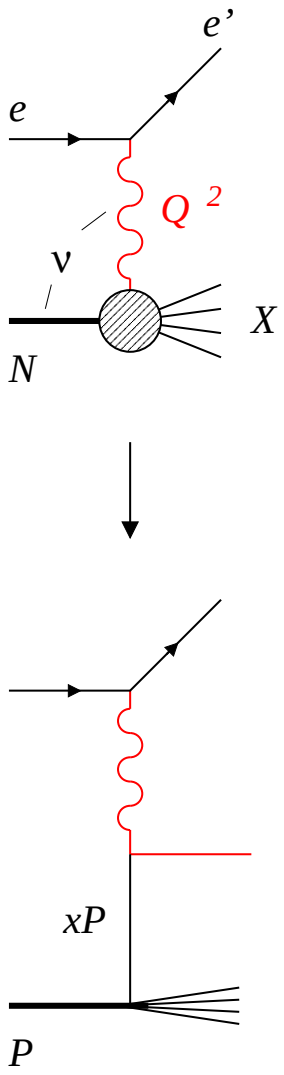
high–Q^2 exclusive processes



generalized parton distributions

longitudinal momentum + transverse spatial distribution of quarks/gluons

Inclusive scattering: Factorization



- Momentum transfer $Q^2 \gg R_{\rm had}^{-2}$:
  Scattering from pointlike constituents (quarks)

- Idea of factorization

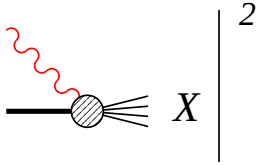
Quark subprocess	short distance	$\sim 1/Q$
Quark distribution	long distance	$\sim R_{\rm hadron}$

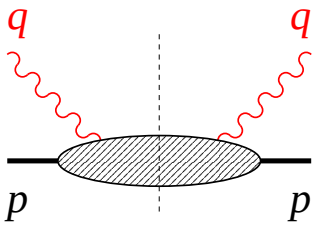
- Frame–dependent approach: Parton model
  - Fast–moving nucleon $P \gg R_{\rm had}^{-1}$
  - $q(x)$ density of quarks with longitudinal momentum xP

$$F_2(x, Q^2) \;=\; e_q^2 \; x \; q(x) \qquad \text{Bjorken scaling}$$

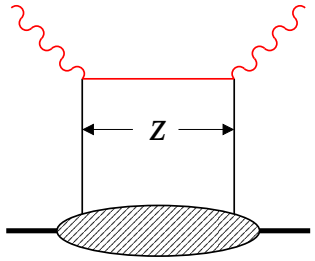
$$x \;=\; x_B \;\equiv\; Q^2/2M\nu \qquad \text{kinem. fixed}$$

Inclusive scattering: QCD factorization

$\sigma^{\gamma^* N} = \sum_X$  $\sum_X |\langle X|J_\mu|p\rangle|^2$ cross section

→  $\langle p|\, \mathrm{T}\, J_\mu(0)\, J_\nu(z)\, |p\rangle$ current–current correlation function

$q \leftarrow$ Fourier $\rightarrow z$ $J_\mu = \bar{\psi}\gamma_\mu\psi$

→  quark subprocess

$\langle p|\, \bar{\psi}(0)...\psi(z)\, |p\rangle_{z^2=0}$ density of quark field light–cone $(z^2 = 0)$

$= \int_0^1 dx\; e^{-ix(pz)}\; q(x)$ spectral representation

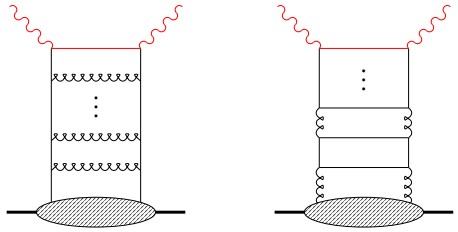
Inclusive scattering: Discussion

- Inclusive scattering at high Q^2 probes quark field density

  $\langle p|\, \bar{\psi}(0) \ldots \psi(z)\, |p\rangle$ light–like separation $z^2 = 0$

- Parton distribution (PDF) as Fourier representation of quark field density
  - Well–defined, frame–independent
  - Universal, process–independent: eN, νN weak current, NN Drell–Yan, . . .
  - Calculable with non-perturbative methods: lattice, instantons, . . .

- QCD radiative corrections:
  $\log Q^2$ dependence of PDFs ("evolution")



- General method: Light–cone expansion in coordinate space
  [here: Balitsky, Braun 89]

PDFs: Components, sum rules

- Parametrization of quark field density ($z^2 = 0$)

$$\langle p|\,\bar{\psi}(0)\,\gamma_\mu\,\psi(z)\,|p\rangle \;=\; \bar{u}\gamma_\mu u \int_0^1 dx\,\left[e^{-ix(pz)}\,q(x) - e^{ix(pz)}\,\bar{q}(x)\right] \qquad \text{quarks/antiquarks}$$

$\gamma_\mu\gamma_5$	$\Delta q(x)$	$-\Delta\bar{q}(x)$	"polarized"

$\psi = \psi_f \;\; (f = u, d, s, \ldots)$ flavor

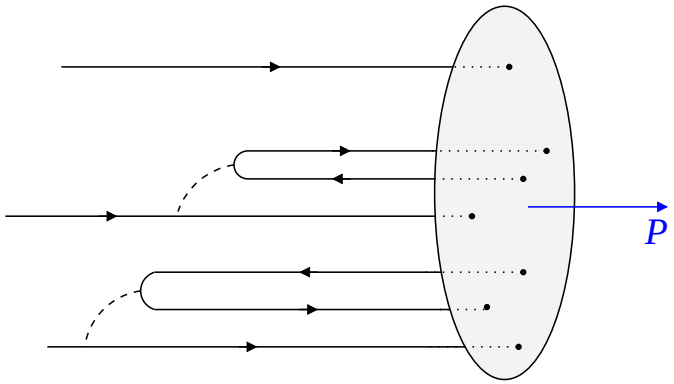
- Partonic sum rules:

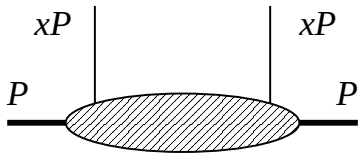
$z_\mu \to 0$: QCD operator becomes local current!

$$\int_0^1 dx[u - \bar{u} + d - \bar{d}] \;=\; g_V \text{ (isoscalar)} \qquad \text{baryon number}$$

$$\int_0^1 dx[\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}] \;=\; g_A \text{ (isovector)} \qquad \text{axial charge [Bjorken SR]}$$

PDFs: Partonic interpretation





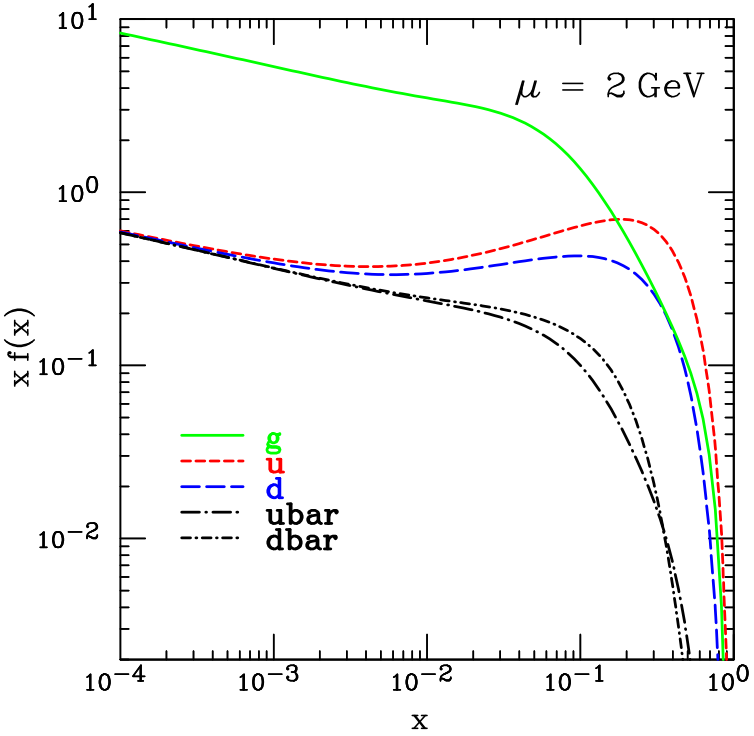
- Fast–moving nucleon $P \gg \mu_{\rm had}$:
  PDF density of quarks/antiquarks
  with longitudinal momentum xP

$$\begin{matrix} q(x) \\ \bar{q}(x) \end{matrix} = \int d^2k_T \, \langle \boldsymbol{P}| \, \underset{b^\dagger b}{a^\dagger a}(\boldsymbol{k} = x\boldsymbol{P} + \boldsymbol{k}_T) \, |\boldsymbol{P}\rangle$$

- Space–time picture of high–Q^2 scattering
  - Partonic configuration frozen ("snapshot")
  - Target probed at transverse resolution $\sim 1/Q$

- No information on transverse spatial structure from inclusive scattering!

PDFs: Global fits



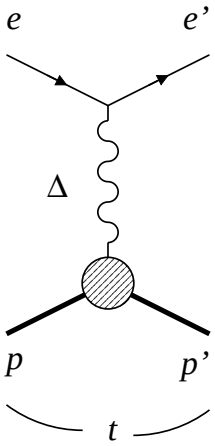
Example: CTEQ6 [hep-ph/0507093]

PDF database:
`http://durpdg.dur.ac.uk/HEPDATA/`

- PDFs extracted from global fits to $eN/\mu N, NN$ (Drell–Yan, jets), and νN data
  - Advanced technology: QCD corrections, uncertainties
  - Many open questions require separate studies: $\bar{d}/\bar{u}, s, \bar{s}$, large x [→ R. Ent]
- Polarized PDFs [→ E. Leader]

Main source of information on quark/antiquark/gluon content of nucleon

Elastic scattering: Form factors

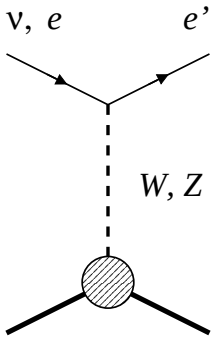


- Vector/axial currents couple to em and weak interactions

  $J_\mu = \bar{\psi}\gamma_\mu\psi, \qquad J^5_\mu = \bar{\psi}\gamma_\mu\gamma^5\psi$ local operators

- Transition matrix elements

$$\langle p'|J_\mu|p\rangle = \bar{u}'\gamma_\mu u\, F_1(t) + \frac{\bar{u}'\sigma_{\mu\nu}\Delta^\nu u}{2M} F_2(t)$$ Dirac/Pauli

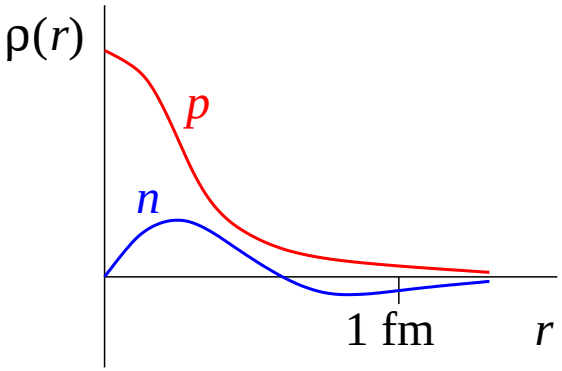


$$J^5_\mu = \bar{u}'\gamma_\mu\gamma_5 u\, G_A(t) + \frac{\bar{u}'\Delta_\mu\gamma_5 u}{2M} G_P(t)$$ axial/pseudosc.

cf. helicity conserving/flip amplitude

$\Delta_\mu \equiv (p' - p)_\mu \quad t \equiv \Delta^2 < 0$ invariant momentum transfer

Form factors: Interpretation



- Spatial interpretation: Charge/current distribution in Breit frame $\Delta_\mu = (0, \boldsymbol{\Delta})$

$$F_1(t = -\boldsymbol{\Delta}^2) \;=\; \int d^3r\, e^{-i(\boldsymbol{\Delta}\boldsymbol{r})}\, \rho(r)$$

  - Frame–dependent!
  - Cf. non-relativistic QM

- Charge radius

$$\langle r^2 \rangle \;\equiv\; \int d^3r\, r^2\, \rho(r) \;=\; -6\, \frac{dF_1}{dt}(t=0)$$

$\langle r^2 \rangle = 0.72$ fm^2 (proton)

-0.11 fm^2 (neutron)

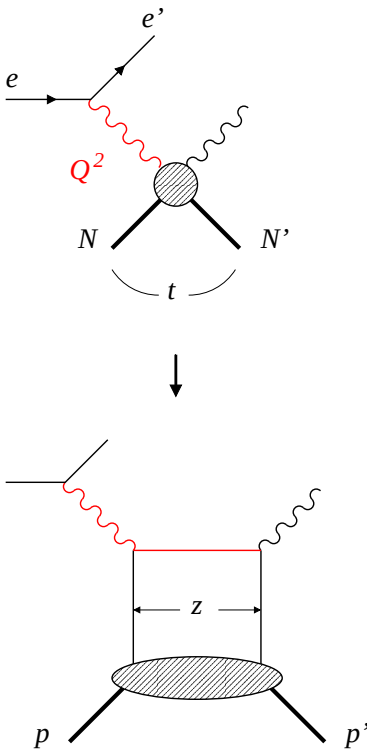
Main source of information on "spatial size" of nucleon

Q:

- Can one extend factorization to processes other than inclusive scattering?
- How can we learn about the transverse spatial distribution of partons?

$\rightarrow$ Exclusive processes
Generalized parton distributions

Exclusive processes: Factorization



[D. Müller et al. 94; Radyushkin 96, Ji 96]

also: Meson production

- Deeply–virtual Compton scattering

$$Q^2, W^2 \gg R_{\rm had}^{-2}, \quad |t| \sim R_{\rm had}^{-2}$$

- Partonic mechanism: Reaction with single quark

- QCD factorization analogous to inclusive

$\langle p'| \, \mathrm{T} \, J_\mu(0) \, J_\nu(z) \, |p\rangle$ Compton amplitude as correlation function

$\langle p'| \, \bar\psi(0) ... \psi(z) \, |p\rangle$ transition quark density $(z^2 = 0)$

$\rightarrow$ GPDs

Same operator as in inclusive scattering but form factor matrix element $p' \neq p$

GPDs: Parametrization of transition quark density

$$\langle p'|\,\bar\psi(0)\,\gamma_\mu\,\psi(z)\,|p\rangle \;=\; \int_{-1}^{1} dx\, e^{-ix(pz)} \left[\bar u'\gamma_\mu u\; H(x,\xi;t) \;+\; \frac{\bar u'\sigma_{\mu\nu}\Delta^\nu u}{2M}\; E(x,\xi;t)\right]$$

$\gamma_\mu\gamma_5$ $\qquad$ $\bar u'\gamma_\mu\gamma_5 u\;\tilde H$ $\qquad$ $\dfrac{\bar u'\Delta_\mu\gamma_5 u}{2M}\;\tilde E$

$\xi \;\equiv\; (p-p')\cdot z/(p+p')\cdot z$ $\qquad$ "skewness"

$t \;\equiv\; \Delta^2$ $\qquad$ inv. momentum transfer $\qquad$ $\Delta = \; p'-p$

- Limiting relations

$p'=p$ momentum transfer zero $\qquad$ $H(x,\xi=0,t=0) \;=\; \begin{cases} q(x) & x>0 \\ -\bar q(-x) & x<0 \end{cases}$ etc.

$z_\mu = 0$ local operator $\qquad$ $\displaystyle\int_{-1}^{1} dx\, H(x,\xi,t) \;=\; F_1(t)$ etc.

GPDs "unify" parton distributions and form factors!

Plan

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- Elastic eN scattering: Form factors
- Exclusive processes: GPDs
- GPDs and nucleon structure: Imaging, sum rules ←

II) Applications: GPDs in high–Q^2 processes

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- GPDs pp scattering

Summary

- Factorization = separation of scales

  short distances $\sim 1/Q$ long distances $\sim R_{\rm hadron}$

- Long–distance information contained in quark field densities at light–like separation

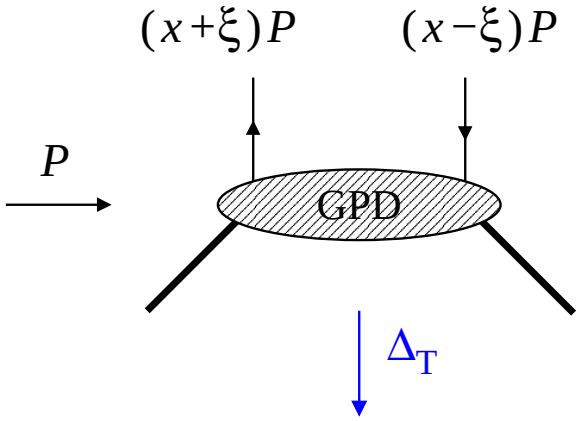
  $\bar\psi(0)\,\Gamma\,\psi(z)$ at $z^2 = 0$, $\Gamma = \gamma_\mu,\ \gamma_\mu\gamma_5$

- Representation of matrix elements

  $\langle p|\ldots|p\rangle$ $\leftrightarrow$ "usual" parton distribution inclusive

  $\langle p'|\ldots|p\rangle$ $\leftrightarrow$ GPD exclusive

GPDs: Partonic interpretation



- Fast–moving nucleon

  "emits" parton with $x_1 = x + \xi$

  "absorbs" $x_2 = x - \xi$

$x_1, x_2 > 0$	quark distribution
$x_1, x_2 < 0$	antiquark distribution
$x_1 > 0,\ x_2 < 0$	emission of $q\bar{q}$ pair . . . new!

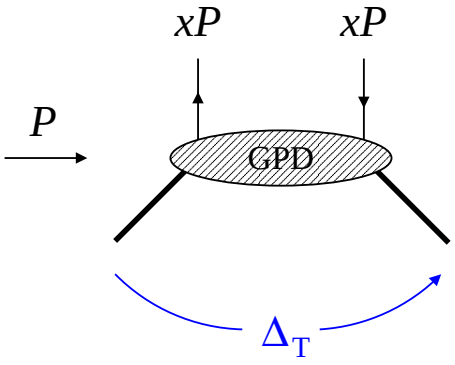
- Parton transverse momenta $\boldsymbol{k_T}$ integrated over as in PDF

- Momentum transfer to nucleon

  longitudinal $-2\xi P$

  transverse $\boldsymbol{\Delta}_\perp$

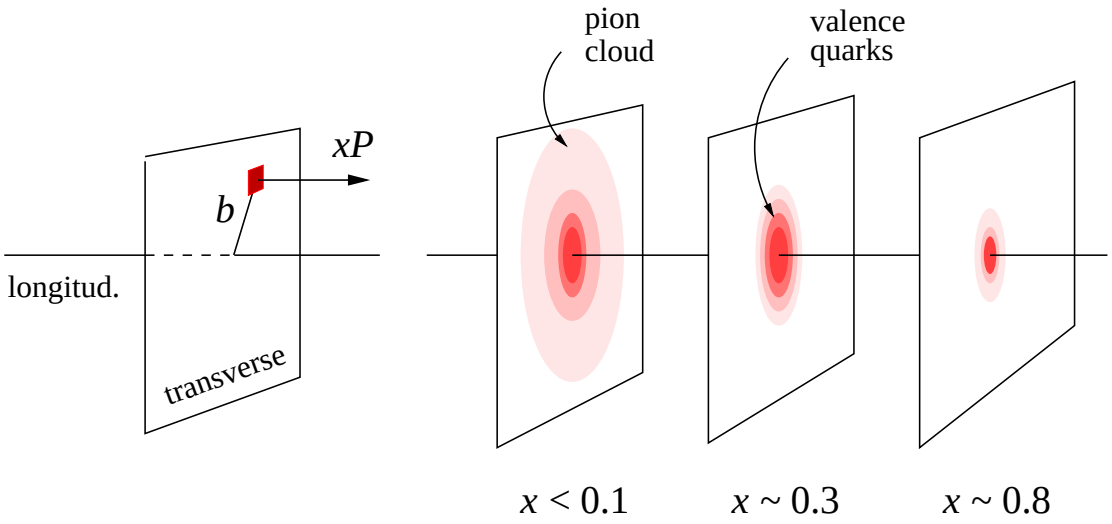
GPDs: Transverse spatial distribution



- Transverse coordinate representation

$$H(x, t = -\boldsymbol{\Delta}_T^2) \;=\; \int d^2b\, e^{-i(\boldsymbol{\Delta}_T \boldsymbol{b})}\, q(x, \boldsymbol{b})$$

- $q(x, \boldsymbol{b})$ transverse spatial distribution of quarks with longitudinal momentum fraction x

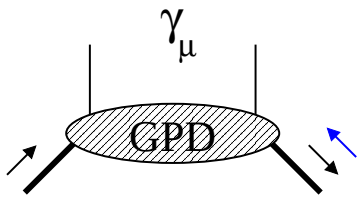


$$\int d^2b\, q(x, \boldsymbol{b}) \;=\; q(x)$$

- Tomographic quark images of nucleon at fixed x
  [Burkardt 02, Diehl 03]

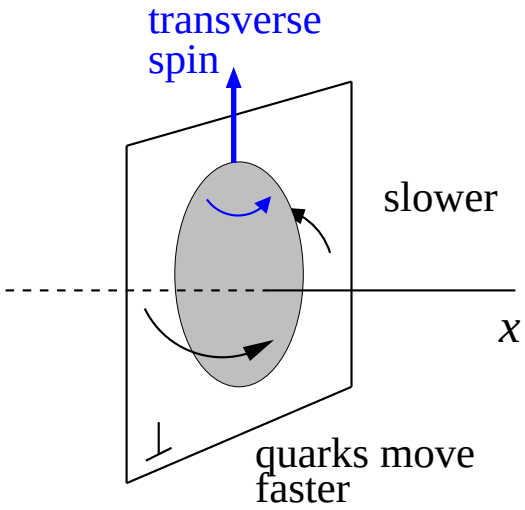
GPDs: Polarization effects

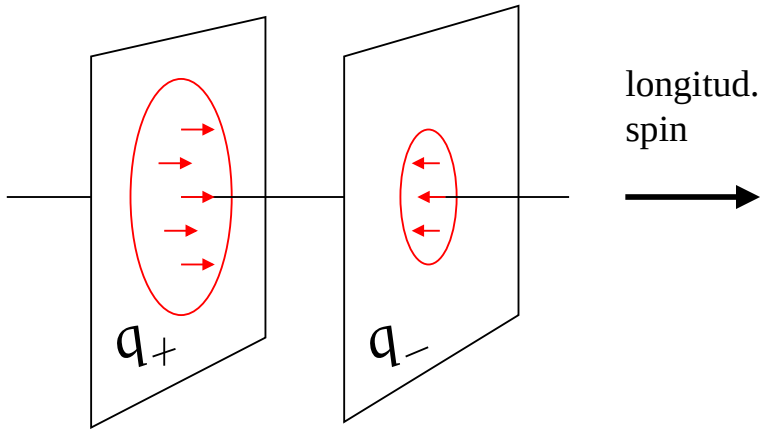
Quarks unpolarized:



$=$ H, Dirac E Pauli (+ Transversity)

polarized: $\gamma_\mu\gamma_5$ $\tilde{H}$, axial $\tilde{E}$ pseudoscalar





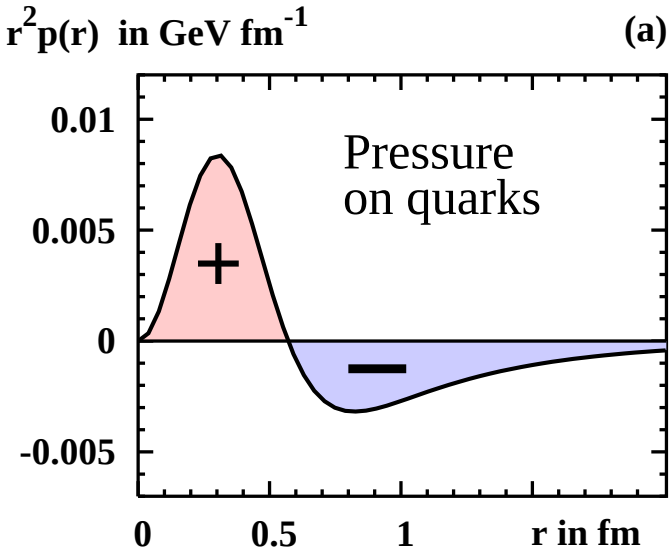
E : Distortion of quark longitudinal motion by transverse spin [Burkardt 03]

$H \pm \tilde{H}$: Spatial distribution of quark helicity

GPDs: Moments, sum rules

$$\int dx\, x\, [H + E](x, \xi; t) \;=\; J^q$$

Quark angular momentum



Stability: Positive $\leftrightarrow$ negative
[Model: Goeke, Schweitzer et al. 07]

- Expansion of quark density in z gives local spin–n operators

$$\begin{aligned} \psi(0)\gamma_\mu\psi(z) \;&=\; \psi(0)\gamma_\mu\psi(0) && \text{spin–1} \\ &+\; z_\nu\; \psi(0)\gamma_\mu\nabla_\nu\psi(0) && \text{spin–2} \\ &+\; \ldots \end{aligned}$$

- Matrix elements as moments of GPDs

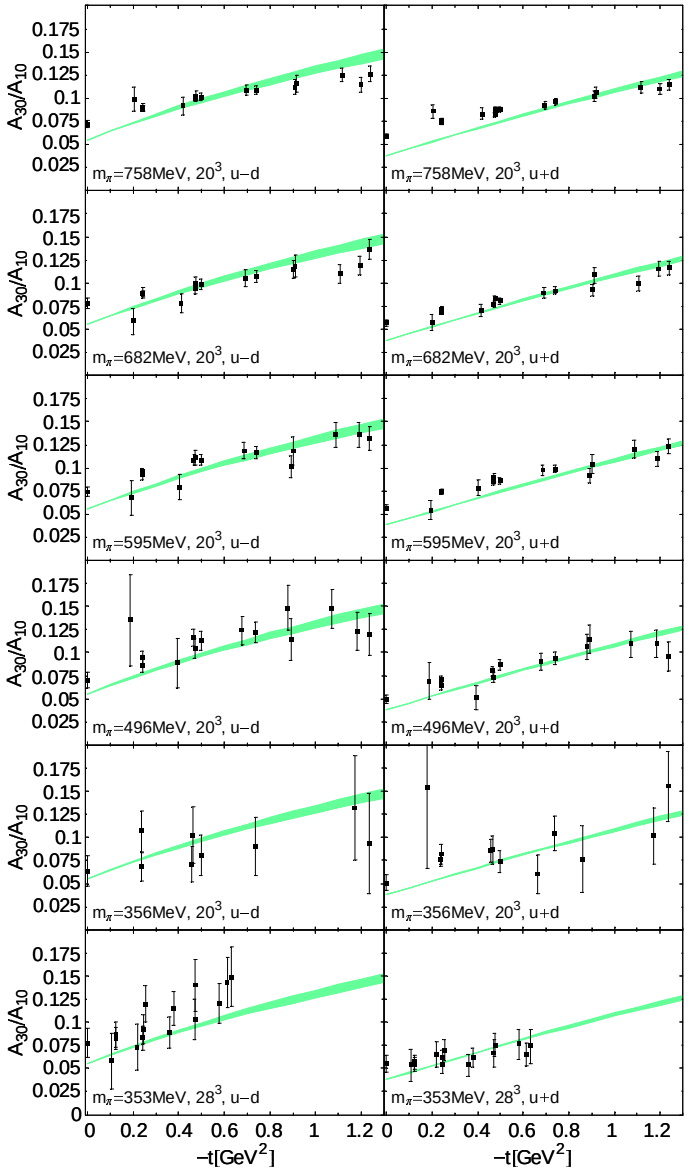
$$\int dx\, x^{n-1}\; H(x, \xi; t) \qquad \text{gen. form factors}$$

- Spin–2: QCD energy–momentum tensor

  $\rightarrow$ Angular momentum J_q [Ji 97]

  $\rightarrow$ Forces on quarks in nucleon rest frame [Polyakov 03]

GPDs: Lattice calculations



- GPDs universal, Twist–2 operator } can be calculated on lattice

  $\leftrightarrow$ TMDs ?

- Lowest moments $n \leq 4$, t–dependence

- Presently non-singlet $q - \bar{q}, u - d$
  Singlet: Disconnected diagrams
  Hägler et al. [LHPC Collab. 07]

- Potential to constrain GPD parametrizations in the future

GPDs: Sources of information

- Basic information from

$$\left.\begin{array}{lll} \text{parton densities} & q(x) & = \quad H(x,\xi=0,t=0) \\ \text{elastic form factors} & F_1(t) & = \quad \int dx\, H(x,\xi,t) \end{array}\right\} \text{GPD models, parametrizations}$$

- New information about $x, \xi \leftrightarrow t$ correlation ("shape of nucleon") from hard exclusive processes

$\rightarrow$ Is partonic description applicable in practice?

Test model–independent predictions: Q^2–scaling, universality, . . .
Implement corrections

$\rightarrow$ What information on GPDs is accessible in exclusive processes?

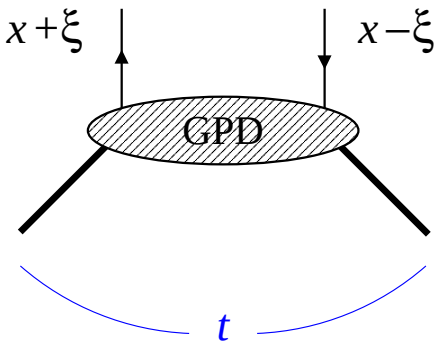
Structure of amplitudes, observables

GPDs: Information accessible in exclusive processes

$$\text{Im}\, A(\xi,t) \quad \to \quad H(\xi,\xi;t)$$

$$\text{Re}\, A(\xi,t) \quad \to \quad \int dx\, \frac{H(x,\xi;t)}{x \pm \xi}$$

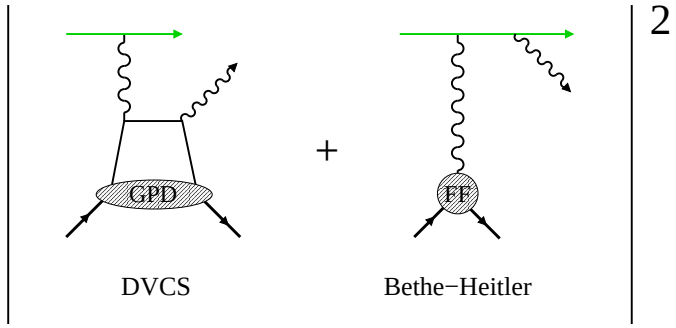
$$\xi = \frac{x_B}{2 - x_B} \qquad \text{kinemat. fixed}$$

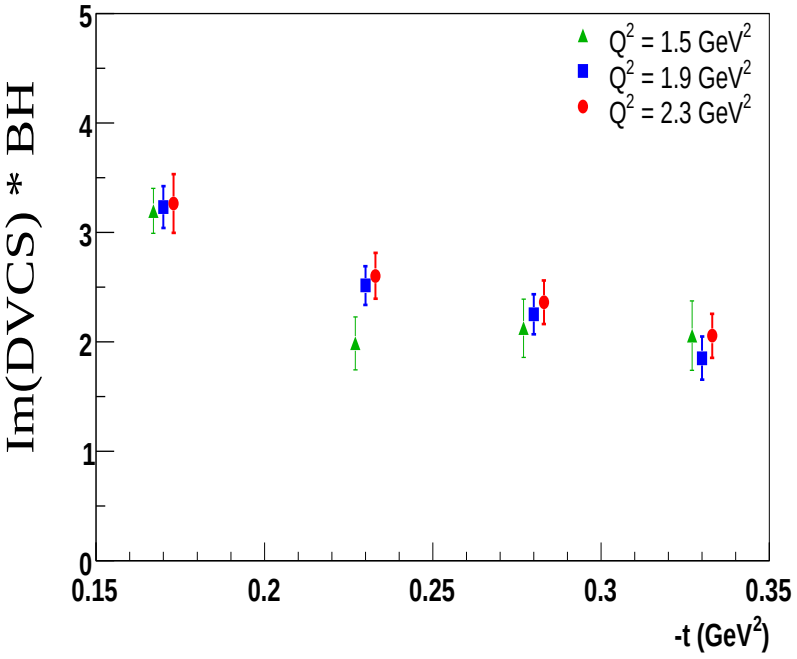


Limited information
from exclusive processes!

- Factorization for exclusive amplitude:
  Im, Re ($\leftrightarrow$ inclusive cross section)
- Only GPDs at $x = \xi$ accessible
  "point by point" from Im A
- Simplifications
  - High energies $\xi \ll 1$: GPDs can be generated by evolution from $\xi = 0$ ("forward")
  - Dispersion relations: Re A restored from Im A + subtraction constant
    . . . new development

DVCS: Observables, scaling





JLab Hall A 2006

- Interference Bethe–Heitler / DVCS in $eN \to e' + N' + \gamma$

  Im(DVCS) $*$ BH from $\overrightarrow{e} - \overleftarrow{e}$ spin

  Re(DVCS) $*$ BH $e^+ - e^-$ charge

  $\to$ measure DVCS amplitude

  $\to$ linear in GPDs H, E!

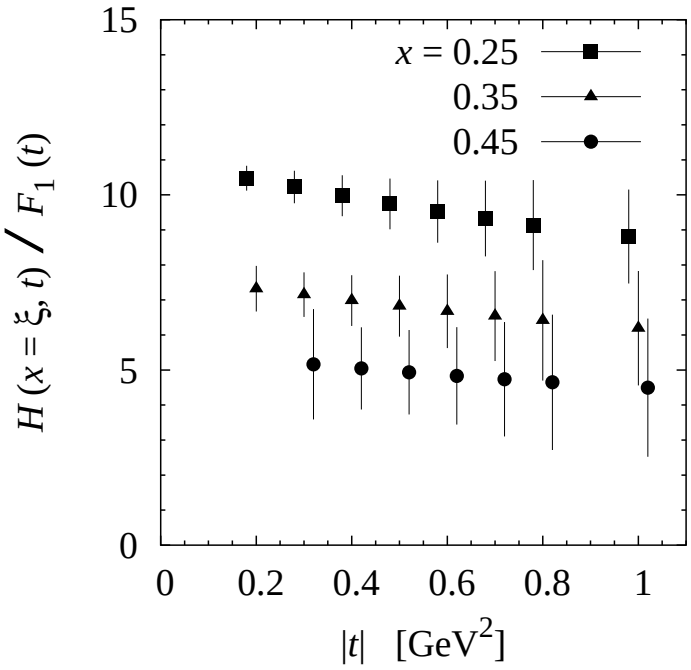
- JLab Hall A data on Im(DVCS) $*$ BH indicate early approach to Q^2–scaling

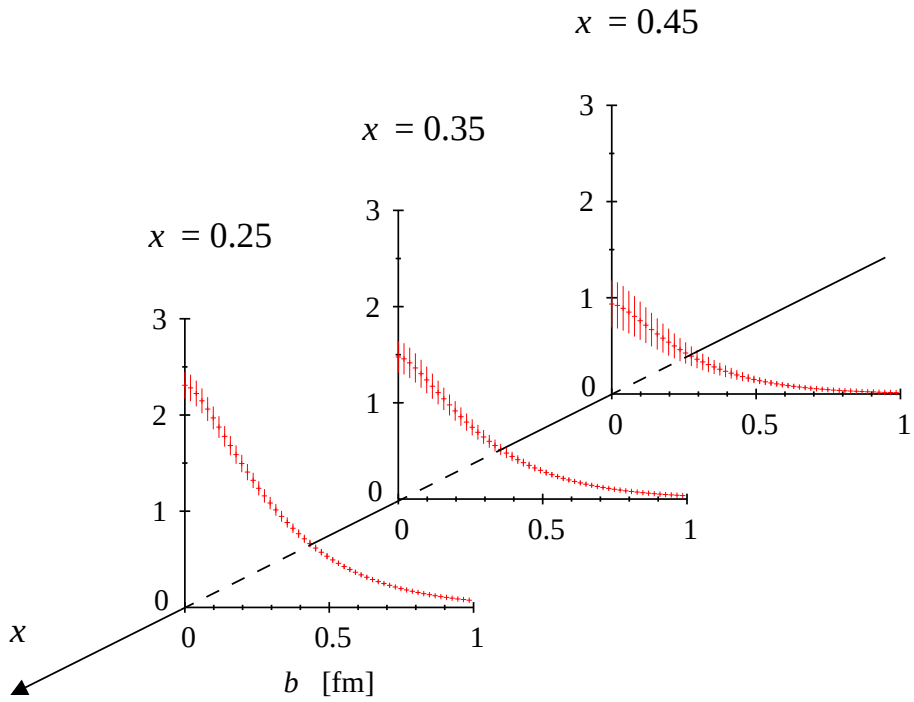
  "Test" of partonic mechanism!

- Separate H, E: p/n, transv. pol. target
  Measure $\tilde{H}, \tilde{E}$: long. pol. target

DVCS: Projections for JLab 12 GeV

- GPD $H(x = \xi; t)$ and "quark image" of nucleon
  from DVCS beam spin asymmetry measurements with CLAS12





H. Avakian, CW; based on Vanderhaeghen–Guichon–Guidal GPD model.
GPD White Paper (H. Abramowicz et al.) 2007

DVCS: Summary

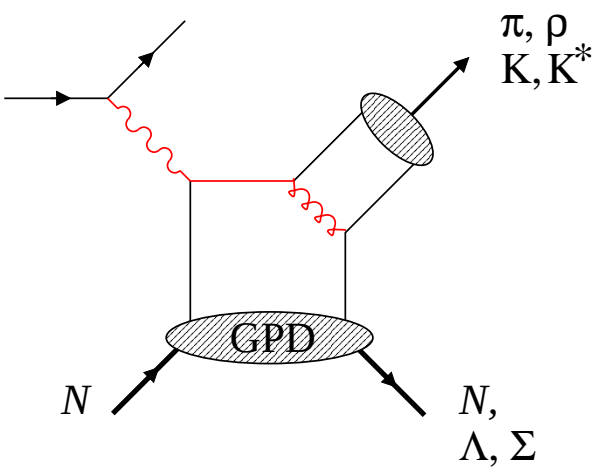
- Seems to approach scaling regime early $Q^2 > 2\,\mathrm{GeV}^2$
  cf. inclusive DIS

- Leading–twist QCD analysis well developed;
  power corrections need more study

- Access to linear combinations of GPDs,
  can be separated by p/n, polarization observables

- New development: Dispersion relations
  Will it become simpler? "Trivial?"
  [Teryaev 05; Anikin, T. 07; Müller et al. 07; Diehl, Ivanov 07]

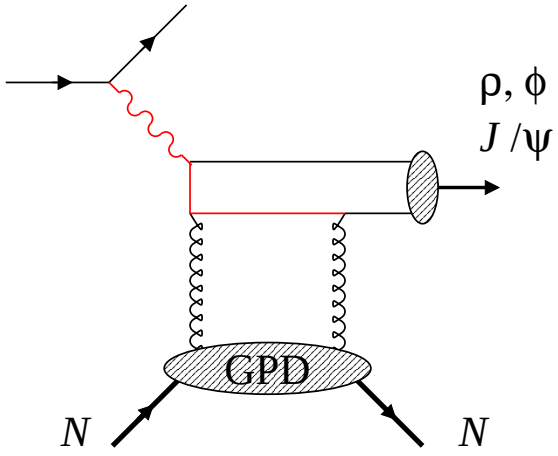
Meson production: QCD factorization



- QCD factorization theorem for σ_L
  [Collins, Fankfurt, Strikman 96]

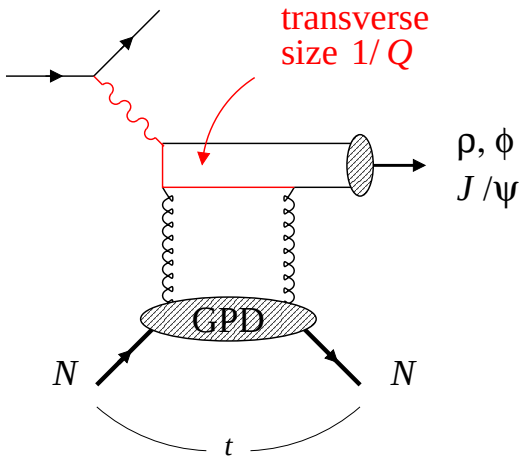
- J^{PC} quantum numbers of meson
  select specific GPD components

$$\left.\begin{array}{lll} 1^- & \text{vector} & q,\ \bar{q},\ g \\ 0^- & \text{pseudoscalar} & \Delta q,\ \Delta\bar{q} \end{array}\right\} + \text{flavor}$$

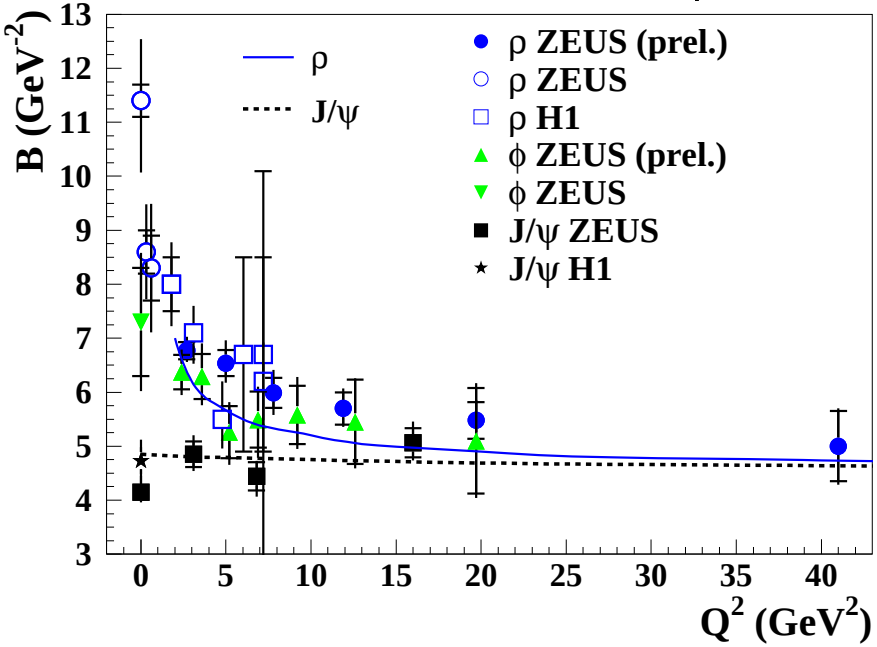


- Partonic process more complex
  - Additional hard interaction required
    to produce collinear $q\bar{q}$ pair
    cf. pion form factor at high t
    [Efremov, Radyushkin 78; Brodsky, Lepage 79]
  - Need $q\bar{q}$ wave function of meson

Meson production: Test of reaction mechanism



Exclusive vector meson t–slope



- Factorization: Meson produced in small–size configuration $\sim 1/Q$

  $\rightarrow$ t–slope of cross section should become independent of Q^2

  $\rightarrow$ $\rho^0, \phi, J/\psi$ should have same t–slope. . . universality!

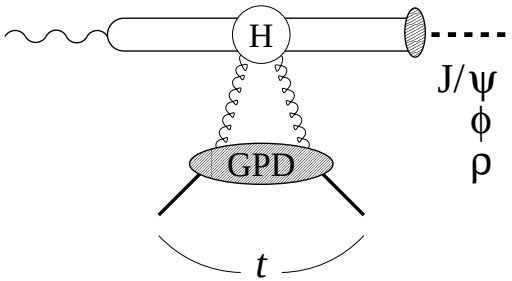
- Seen in HERA vector meson data

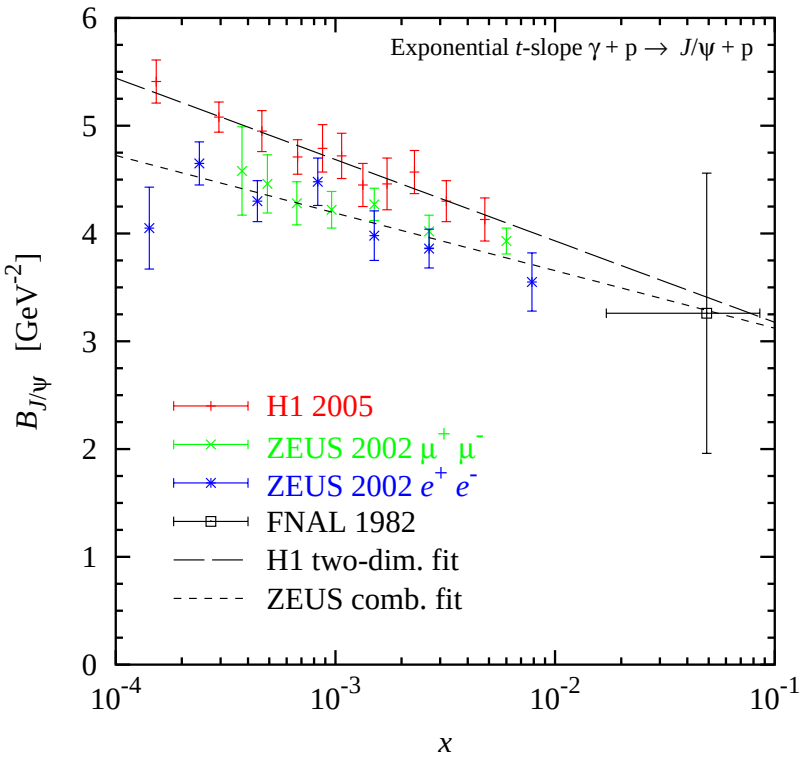
  Tests approach to small–size regime!

- $Q^2 \sim$ few GeV^2: Substantial finite–size corrections

  – Dynamical models w. intrinsic k_T [Frankfurt et al. 96; Vanderhaeghen et al. 98; Kroll, Goloskokov 05]

Meson production: Transverse gluon imaging





- J/ψ production unique channel
  - Couples only to gluon GPD, clean!
  - Factorization already at $Q^2 = 0$ because of small size $(1/M_c)$

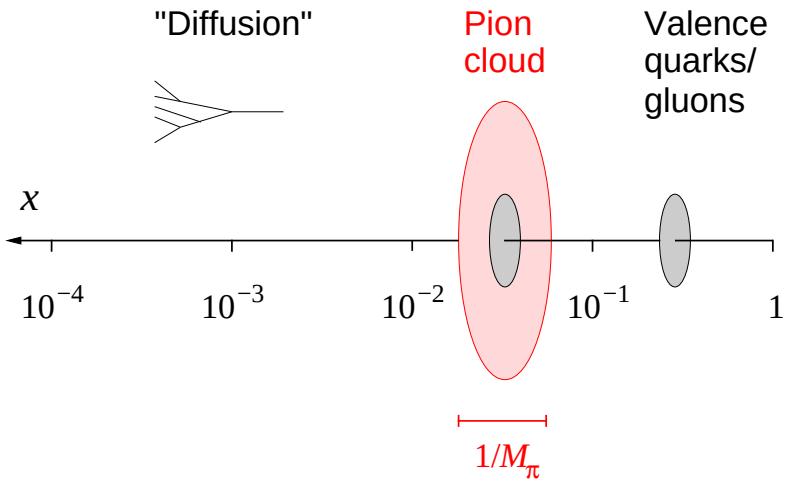
- Transverse spatial distribution of gluons

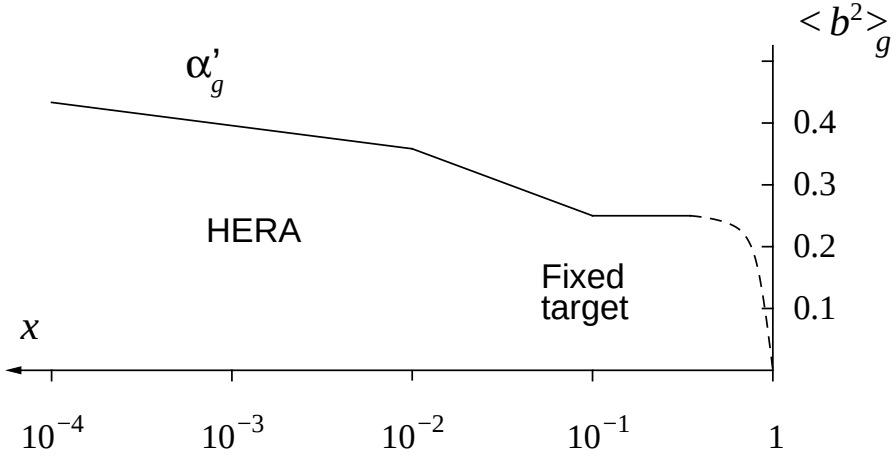
$$\frac{d\sigma}{dt} \propto \left[\frac{H_g(x,\xi;t)}{H_g(x,\xi;0)}\right]^2 \xrightarrow{\text{FT}} \text{spatial distribution}$$

Data: HERA, FNAL

also: Cornell, SLAC, CERN

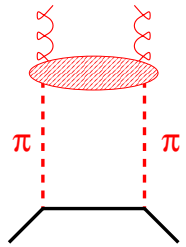
Meson production: Transverse gluon imaging II





Scale $Q^2 \approx 3\,\text{GeV}^2$

- Gluonic transverse size increases with decreasing x
- Pion cloud contributes at $x < M_\pi/M_N$ [Strikman, CW 03]



cf. "Yukawa tail"

- Small x: Logarithmic growth with $\alpha'_g \ll \alpha'_{\text{soft}}$ ("diffusion")

Much interesting information on "gluonic size" of nucleon + x–dep.

Meson production: Cross section ratios

$\phi \quad \longleftrightarrow \quad \rho^0 \quad \longleftrightarrow \quad \rho^+$

gluon — gluon, quark — quark

$\pi^0 \quad \longleftrightarrow \quad \eta \quad \longleftrightarrow \quad \eta'$

combinations of $\Delta u,\ \Delta d,\ \Delta s$

$K^0 \quad \longleftrightarrow \quad \pi^0$

$SU(3)$ flavor symmetry

Probe GPDs "selectively"

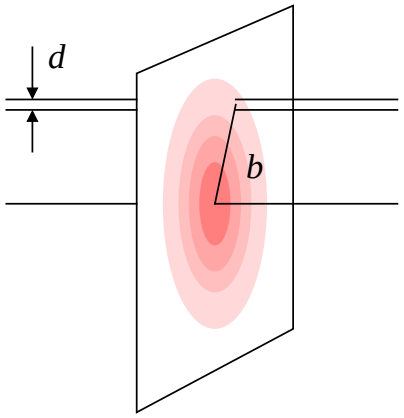
- Factorized analysis of absolute cross sections challenging:
  - Partonic scattering process (effecive QCD scale, higher twist)?
  - Meson wave functions?
- Easier: Cross section ratios!
  - Quasi model–independent!
  - Many interesting possibilities for JLab 6 & 12 GeV
- Also: Target spin asymmetries $(\sigma^+ - \sigma^-)/(\sigma^+ + \sigma^-)$

  Relative t–dependences $[d\sigma/dt(t)]$ / $[d\sigma/dt(0)]$

Meson production: Summary

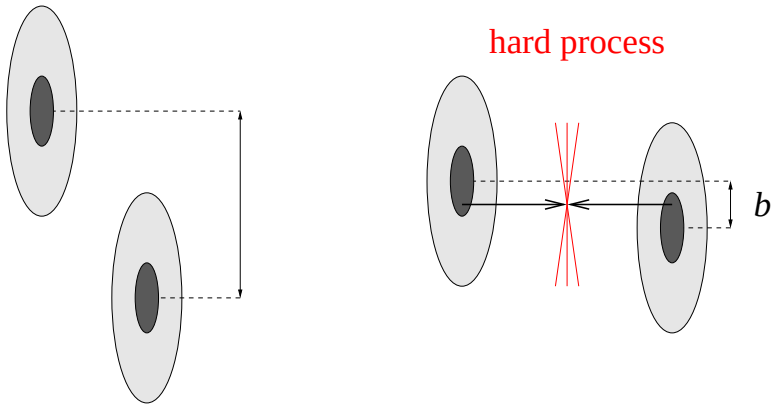
- Access to individual spin/flavor components of GPDs
- QCD factorization = meson produces in small–size configuration $\sim 1/Q$
  cf. color transparency
- Seems to require $Q^2 \sim 10\,\text{GeV}^2$ to become fully effective
- $Q^2 \sim$ few GeV^2: GPD–based models with finite–size corrections
  [Frankfurt et al. 96; Vanderhaeghen et al. 98; Kroll, Goloskokov 05]
- Many opportunities for model–independent comparative studies:
  Cross section ratios

GPDs: High–energy physics, pp scattering



- Transverse gluon distribution essential ingredient in studies of unitarity limit/saturation at small x
  - QCD dipole model
  - "Saturation scale" $Q_s(b)$

[Frankfurt, Strikman, Guzey/Rogers/CW 02+; Kowalski, Teaney 04]



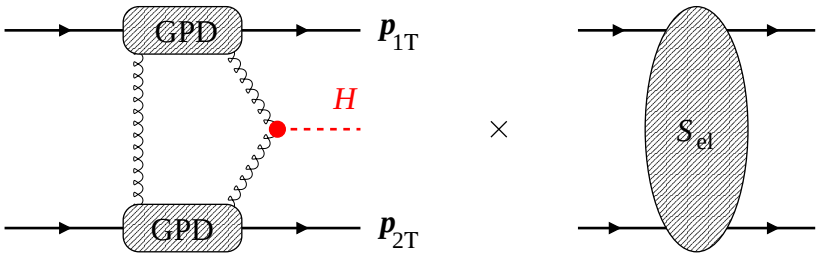
- pp collisions with hard processes
  - centrality dependence
  - spectator interactions, underlying event structure

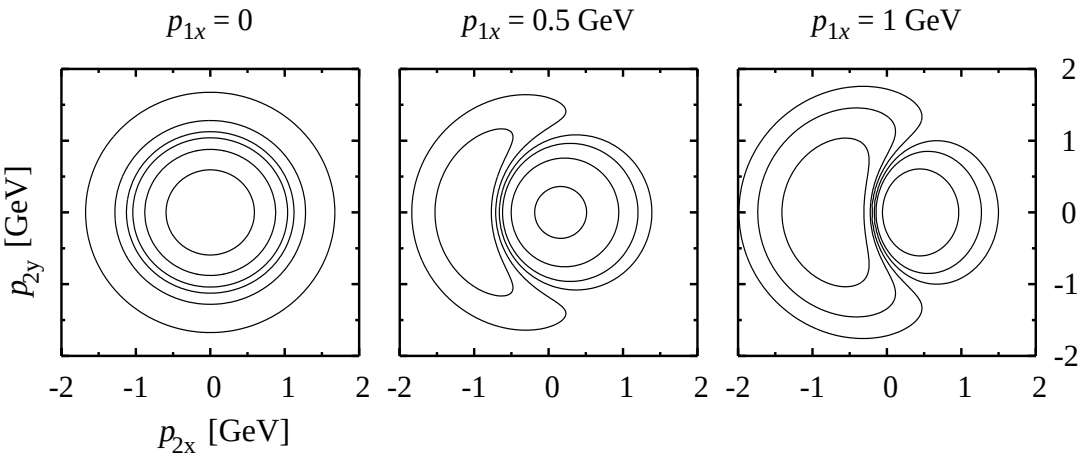
[Frankfurt, Strikman, CW 04/05]

GPDs: Exclusive diffraction in pp

$$pp \to p + \text{gap} + H + \text{gap} + p$$

$(= \text{dijet, Higgs, } Q\bar{Q}, \ldots)$





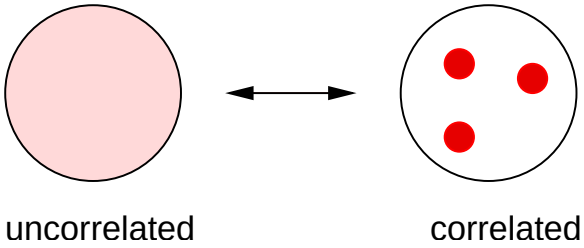
- Diffraction pattern in $\boldsymbol{p}_{1T}, \boldsymbol{p}_{2T}$ sensitive to gluon GPD
- Could be observed in pp with forward detectors

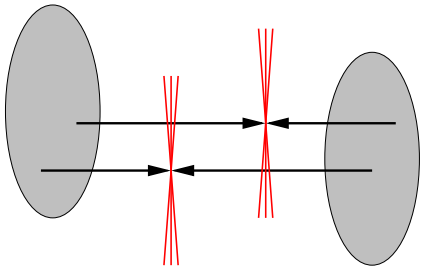
  CMS/TOTEM at LHC
  STAR pp2pp @ $\sqrt{s} = 500$ GeV ?

[Frankfurt, Hyde, Strikman, CW 07]

Probe gluon GPD in pp
. . . New direction!

Outlook: Transverse correlations of partons





- GPDs single–particle distributions
  Next step: Two–particle correlations

- Fermilab CDF data on 3 jet + photon compatible with strong transverse correlations of size $\rho \sim 0.3\,\mathrm{fm}$
  [Frankfurt, Strikman, CW 04]

  . . . Constituent quarks?

  cf. Instanton liquid picture of QCD vacuum [Diakonov, Petrov 84]

- Correlations could substantially modify rapidity gap survival in diffraction
  [Frankfurt, Hyde, Strikman, CW 07]

Summary

- GPDs unifying framework for discussing single–particle quark/gluon structure of hadrons in QCD

  "Quark/gluon imaging" of nucleon

- Exclusive processes in eN scattering at high Q^2:
  Challenge for future experimental facilities [JLab 12 GeV, EIC]
  and theory/phenomenology

- Interesting new connections $eN \leftrightarrow NN$!