THE TRANSVERSE STRUCTURE OF THE NUCLEON

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The Nucleon Structure – 12th HANUC Lecture Week – Torino

Lecture I

- Transverse spin physics: general motivations
- Transversity distribution function (collinear factorization)
- Azimuthal and single spin asymmetries (phenomenological motivations)
- Theoretical approaches

  TMD generalized parton model (inclusion of spin and k_T effects)

  TMD color gauge invariant approach (pQCD gauge links)

  Twist-three collinear factorization formalism
- Open points, possible future developments

Lecture II: Phenomenological applications

Transverse spin physics

It was an early common belief that transverse spin effects should play a negligible role in high-energy hadronic reactions

In a frame where a particle with mass m is moving very fast (with energy $E \gg m$) the transverse spin components are suppressed with respect to the longitudinal one by a factor m/E

$$s^\mu = s^\mu_\parallel + s^\mu_\perp = \frac{\lambda}{m}\, p^\mu + s^\mu_\perp \qquad s^2 = \lambda^2 + s^2_\perp = -1$$

However, this does not mean that ALL transverse polarization phenomena are subleading. Some of them are neither kinematically nor dynamically suppressed

In fact, there are several transverse spin effects contradicting this prejudice: transverse single spin asymmetries, transverse hyperon polarization in unpol. hadronic collisions; spin-spin correlations in pp elastic scattering

Transverse spin physics

A new class of leading twist (dominant term in a 1/Q power expansion, with Q the large energy scale) spin and transverse momentum dependent (TMD) partonic distribution and fragmentation functions has been shown to play a fundamental role in this game

Polarized TMD distributions are intimately related to
parton orbital motion inside hadrons,
hadron structure in the impact parameter space,
generalized parton distributions in deeply virtual Compton scattering
[Lectures by C. Weiss]
light-cone hadronic wave functions

A continued careful study of these polarized observables in different kinematical situations and in different processes will hopefully help clarifying in much more detail the dynamical structure of hadrons

Transversity distribution

In the collinear (transverse momentum integrated) case the leading twist quark-quark correlator reads

$$\Phi(x;P,S) = \frac{1}{2}\{ f_1(x)\, \not{P} + S_L\, g_1(x)\, \gamma_5 \not{P} + \frac{1}{2} h_1(x)\, \gamma_5 [\not{S}_T, \not{P}] \}$$

Unpolarized distribution — helicity distribution — transversity distribution

$h_T(x)$ Ralston & Soper (1979)

$\Delta_1 q(x)$ Artru & Mekhfi (1990)

$h_1(x)$ Jaffe & Ji (1991)

Today: $h_1(x)$, $\delta q(x)$, $\Delta_T q(x)$

$$\Delta_T q(x) = f_{q^\uparrow/p^\uparrow}(x) - f_{q^\downarrow/p^\uparrow}(x)$$

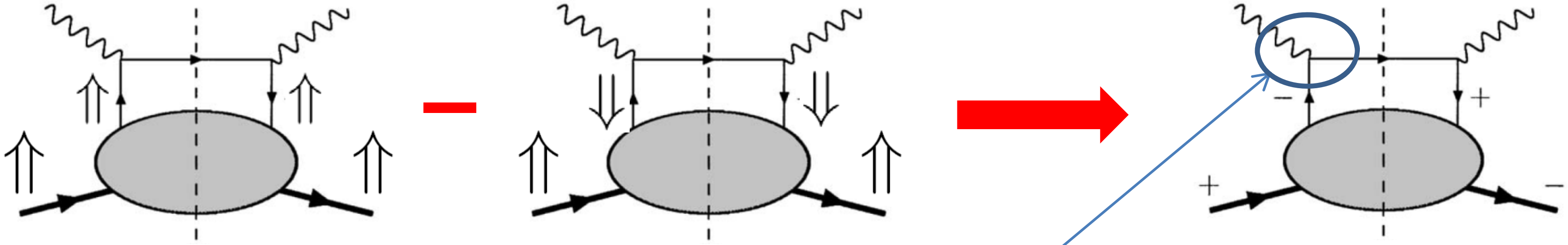
$$|\uparrow,\downarrow\rangle_x = (1/\sqrt{2})(|+\rangle \pm |-\rangle)$$

$$|\uparrow,\downarrow\rangle_y = (1/\sqrt{2})(|+\rangle \pm i|-\rangle)$$

In the proton rest frame there is nothing discriminating between longitudinal and transverse spin direction $\Rightarrow$ helicity distribution $\equiv$ transversity distribution

Transversity distribution

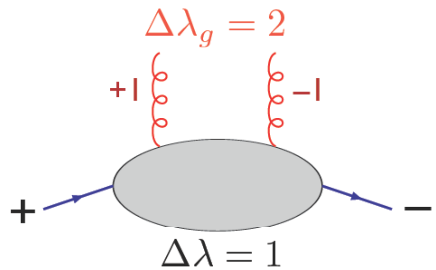
Transversity is chiral-odd ⇒decouples from transversely polarized Deeply Inelastic Scattering



QCD and electroweak interactions preserve helicity in the fermion-vector boson interaction vertex (to all orders in the coupling constant power expansion), therefore helicity-flip terms are forbidden (m/E suppressed)

This explains why transversity is so poorly known as compared to the other two leading twist (unpolarized and helicity) distributions accessible in DIS

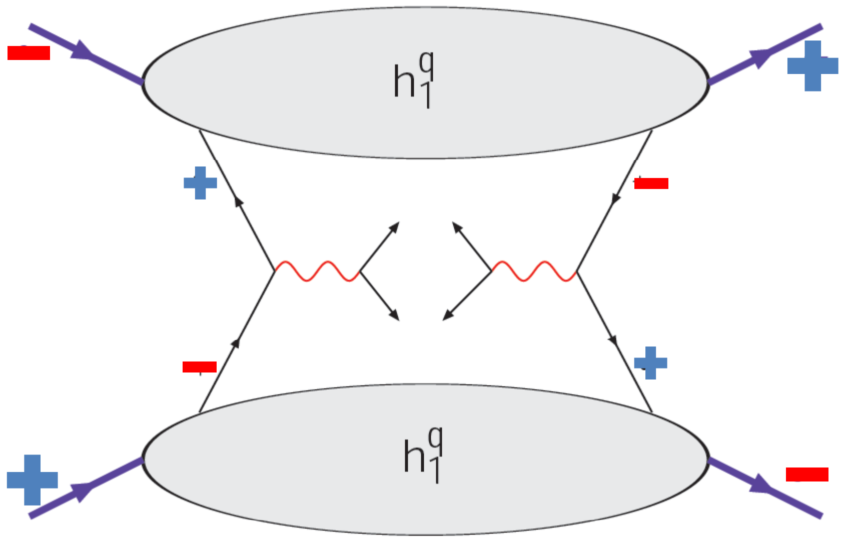
In collinear configuration no analogue of transversity for gluons in spin 1/2 hadrons



Are there alternative ways to measure transversity?

Double transverse spin asymmetry A_{TT} in Drell-Yan dimuon production processes

$$p^\uparrow p^\uparrow \to \ell^+\ell^- + X, \qquad p^\uparrow \bar{p}^\uparrow \to \ell^+\ell^- + X$$



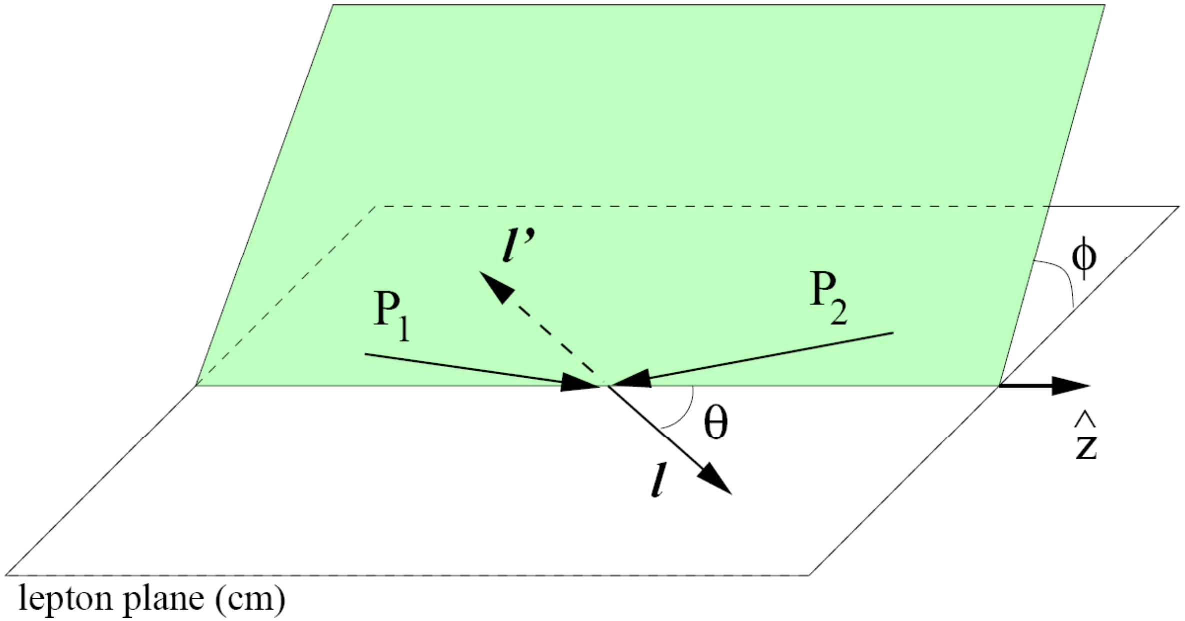
Soon at RHIC
However in pp collisions always a sea quark distribution is involved; estimates indicate a very small asymmetry

On a longer time scale
p-pbar collisions at PAX
Needs efficiently polarized antiprotons
(a formidable task by itself)

$$A_{TT}^{DY} = \frac{d\sigma(\boldsymbol{S}_{1\perp},\boldsymbol{S}_{2\perp}) - d\sigma(\boldsymbol{S}_{1\perp},-\boldsymbol{S}_{2\perp})}{d\sigma(\boldsymbol{S}_{1\perp},\boldsymbol{S}_{2\perp}) + d\sigma(\boldsymbol{S}_{1\perp},-\boldsymbol{S}_{2\perp})}$$

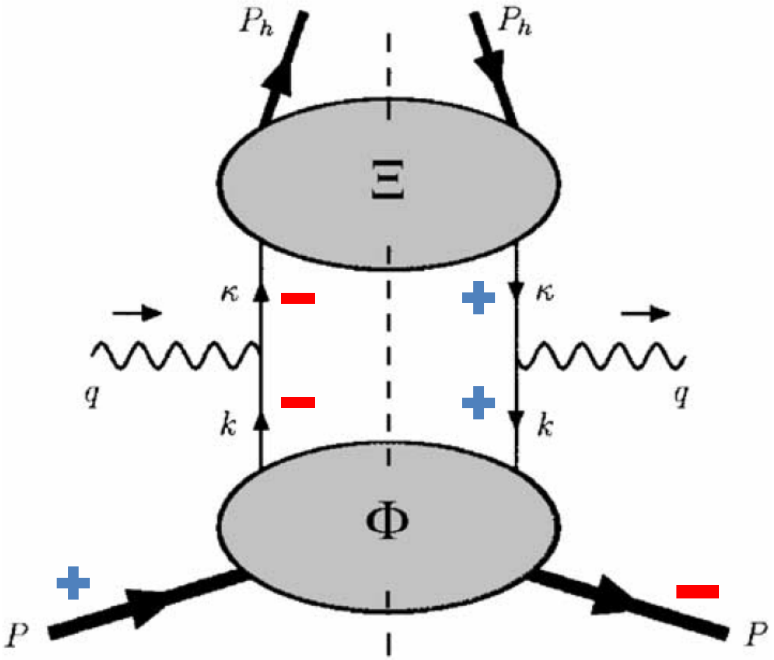
$$= |\boldsymbol{S}_{1\perp}|\,|\boldsymbol{S}_{2\perp}|\,\frac{\sin^2\theta\,\cos(2\phi-\phi_{S_1}-\phi_{S_2})}{1+\cos^2\theta}\,\frac{\sum_q e_q^2\,\Delta_T q(x_q)\Delta_T\bar{q}(x_{\bar{q}}) + [q\leftrightarrow\bar{q}]}{\sum_q e_q^2\,q(x_q)\bar{q}(x_{\bar{q}}) + [q\leftrightarrow\bar{q}]}$$

Kinematics of the Drell-Yan process in the dilepton c.m. frame



Azimuthal spin asymmetries in single particle SIDIS production, with a chiral-odd and transverse momentum dependent fragmentation function (Collins effect, see the sequel)

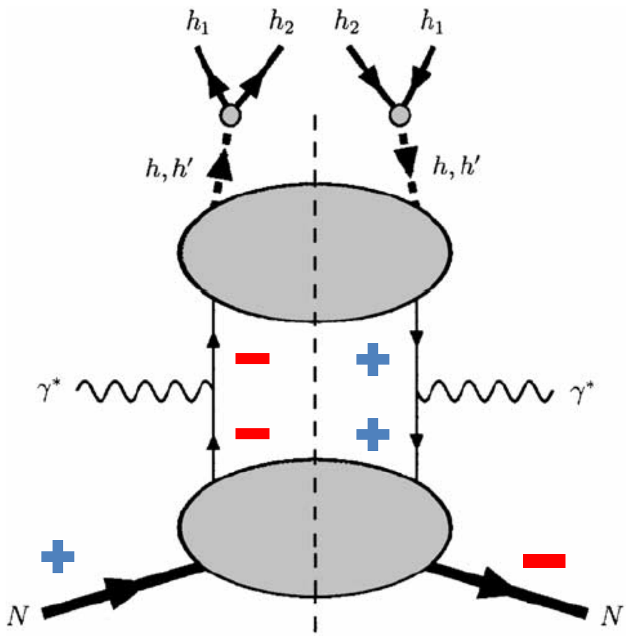
$$\ell\, p^{\uparrow}(P) \to \ell' + h(P_h) + X$$



Azimuthal correlations in two-particle production in SIDIS and e+e- collisions via interference (dihadron) fragmentation functions

$$\ell\, p^{\uparrow}(P) \to \ell' + h_1(P_{h_1}) + h_2(P_{h_2}) + X$$

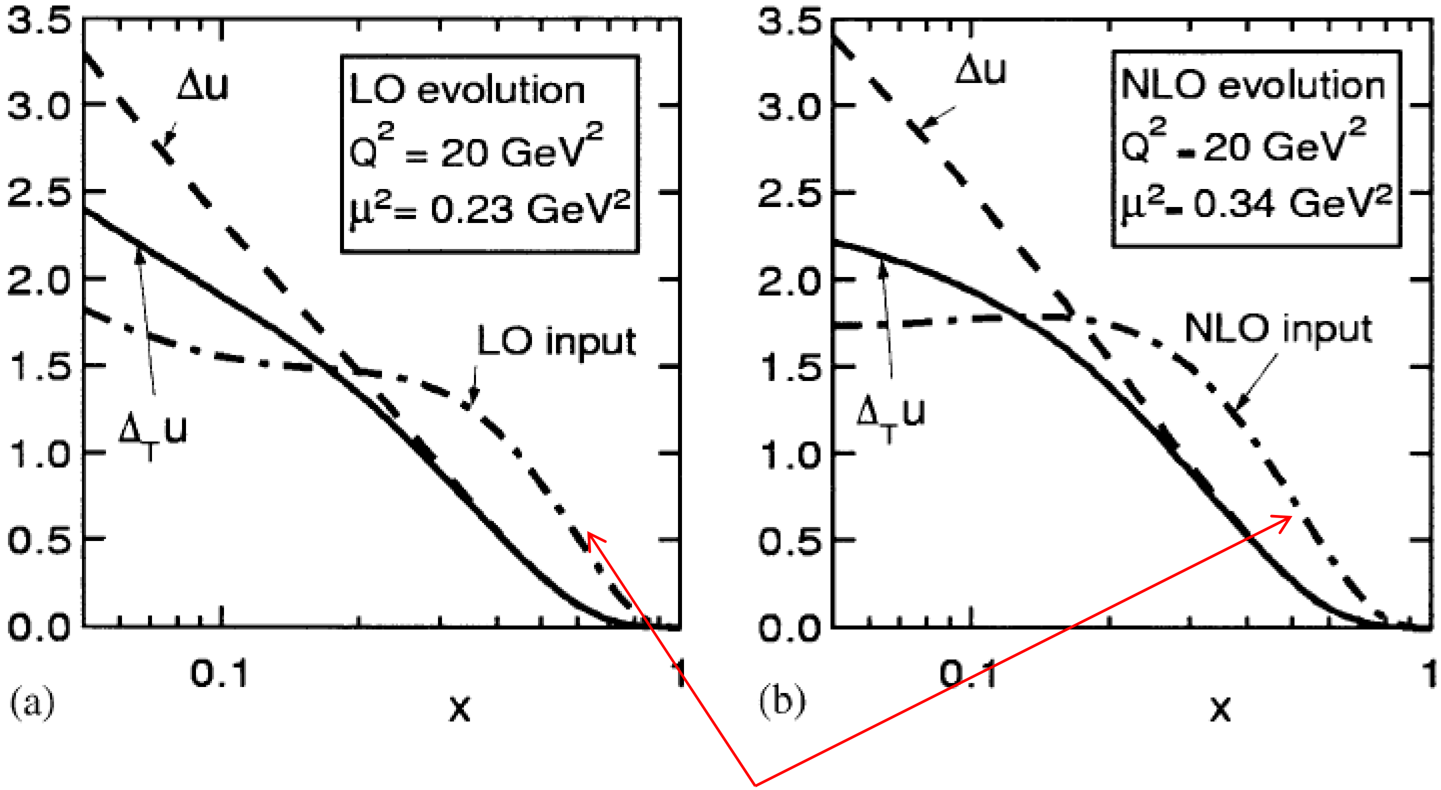
$$(\boldsymbol{P}_{h_1} \times \boldsymbol{P}_{h_2}) \cdot \boldsymbol{s}_{q\perp}$$



Present information on transversity

Relatively simple (non-singlet like) QCD evolution with scale
(no gluon contribution to transversity for spin 1/2 hadrons)

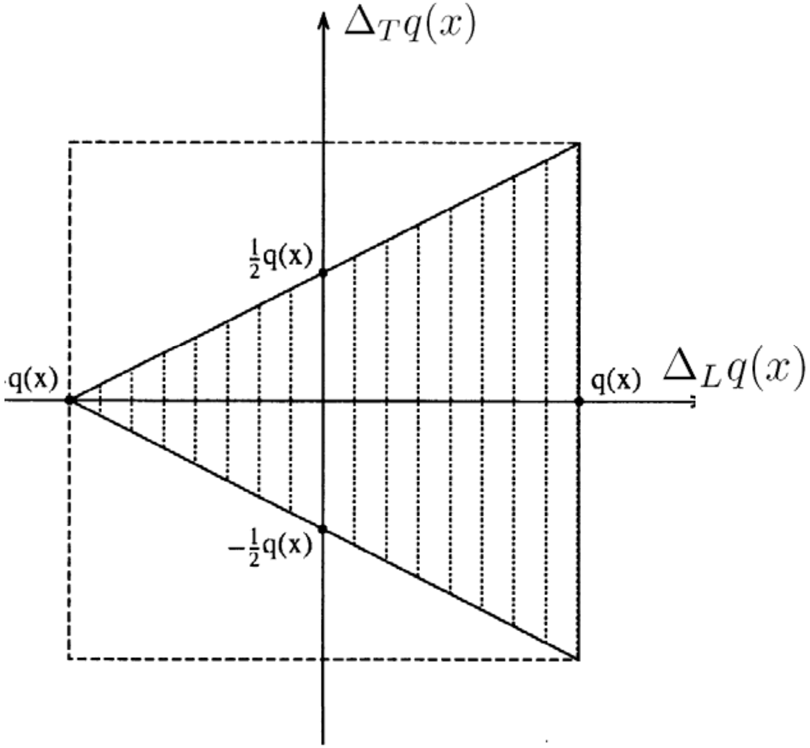
Hayashigaki, Kanazawa, Koike PRD 56 (1997)



$$\Delta_T q(x, \mu) \equiv \Delta_L q(x, \mu)$$

Soffer bound (preserved by pQCD evolution)

$$|\Delta_T q(x,Q)| \le \frac{1}{2}\left[\, q(x,Q) + \Delta_L q(x,Q) \,\right]$$



Tensor charge

$$\langle PS|\bar{\psi}(0)\mathrm{i}\sigma^{\mu\nu}\gamma_5\psi_i(0)|PS\rangle = 2g_T(S^\mu P^\nu - S^\nu P^\mu)$$

$$\delta q(Q^2) = \int_0^1 \mathrm{d}x \left[\, \Delta_T q(x,Q^2) - \Delta_T \bar{q}(x,Q^2) \,\right]$$

Model calculations for the transversity distribution

Relativistic MIT bag models
Colour dielectric models
Chiral quark soliton models
Light-cone models and Melosh rotation
Diquark spectator models

Problems with model scale determination

Non perturbative QCD calculations
(tensor charge)
QCD sum rules
Lattice calculations

First parameterizations by fitting spin and azimuthal asymmetries in SIDIS hadron production with polarized and transverse momentum dependent PDFs and FFs (see sequel)

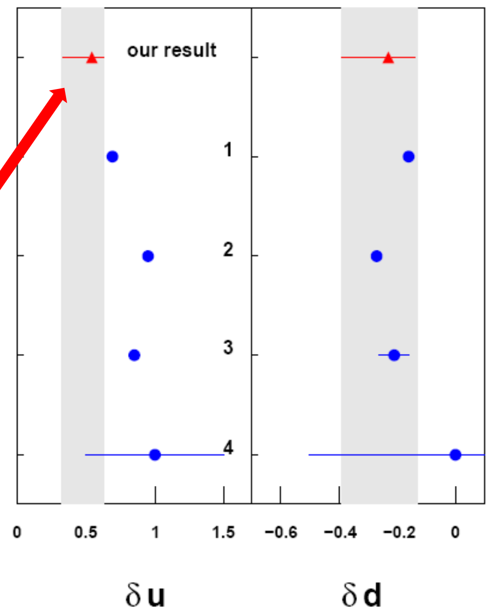
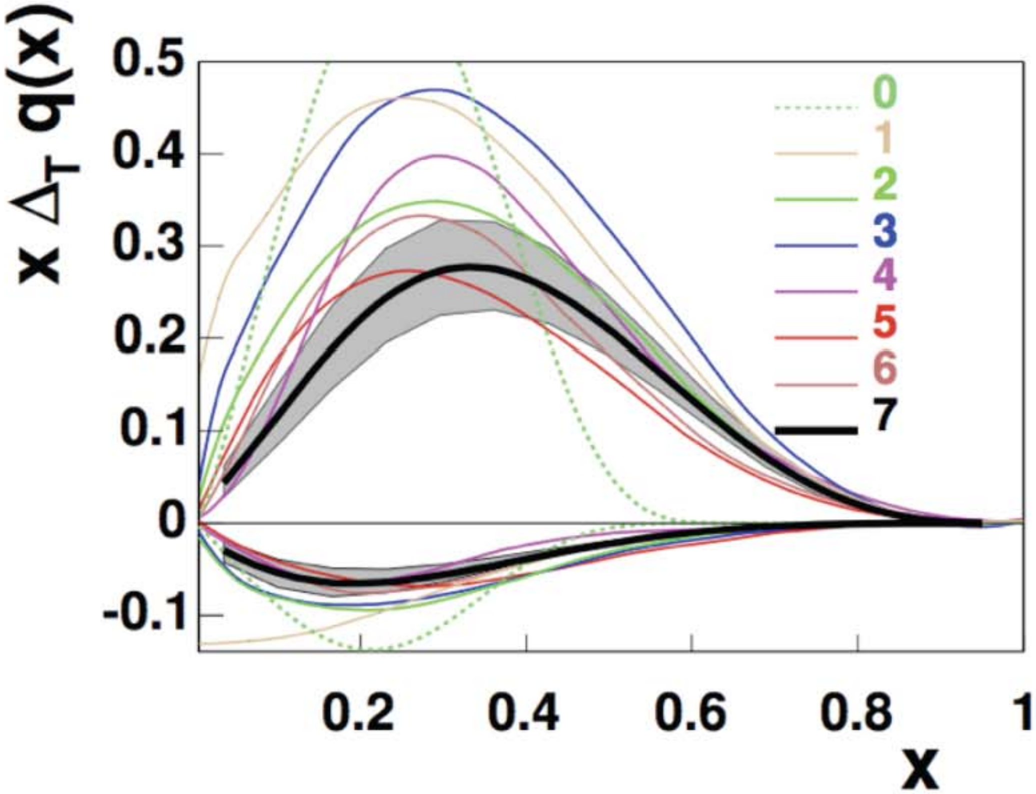


Figure 8. Tensor charge from different models compared to our result. 1: Quark-diquark model of Ref. [47], 2: Chiral quark soliton model of Ref. [48], 3: Lattice QCD [49], 4: QCD sum rules [50].

Anselmino et al

Model calculations for the transversity distribution



0. Barone, Calarco, Drago PLB 390 287 (97)
1. Soffer et al. PRD 65 (02)
2. Korotkov et al. EPJC 18 (01)
3. Schweitzer et al. PRD 64 (01)
4. Wakamatsu, PLB B653 (07)
5. Pasquini et al., PRD 72 (05)
6. Cloet, Bentz and Thomas PLB 659 (08)
7. This analysis.

Azimuthal and spin asymmetries in high-energy inclusive and semi-inclusive particle production

Theoretical expectactions (circa 1980)

Longitudinal single spin asymmetries

Forbidden by rotational and parity invariance of strong interactions
can arise in weak processes; we will not consider them here

Transverse spin asymmetries (SSA)

Expected to be negligible in high-energy hadronic processes. In the framework of pQCD and collinear factorization, at leading twist SSAs are related to the imaginary part of interference terms between two elementary scattering amplitudes, off-diagonal (helicity-flip) in the helicity indices of the involved (transversely polarized) parton

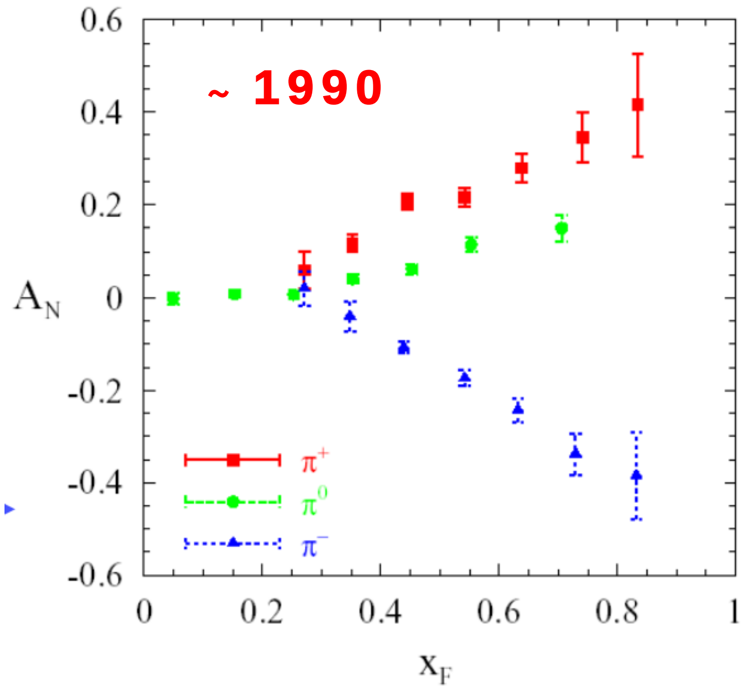
At tree level, all helicity amplitudes are real;
imaginary parts can only arise through higher-order (loop) contributions

The QCD massless quark –gluon coupling preserves helicity at all perturbative orders
Helicity flip contributions in the amplitudes must be proportional to (powers of) m_q/E_q

$$\hat{a}_N = \frac{d\hat{\sigma}^{a^\uparrow b \to cd} - d\hat{\sigma}^{a^\downarrow b \to cd}}{d\hat{\sigma}^{a^\uparrow b \to cd} + d\hat{\sigma}^{a^\downarrow b \to cd}} \propto \alpha_s(\hat{s})\,\frac{m_q}{\hat{s}} \sim \alpha_s(p_T)\,\frac{m_q}{p_T}$$

Kane, Pumplin, Repko PRL 41 (1978)

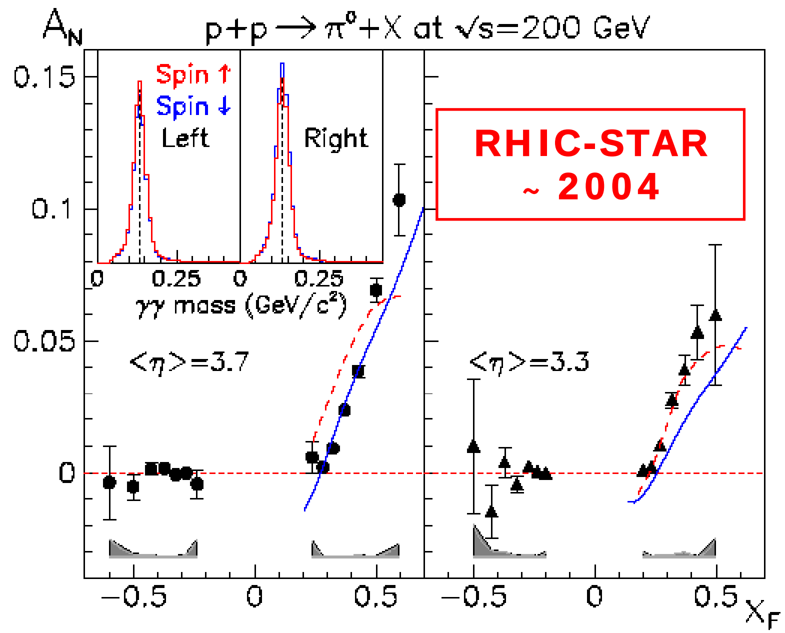
Unexpected experimental results for pion SSAs in pp collisions

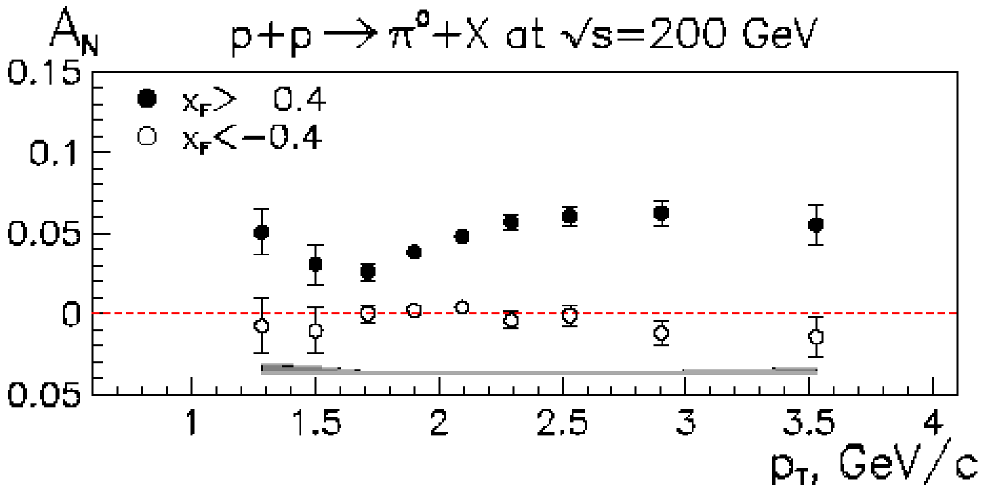


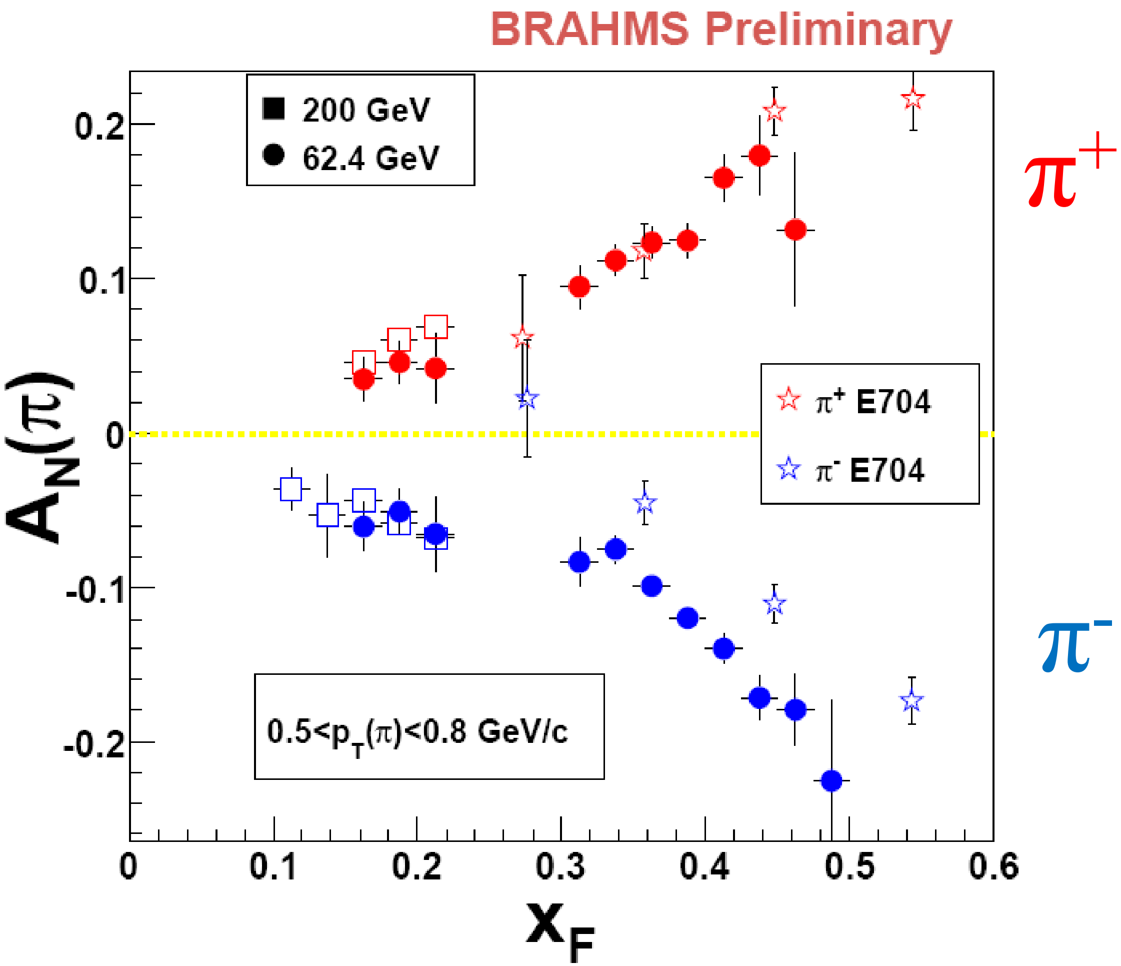
Fermilab E704: $A_N(p^\uparrow p \to \pi X)$

$\sqrt{s} = 20$ GeV, $0.7 < p_T < 2.0$ GeV

$$x_F = 2p_L/\sqrt{s}$$

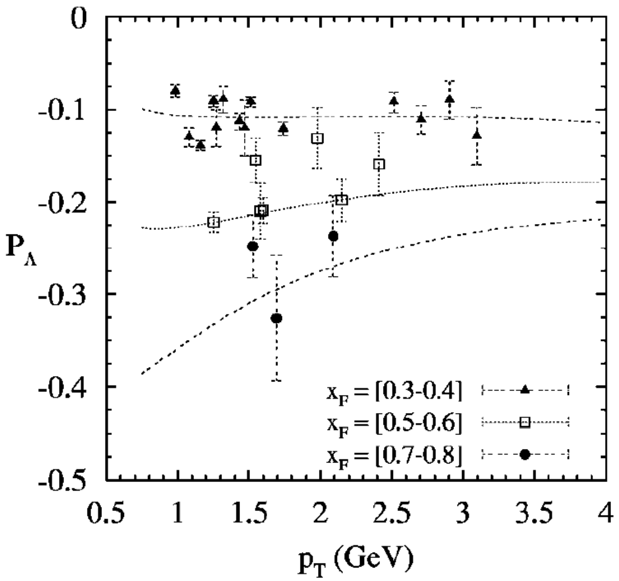






C. Aidala, Spin2008
see lectures on Friday

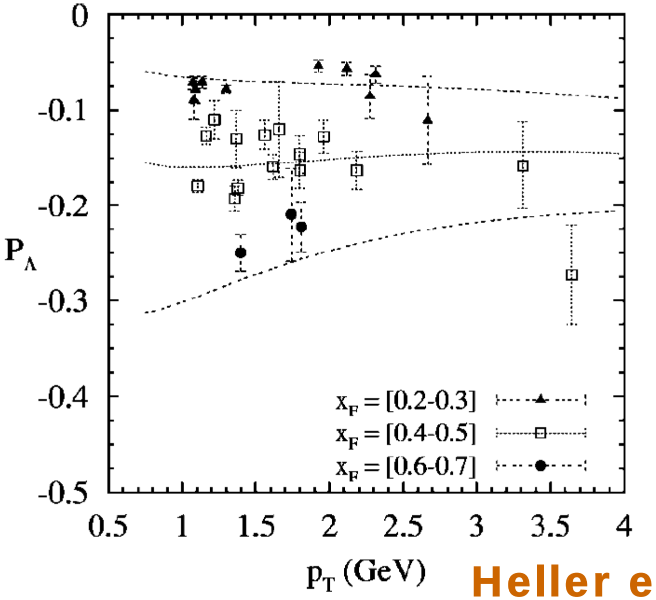
Transverse hyperon polarization in unpolarized pN collisions

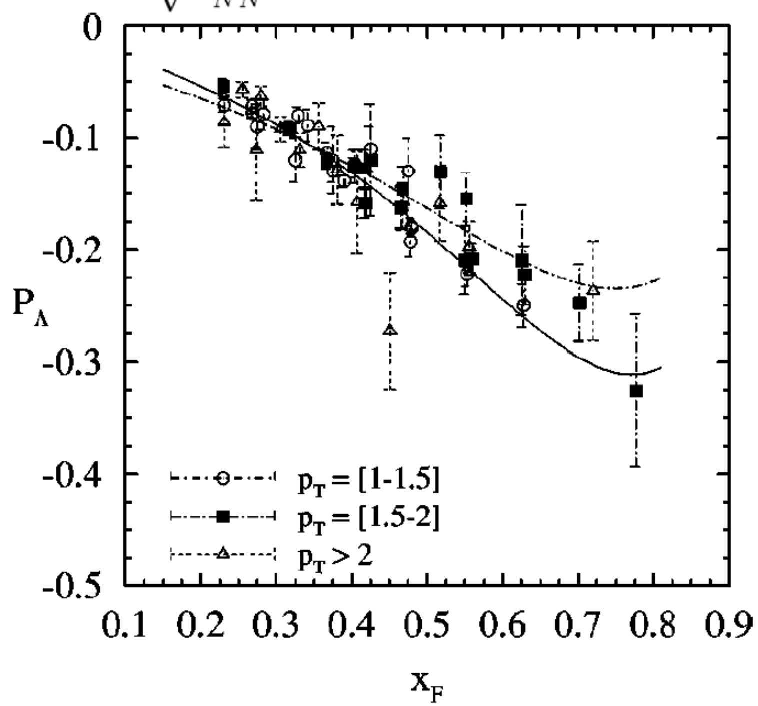


$$P_\Lambda = \frac{d\sigma^{AB\to\Lambda^\uparrow X} - d\sigma^{AB\to\Lambda^\downarrow X}}{d\sigma^{AB\to\Lambda^\uparrow X} + d\sigma^{AB\to\Lambda^\downarrow X}}$$

$$P_\Lambda(p\,Be \to \Lambda^\uparrow X)$$

$$\sqrt{s_{NN}} \simeq 27 \text{ GeV and } 39 \text{ GeV}$$





Heller et al - Lundberg et al - Ramberg et al (1978-1994)

1991

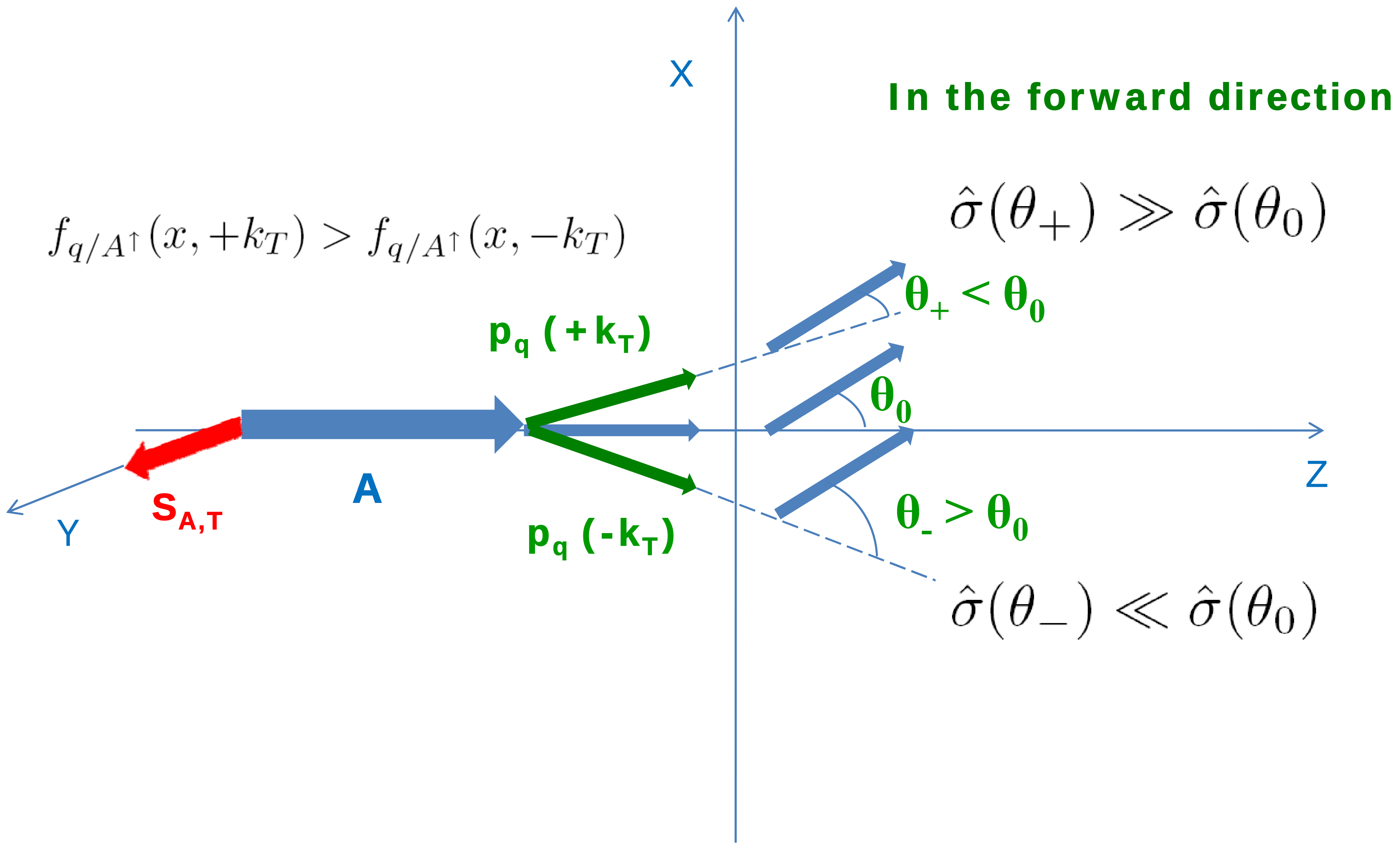
D. Sivers suggests that the origin of the huge hadronic SSAs observed at large positive Feynman x and intermediate hadronic p_T, could not be due to the underlining partonic asymmetry. Rather, they could be related to:

A transverse momentum dependent (TMD) PDF significantly asymmetric in the azimuthal distribution of the (un)polarized partons around the direction of motion of the transversely polarized parent hadron
(The leading-twist TMD Sivers distribution function)

$$\Delta \hat{f}_{a/A^\uparrow}(x, \boldsymbol{k}_\perp) \equiv \hat{f}_{a/A^\uparrow}(x, \boldsymbol{k}_\perp) - \hat{f}_{a/A^\downarrow}(x, \boldsymbol{k}_\perp) = \hat{f}_{a/A^\uparrow}(x, \boldsymbol{k}_\perp) - \hat{f}_{a/A^\uparrow}(x, -\boldsymbol{k}_\perp)$$

Chiral –even (no quark helicity-flip required, naively Time Reversal odd (T-odd)

An azimuthally coherent [higher-twist in (k_T/p_T)] dependence of the unpol. partonic cross sections from intrinsic parton momenta (k_T), relevant at the intermediate-large hadronic p_T observed in the experiments ($p_T \sim$1-4 GeV)


X
In the forward direction
$\hat{\sigma}(\theta_+) \gg \hat{\sigma}(\theta_0)$
$f_{q/A^\uparrow}(x,+k_T) > f_{q/A^\uparrow}(x,-k_T)$
$\theta_+ < \theta_0$
$p_q\ (+k_T)$
θ_0
Z
A
$S_{A,T}$
Y
$p_q\ (-k_T)$
$\theta_- > \theta_0$
$\hat{\sigma}(\theta_-) \ll \hat{\sigma}(\theta_0)$

1993

J. Collins suggests that the naively T-odd Sivers distribution function should vanish because of time-reversal invariance (see however the sequel!)

He proposes an alternative mechanism, acting in the fragmentation process, for SSAs in semi-inclusive DIS with a transv. polarized proton target, $lp^{\uparrow} \longrightarrow l' h X$. This mechanism involves:

the TMD transversity distribution in the initial polarized hadron (opening an alternative way to measure it!)

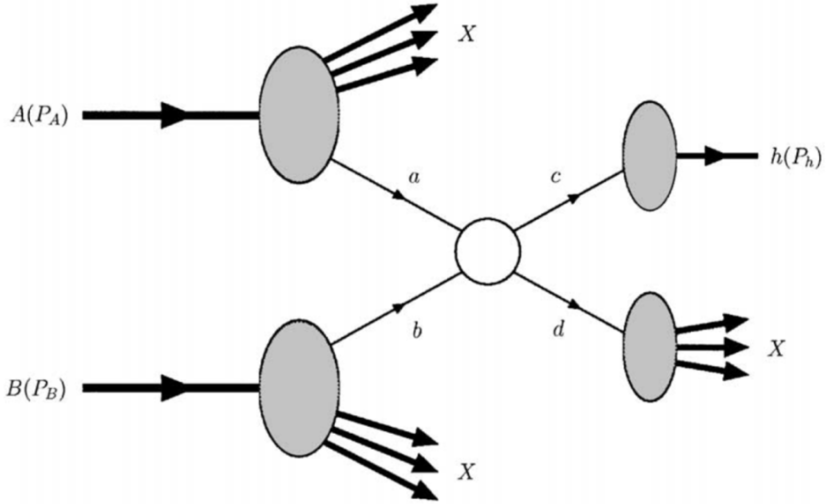
The partonic transverse polarization transfer from the initial polarized parton to the final fragmenting parton

An azimuthal asymmetry around the jet axis in the distribution of the observed (un)polarized hadrons (the Collins effect)

$$\Delta\hat{D}_{C/q^{\uparrow}}(z,\boldsymbol{k}_{\perp}) \equiv \hat{D}_{C/q^{\uparrow}}(z,\boldsymbol{k}_{\perp}) - \hat{D}_{C/q^{\downarrow}}(z,\boldsymbol{k}_{\perp}) = \hat{D}_{C/q^{\uparrow}}(z,\boldsymbol{k}_{\perp}) - \hat{D}_{C/q^{\uparrow}}(z,-\boldsymbol{k}_{\perp})$$

Chiral-odd, naively T-odd

Single particle inclusive production in pp collisions in a QCD-inspired parton model



$$\frac{E_C\, d\sigma^{AB\to C\,X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d} \int dx_a\, dx_b\, dz\, f_{a/A}(x_a, Q^2)\, f_{b/B}(x_b, Q^2)\, \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}}\, \frac{\hat{s}}{\pi z^2}\, \delta(\hat{s}+\hat{t}+\hat{u})\, D_{C/c}(z, Q^2)$$

Generalized parton model: Leading twist TMD PD functions and FFs

Consider the doubly polarized invariant differential cross section for single-inclusive particle production, A(S_A) B(S_B) $\longrightarrow$ C + X:

$$\frac{E_C\, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d,\{\lambda\}} \int \frac{dx_a\, dx_b\, dz}{16\pi^2 x_a x_b z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_{\perp C}\, \delta(\boldsymbol{k}_{\perp C}\cdot\hat{\boldsymbol{p}}_c)\, J(\boldsymbol{k}_{\perp C})$$
$$\times\ \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\, \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\, \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b})$$
$$\times\ \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\, \delta(\hat{s}+\hat{t}+\hat{u})\, \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})\,,$$

- A, B are the initial spin 1/2 hadrons, typically two protons in pure spin states S_A, S_B
- C is the observed unpolarized particle, typically a pion, kaon or photon; can be extended to polarized spin 1/2 or spin 1 hadrons (hyperons, ρ mesons, etc.)
- J(k⊥c) is a phase-space kinematical factor [J(k⊥c) $\longrightarrow$ 1 for massless collinear partons/hadrons]
- Consider the process in the hadronic (AB) c.m. frame, with hadron A moving along the +Z_{cm} axis and particle C produced in the $(XZ)_{cm}$ upper-half plane
- The notation $\{\lambda\}$ indicates a sum over all helicity indices
- x_a, x_b, z, are usual light-cone momentum fractions
- The ρ's are the helicity density matrices of partons a(b) inside the polarized hadrons A(B)

$\hat{f}_{a/A,S_A}(x_a, \boldsymbol{k}_{\perp a})$ are the leading-twist spin and transverse momentum dependent (TMD) PDF of the unpolarized parton a inside hadron A with spin S_A (analogously for b/B); they generalize the usual partonic distributions in collinear (k_T-integrated) configuration

$\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$ are the helicity amplitudes for the hard elementary process ab $\longrightarrow$ cd

$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z, \boldsymbol{k}_{\perp C})$ are the leading twist TMD fragmentation functions for the process c $\longrightarrow$ C + X [for unpolarized C they are diagonal in the hadron helicity indices]

The polarization of parton a (with spin s_a) is determined by the polarization state of the parent hadron A (with spin S_A, fixed by experimental conditions) and by the soft nonperturbative processes related to the hadron structure (analogously for parton b inside B). Complete information is encoded in

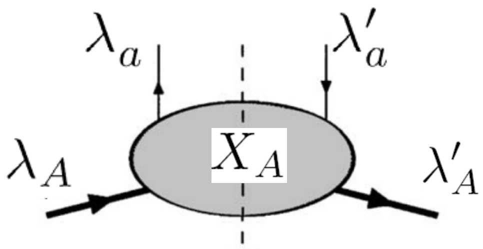
$$\rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\,\hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a}) = \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A} \,\not{\int}_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}\hat{\mathcal{F}}^{*}_{\lambda'_a,\lambda_{X_A};\lambda'_A} \equiv \sum_{\lambda_A,\lambda'_A} \rho^{A,S_A}_{\lambda_A,\lambda'_A}\,\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}$$

$$\rho^{A,S_A}_{\lambda_A,\lambda'_A} = \frac{1}{2}\begin{pmatrix} 1+P^A_Z & P^A_X - \mathrm{i}P^A_Y \\ P^A_X + \mathrm{i}P^A_Y & 1-P^A_Z \end{pmatrix}_{A,S_A} = \frac{1}{2}\begin{pmatrix} 1+P^A_L & P^A_T e^{-\mathrm{i}\phi_{S_A}} \\ P^A_T e^{\mathrm{i}\phi_{S_A}} & 1-P^A_L \end{pmatrix}_{A,S_A}$$

Helicity Density Matrix of hadron A

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,\boldsymbol{k}_{\perp a}) \equiv \not{\int}_{X_A,\lambda_{X_A}} \hat{\mathcal{F}}_{\lambda_a,\lambda_{X_A};\lambda_A}(x_a,\boldsymbol{k}_{\perp a})\hat{\mathcal{F}}^{*}_{\lambda'_a,\lambda_{X_A};\lambda'_A}(x_a,\boldsymbol{k}_{\perp a})$$

Related do the leading twist hadronic correlator



Due to rotational invariance

$$\hat{F}^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,\boldsymbol{k}_{\perp a}) = F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}(x_a,k_{\perp a})\exp[i(\lambda_A-\lambda'_A)\phi_a]$$

And to parity invariance of strong interactions

$$F^{-\lambda_a,-\lambda'_a}_{-\lambda_A,-\lambda'_A} = (-1)^{2(S_A-s_a)}(-1)^{(\lambda_A-\lambda_a)+(\lambda'_A-\lambda'_a)}F^{\lambda_a,\lambda'_a}_{\lambda_A,\lambda'_A}$$

Some parity relations are different for quarks ($s_q=1/2$) and gluons ($s_g=1$)

There are eight independent leading twist TMD functions

$$F^{++}_{++},\quad F^{--}_{++},\quad F^{+-}_{+-},\quad F^{-+}_{+-},\quad F^{++}_{+-},\quad F^{--}_{+-},\quad F^{+-}_{++},\quad F^{+-}_{--}$$

e.g. for quarks [upper (lower) indices refer to parton (hadron) helicities]

$F^{++}_{++} \pm F^{--}_{++}$ $\Rightarrow$ Obviously purely real quantities
Unpolarized and longitudinally polarized PDFs, f_1 and g_{1L}

$F^{+-}_{+-} \pm F^{-+}_{+-}$ $\Rightarrow$ Purely real(imaginary) quantities for quarks(gluons)
Contributions to the quark TMD transversity DF,: $h_1,\ \ h_{1T}^{\perp}$

$F^{++}_{+-} \pm F^{--}_{+-}$ $\Rightarrow$ Sivers function and $g_{1T}^{\perp}$ PDF

$F^{+-}_{++} \pm F^{+-}_{--}$ $\Rightarrow$ Boer-Mulders function and $h_{1L}^{\perp}$ PDF

These last 4 complex functions are not independent; can also be written in terms of the real and imaginary parts of two of them

quark sector

$$
\begin{aligned}
\hat{f}_{a/A} &= \hat{f}_{a/A,S_L} = (F_{++}^{++} + F_{--}^{++}) \\
\hat{f}_{a/A,S_T} &= \hat{f}_{a/A} + \frac{1}{2}\,\Delta\hat{f}_{a/S_T} = (F_{++}^{++} + F_{--}^{++}) + 2\,\left[\mathrm{Im}F_{+-}^{++}\,\sin(\phi_{S_A} - \phi_a)\right] \\
P_x^a\,\hat{f}_{a/A,S_L} &= \Delta\hat{f}_{s_x/S_L} = 2\,\mathrm{Re}F_{++}^{+-} \\
P_x^a\,\hat{f}_{a/A,S_T} &= \Delta\hat{f}_{s_x/S_T} = \left[F_{+-}^{+-} + F_{+-}^{-+}\right]\,\cos(\phi_{S_A} - \phi_a) \\
P_y^a\,\hat{f}_{a/A,S_L} &= P_y^a\,\hat{f}_{a/A} = \Delta\hat{f}_{s_y/S_L} = -2\,\mathrm{Im}F_{++}^{+-} \\
P_y^a\,\hat{f}_{a/A,S_T} &= \Delta\hat{f}_{s_y/S_T} = -2\,\mathrm{Im}F_{++}^{+-} + \left[F_{+-}^{+-} - F_{+-}^{-+}\right]\,\sin(\phi_{S_A} - \phi_a) \\
P_z^a\,\hat{f}_{a/A,S_L} &= \Delta\hat{f}_{s_z/S_L} = (F_{++}^{++} - F_{--}^{++}) \\
P_z^a\,\hat{f}_{a/A,S_T} &= \Delta\hat{f}_{s_z/S_T} = 2\,\left[\mathrm{Re}F_{+-}^{++}\,\cos(\phi_{S_A} - \phi_a)\right]
\end{aligned}
$$

Amsterdam Group Notation

$$
\begin{aligned}
f_1(x_a, k_{\perp a}) &= F_{++}^{++} + F_{--}^{++} \\
\frac{k_{\perp a}}{M}\,f_{1T}^{\perp}(x_a, k_{\perp a}) &= -2\,\mathrm{Im}F_{+-}^{++} \\
g_{1L}(x_a, k_{\perp a}) &= F_{++}^{++} - F_{--}^{++} \\
\frac{k_{\perp a}}{M}\,g_{1T}^{\perp}(x_a, k_{\perp a}) &= 2\,\mathrm{Re}F_{+-}^{++}
\end{aligned}
$$

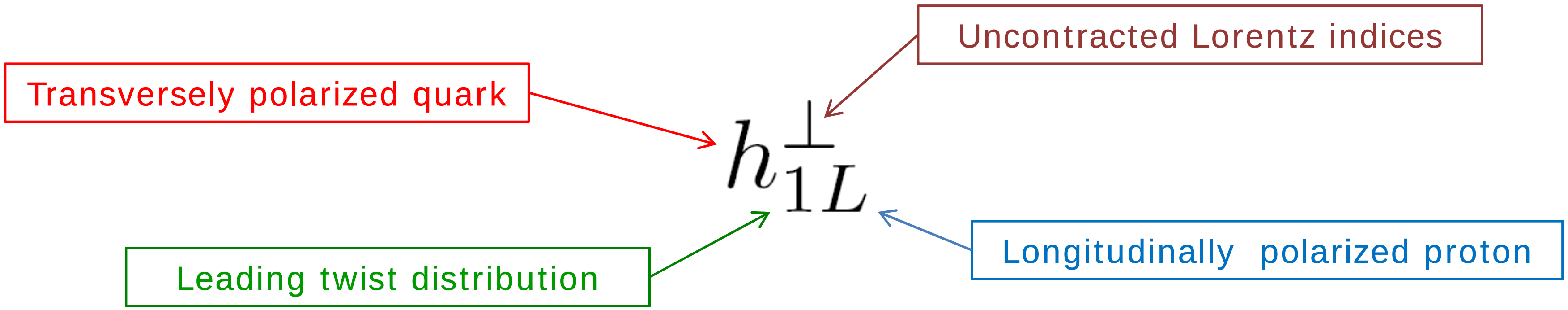
$$
\begin{aligned}
\frac{k_{\perp a}}{M}\,h_{1L}^{\perp}(x_a, k_{\perp a}) &= 2\mathrm{Re}F_{++}^{+-} \\
\frac{k_{\perp a}}{M}\,h_{1}^{\perp}(x_a, k_{\perp a}) &= 2\,\mathrm{Im}F_{++}^{+-} \\
h_1(x_a, k_{\perp a}) &= F_{+-}^{+-} \\
\frac{k_{\perp a}^2}{2M^2}\,h_{1T}^{\perp}(x_a, k_{\perp a}) &= F_{+-}^{-+}
\end{aligned}
$$

Jaffe, Ji – Amsterdam group notation

f, g, h : unpolarized, longitudinally pol., transversely pol. quark

Subscript 1: leading twist distribution
Subscript L: longitudinally polarized hadron
Subscript T: transversely polarized hadron

Apex $\perp$: presence of transverse momenta with uncontracted Lorentz indices



$$h_{1L}^{\perp}$$

gluon sector

$$\rho^{g/A,S_A}_{\lambda_g,\lambda'_g} = \frac{1}{2}\begin{pmatrix} 1+P^g_z & \mathcal{T}^g_1 - i\mathcal{T}^g_2 \\ \mathcal{T}^g_1 + i\mathcal{T}^g_2 & 1-P^g_z \end{pmatrix}_{A,S_A} = \frac{1}{2}\begin{pmatrix} 1+P^g_{circ} & -P^g_{lin}\, e^{-2i\phi} \\ -P^g_{lin}\, e^{2i\phi} & 1-P^g_{circ} \end{pmatrix}_{A,S_A}$$

$$\begin{aligned}
\hat{f}_{g/A} &= \hat{f}_{g/A,S_L} = (F^{++}_{++} + F^{++}_{--}) \\
\hat{f}_{g/A,S_T} &= \hat{f}_{g/A} + \frac{1}{2}\Delta\hat{f}_{g/A,S_T} = (F^{++}_{++} + F^{++}_{--}) + 2\,\left[\mathrm{Im}F^{++}_{+-}\sin(\phi_{S_A} - \phi_a)\right] \\
\mathcal{T}^g_1\,\hat{f}_{g/A,S_L} &= \mathcal{T}^g_1\,\hat{f}_{g/A} = \Delta\hat{f}^g_{\mathcal{T}_1/S_L} = 2\,\mathrm{Re}F^{+-}_{++} \\
\mathcal{T}^g_1\,\hat{f}_{g/A,S_T} &= \Delta\hat{f}^g_{\mathcal{T}_1/S_T} = 2\,\mathrm{Re}F^{+-}_{++} + \mathrm{Im}\left[F^{+-}_{+-} + F^{-+}_{+-}\right]\sin(\phi_{S_A} - \phi_a) \\
\mathcal{T}^g_2\,\hat{f}_{g/A,S_L} &= \Delta\hat{f}^g_{\mathcal{T}_2/S_L} = -2\,\mathrm{Im}F^{+-}_{++} \\
\mathcal{T}^g_2\,\hat{f}_{g/A,S_T} &= \Delta\hat{f}^g_{\mathcal{T}_2/S_T} = -\mathrm{Im}\left[F^{+-}_{+-} - F^{-+}_{+-}\right]\cos(\phi_{S_A} - \phi_a) \\
P^g_z\,\hat{f}_{g/A,S_L} &= \Delta\hat{f}^g_{s_L/S_L} = (F^{++}_{++} - F^{++}_{--}) \\
P^g_z\,\hat{f}_{g/A,S_T} &= \Delta\hat{f}^g_{s_L/S_T} = 2\,\left[\mathrm{Re}F^{++}_{+-}\cos(\phi_{S_A} - \phi_a)\right]
\end{aligned}$$

Mulders, Rodrigues: PRD63 (2001) 094021

TMD fragmentation functions into unpol. hadrons (leading twist)

Introducing the soft, nonperturbative helicity fragmentation amplitudes for the process c → C + X, the following properties hold for their products:

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = \sum\!\!\!\!\!\!\int_{X,\lambda_X} \hat{\mathcal{D}}_{\lambda_C,\lambda_X;\lambda_c}(z,\boldsymbol{k}_{\perp C})\,\hat{\mathcal{D}}^*_{\lambda'_C,\lambda_X;\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

$$\hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = D^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})\, e^{i(\lambda_c-\lambda'_c)\phi^H_C}$$

$$\hat{D}^{C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = \sum_{\lambda_C,\lambda_C} \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C}) = D^{C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})e^{i(\lambda_c-\lambda'_c)\phi^H_C}$$

$$D^{C}_{-\lambda_c,-\lambda'_c}(z,k_{\perp C}) = (-1)^{2s_c}(-1)^{\lambda_c+\lambda'_c}D^{C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})$$

Quark sector

$$\hat{D}^{C/q}_{++}(z,\boldsymbol{k}_{\perp C}) = D^{C/q}_{++}(z,k_{\perp C}) \equiv \hat{D}_{C/q}(z,k_{\perp C})$$

$$2\,\mathrm{Im}\,D^{C/q}_{+-}(z,k_{\perp C}) \equiv \Delta^N\hat{D}_{C/q^\uparrow}(z,k_{\perp C})$$

Gluon sector

$$\hat{D}^{C/g}_{++}(z,\boldsymbol{k}_{\perp C}) = D^{C/g}_{++}(z,k_{\perp C}) \equiv \hat{D}_{C/g}(z,k_{\perp C})$$

$$2\,\mathrm{Re}\,D^{C/g}_{+-}(z,k_{\perp C}) \equiv \Delta^N\hat{D}_{C/\mathcal{T}^g_1}(z,k_{\perp C})$$

TMD fragmentation functions – spin 1/2 hadrons, LT

$$\rho^{C}_{\lambda_C,\lambda'_C}\,\hat{D}_{C/c,s_c}(z,\boldsymbol{k}_{\perp C}) = \sum_{\lambda_c,\lambda'_c} \hat{D}^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})\rho^{c,s_c}_{\lambda_c,\lambda'_c}$$

$$D^{-\lambda_C,-\lambda'_C}_{-\lambda_c,-\lambda'_c}(z,k_{\perp C}) = (-1)^{2(s_c-S_C)}\,(-1)^{(\lambda_c+\lambda'_c)-(\lambda_C+\lambda'_C)}\,D^{\lambda_C,\lambda'_C}_{\lambda_c,\lambda'_c}(z,k_{\perp C})$$

Summary of quark and gluons
helicity fragmentation amplitudes for spin 1/2 hadrons

$$\hat{D}^{C}_{c}(z,k_{\perp C}) = D^{++}_{++} + D^{++}_{--}$$
$$\Delta^N\hat{D}_{C/c^\uparrow}(z,\boldsymbol{k}_{\perp C}) = 4\,\mathrm{Im}D^{++}_{+-}\ \sin(\phi_{s_c}-\phi^H_C)$$
$$\Delta^N\hat{D}_{C/T^c_1}(z,\boldsymbol{k}_{\perp C}) = -4\,P^c_{lin}\mathrm{Re}D^{++}_{+-}\ \cos(2\phi_{l_c}-2\phi^H_C)$$
$$\Delta\hat{D}^{C/c}_{S_{Z_C}/s_z}(z,k_{\perp C}) = D^{++}_{++} - D^{++}_{--}$$
$$\Delta\hat{D}^{C/c}_{S_{Z_C}/s_T}(z,\boldsymbol{k}_{\perp C}) = 2\,\mathrm{Re}D^{++}_{+-}\ \cos(\phi_{s_c}-\phi^H_C)$$
$$\Delta\hat{D}_{S_{Z_C}/c,s_z}(z,\boldsymbol{k}_{\perp C}) = -2\,P^c_{lin}\mathrm{Im}(D^{++}_{+-})\ \sin(2\phi_{l_c}-2\phi^H_C)$$
$$\Delta\hat{D}^{C/c}_{S_{X_C}/s_z}(z,k_{\perp C}) = 2\,\mathrm{Re}D^{+-}_{++}$$
$$\Delta\hat{D}^{C/c}_{S_{X_C}/s_T}(z,\boldsymbol{k}_{\perp C}) = \left(D^{+-}_{+-} + D^{-+}_{+-}\right)\ \cos(\phi_{s_c}-\phi^H_C)$$
$$\Delta\hat{D}_{S_{X_C}/P_{lin}}(z,\boldsymbol{k}_{\perp C}) = -\left(\mathrm{Im}D^{+-}_{+-} + \mathrm{Im}D^{-+}_{+-}\right)\ \sin(2\phi_{l_c}-2\phi^H_C)$$
$$\Delta\hat{D}^{C/c}_{S_{Y_C}/c}(z,k_{\perp C}) = -2\,\mathrm{Im}D^{+-}_{++}$$
$$\Delta^-\hat{D}^{C/c}_{S_{Y_C}/s_T}(z,\boldsymbol{k}_{\perp C}) = \left(D^{+-}_{+-} - D^{-+}_{+-}\right)\ \sin(\phi_{s_c}-\phi^H_C)$$
$$\Delta^-\hat{D}_{S_{Y_C}/P_{lin}}(z,\boldsymbol{k}_{\perp C}) = \left(\mathrm{Im}D^{+-}_{+-} - \mathrm{Im}D^{-+}_{+-}\right)\cos(2\phi_{l_c}-2\phi^H_C)$$

unpolarized FF

Collins FF

Polarizing FF

Sivers distribution function (chiral-even, naively T-odd)

$$\Delta \hat{f}_{a/A^\uparrow}(x, \boldsymbol{k}_\perp) \equiv \hat{f}_{a/A^\uparrow}(x, \boldsymbol{k}_\perp) - \hat{f}_{a/A^\downarrow}(x, \boldsymbol{k}_\perp) = \hat{f}_{a/A^\uparrow}(x, \boldsymbol{k}_\perp) - \hat{f}_{a/A^\uparrow}(x, -\boldsymbol{k}_\perp)$$

Boer-Mulders function (chiral odd, naively T-odd)

$$\Delta \hat{f}_{a^\uparrow/A}(x, \boldsymbol{k}_\perp) \equiv \hat{f}_{a^\uparrow/A}(x, \boldsymbol{k}_\perp) - \hat{f}_{a^\downarrow/A}(x, \boldsymbol{k}_\perp) = \hat{f}_{a^\uparrow/A}(x, \boldsymbol{k}_\perp) - \hat{f}_{a^\uparrow/A}(x, -\boldsymbol{k}_\perp)$$

Collins fragmentation function (chiral-odd, naively T-odd)

$$\Delta \hat{D}_{C/q^\uparrow}(z, \boldsymbol{k}_\perp) \equiv \hat{D}_{C/q^\uparrow}(z, \boldsymbol{k}_\perp) - \hat{D}_{C/q^\downarrow}(z, \boldsymbol{k}_\perp) = \hat{D}_{C/q^\uparrow}(z, \boldsymbol{k}_\perp) - \hat{D}_{C/q^\uparrow}(z, -\boldsymbol{k}_\perp)$$

"Polarizing" fragmentation function (chiral even, naively T-odd)

$$\Delta \hat{D}_{\Lambda^\uparrow/q}(z, \boldsymbol{k}_\perp) \equiv \hat{D}_{\Lambda^\uparrow/q}(z, \boldsymbol{k}_\perp) - \hat{D}_{\Lambda^\downarrow/q}(z, \boldsymbol{k}_\perp) = \hat{D}_{\Lambda^\uparrow/q}(z, \boldsymbol{k}_\perp) - \hat{D}_{\Lambda^\uparrow/q}(z, -\boldsymbol{k}_\perp)$$

Theoretical information on TMD functions

Simple positivity bounds, e.g.

$$\left|\frac{\Delta^N f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp)}{2\,f_{q/p}(x,\boldsymbol{k}_\perp)}\right| = \left|\frac{f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp)-f_{q/p^\downarrow}(x,\boldsymbol{k}_\perp)}{f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp)+f_{q/p^\downarrow}(x,\boldsymbol{k}_\perp)}\right| \le 1$$

Soffer bound for the k_T dependent transversity distribution

Generalized positivity bounds for k_T moments of TMD distribution and fragmentation functions [Bacchetta, Boglione, Henneman, Mulders, PRL 85 (2000)]

$$|h_1| \le \frac{1}{2}(f_1 + g_{1L}) \le f_1$$

$$|h_{1T}^{\perp(1)}| \le \frac{1}{2}(f_1 - g_{1L}) \le f_1$$

$$\begin{aligned}(g_{1T}^{(1)})^2 + (f_{1T}^{\perp(1)})^2 &\le \frac{\boldsymbol{p}_T^2}{4M^2}(f_1 + g_{1L})(f_1 - g_{1L}) \\ &\le \frac{\boldsymbol{p}_T^2}{4M^2} f_1^2\,, \\ (h_{1L}^{\perp(1)})^2 + (h_1^{\perp(1)})^2 &\le \frac{\boldsymbol{p}_T^2}{4M^2}(f_1 + g_{1L})(f_1 - g_{1L}) \\ &\le \frac{\boldsymbol{p}_T^2}{4M^2} f_1^2\,.\end{aligned}$$

Theoretical information on TMD functions

Burkardt sum rule for the Sivers distribution

$$\langle \boldsymbol{k}_\perp \rangle = \sum_{a=q,\bar{q},g} \langle \boldsymbol{k}_\perp \rangle_a = \int dx \int d^2\boldsymbol{k}_\perp \boldsymbol{k}_\perp \sum_{a=q,\bar{q},g} \Delta \hat{f}_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) = 0$$

Intuitively expected, the non trivial fact is the proof of its validity in presence of final state interactions that might spoil the simple partonic interpretation

[Burkardt PRD 69 (2004)]

Schäfer – Teryaev sum rule for the Collins function

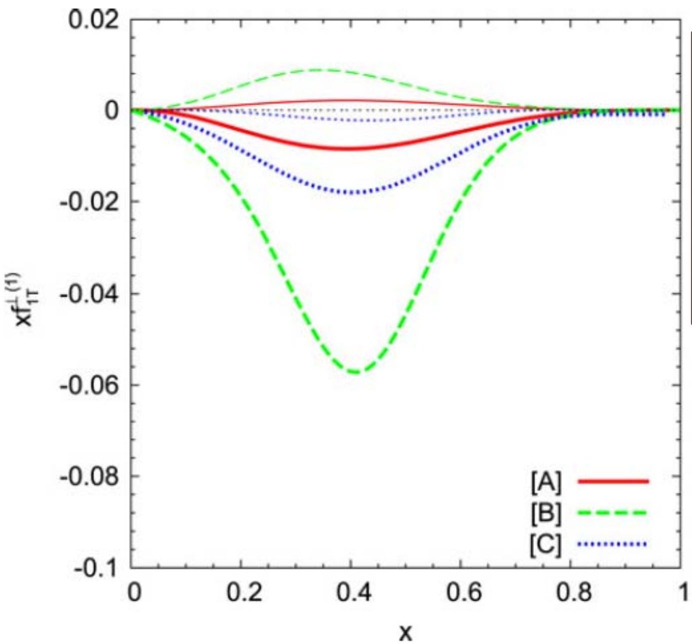
[Schäfer Teryaev PRD 61 (2000)]

$$\sum_h \int dz\, z H_1^{\perp(1)q}(z) = \sum_h \int dz \int d^2\boldsymbol{k}_\perp \frac{|\boldsymbol{k}_\perp|}{4M_h} \Delta^N D_{h/q^\uparrow}(z, |\boldsymbol{k}_\perp|) = 0$$

Theoretical information on TMD PDFs and FFs: models

[see also talk by F. Conti this afternoon]

TMD distributions:
spectator model for the nucleon with scalar and axial-vector diquarks [B]
single gluon rescattering to generate T-odd distributions (Sivers, Boer-Mulders)
Different choices for the nucleon-quark-diquark form factor (point-like, dipolar, Gaussian)
MIT bag model wave functions for quarks [A]
Instanton liquid model for QCD vacuum + MIT bag model [C]



Sivers function (first k_T moment)
[A] Yuan PLB 575 (2003)
[B] Bacchetta, Schäfer, Yang PLB 578 (2004)
[C] Cherednikov et al PLB 642 (2006)
Thik lines: u quark
Thin lines: d quark

Gamberg, Goldstein et al
Lu, Ma
Brodsky, Yuan

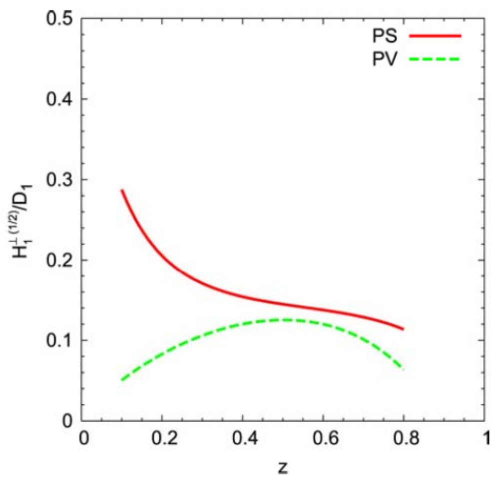
Theoretical information on TMD PDFs and FFs: models

Boer-Mulders function

All models predict a negative BM function for both u and d quarks
[Gamberg Goldstein - Bacchetta Schäfer, Yang]
Same conclusion reached relating it with the Fourier transform of chirally odd GPDs
[Burkardt, Hannafious ,PLB 658 (2008)]

Collins fragmentation function

Different calculations modelling the fragmentation process
at tree level with insertion of one-loop corrections
pseudoscalar/pseudovector pion-quark couplings with pion/gluon loops
[Bacchetta Kundu Metz Mulders - Gamberg Goldstein Oganessyan]



Amrath, Bacchetta, Metz
PRD 71 (2005)

Helicity amplitudes for the elementary partonic process ab → cd

All intrinsic parton motions are explicitely taken into account: all soft elementary processes, A(B) → a(b) + X and c → C + X, and the elementary QCD process ab → cd (which is not planar in the hadronic c.m. frame) take place out of the hadronic production plane (the XZ_{cm} plane). This introduces several non trivial azimuthal phases in the PDF and FF and in the helicity amplitudes for the elementary process. The helicity amplitudes in the hadronic c.m. frame and those in the canonical partonic c.m. frame (no azimuthal phases) are related in the following way [see PRD71 (2005) 041002 for details]:

$$\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} = \hat{M}^0_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, e^{-i(\lambda_a\xi_a+\lambda_b\xi_b-\lambda_c\xi_c-\lambda_d\xi_d)}\, e^{-i[(\lambda_a-\lambda_b)\tilde{\xi}_a-(\lambda_c-\lambda_d)\tilde{\xi}_c]}\, e^{i(\lambda_a-\lambda_b)\phi''_c}$$

Well-known parity properties hold for the canonical helicity amplitudes:

$$\hat{M}^0_{-\lambda_c,-\lambda_d;-\lambda_a,-\lambda_b} = \eta_a\,\eta_b\,\eta_c\,\eta_d\,(-1)^{s_a+s_b-s_c-s_d}(-1)^{(\lambda_a-\lambda_b)-(\lambda_c-\lambda_d)}\,\hat{M}^0_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}$$

For massless partons there are only three independent helicity amplitudes:

$$\hat{M}_{++;++} \equiv \hat{M}^0_1\, e^{i\varphi_1}\,, \quad \hat{M}_{-+;-+} \equiv \hat{M}^0_2\, e^{i\varphi_2}\,, \quad \hat{M}_{-+;+-} \equiv \hat{M}^0_3\, e^{i\varphi_3}$$

At LO there are eight elementary contributions ab → cd which must be considered separately, since they get different PDF and FF weights in the phase-space integrations:

$$q_a q_b \to q_c q_d\,, \quad q\bar{q} \to g_c g_d\,, \quad g_a g_b \to q\bar{q}\,, \quad g_a g_b \to g_c g_d$$
$$qg \to qg\,, \quad gq \to gq\,, \quad qg \to gq\,, \quad gq \to qg$$

Canonical helicity amplitudes for processes which only differ by the exchange of the two initial particles, a ↔ b, or of the two final partons, c ↔ d, are related as follows:

$$\hat{M}^{0,\,ba\to cd}_{\lambda_c,\lambda_d;\lambda_b,\lambda_a}(\theta) = \hat{M}^{0,\,ab\to cd}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}(\pi-\theta)\,e^{-i\pi(\lambda_c-\lambda_d)}$$

$$\hat{M}^{0,\,ab\to dc}_{\lambda_d,\lambda_c;\lambda_a,\lambda_b}(\theta) = \hat{M}^{0,\,ab\to cd}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}(\pi-\theta)\,e^{-i\pi(\lambda_a-\lambda_b)}$$

up to an overall, helicity independent, phase, which is irrelevant in the expressions of the physical observables, where only bilinear combinations of the helicity amplitudes occurr.

Kernels for the process $A(S_A)+B(S_B) \to C + X$

$$\Sigma(S_A,S_B)^{ab\to cd} = \sum_{\{\lambda\}} \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\,\hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\,\rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\,\hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b})$$
$$\times\ \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\,\hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\,\hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})$$

Unpolarized cross section and SSA for pp → π + X

$$\frac{E_C\, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d} \int \frac{dx_a\, dx_b\, dz}{16\pi^2 x_a x_b z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_{\perp C}\, \delta(\boldsymbol{k}_{\perp C}\cdot \hat{\boldsymbol{p}}_c)\, J(\boldsymbol{k}_{\perp C})$$
$$\times\ \Sigma(S_A, S_B)^{ab\to cd}(x_a, x_b, z, \boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}, \boldsymbol{k}_{\perp C})\, \delta(\hat{s}+\hat{t}+\hat{u})$$

$$A_N(pp\to\pi+X) = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

The calculation requires the difference and sum of single-spin kernels for opposite transverse polarizations [only $q_a q_b \to q_c q_c$ and gq → gq contributions shown]

$$[\Sigma(\uparrow,0)-\Sigma(\downarrow,0)]^{q_a q_b\to q_c q_d} =$$
$$\frac{1}{2}\,\Delta\hat{f}_{a/A^\uparrow}(x_a,\boldsymbol{k}_{\perp a})\,\hat{f}_{b/B}(x_b,k_{\perp b})\left[|\hat{M}_1^0|^2+|\hat{M}_2^0|^2+|\hat{M}_3^0|^2\right]\hat{D}_{C/c}(z,k_{\perp C})$$
$$+\ 2\left[\Delta^-\hat{f}^a_{s_y/\uparrow}(x_a,\boldsymbol{k}_{\perp a})\cos(\varphi_3-\varphi_2)-\Delta\hat{f}^a_{s_x/\uparrow}(x_a,\boldsymbol{k}_{\perp a})\sin(\varphi_3-\varphi_2)\right]$$
$$\times\ \Delta\hat{f}^b_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})\,\hat{M}_2^0\,\hat{M}_3^0\,\hat{D}_{C/c}(z,k_{\perp C})$$
$$-\ \left[\Delta\hat{f}^a_{s_x/\uparrow}(x_a,\boldsymbol{k}_{\perp a})\sin(\varphi_1-\varphi_2+\phi_C^H)-\Delta^-\hat{f}^a_{s_y/\uparrow}(x_a,\boldsymbol{k}_{\perp a})\cos(\varphi_1-\varphi_2+\phi_C^H)\right]$$
$$\times\ \hat{f}_{b/B}(x_b,k_{\perp b})\,\hat{M}_1^0\,\hat{M}_2^0\,\Delta^N\hat{D}_{C/c^\uparrow}(z,k_{\perp C})$$
$$+\ \frac{1}{2}\,\Delta\hat{f}_{a/A^\uparrow}(x_a,\boldsymbol{k}_{\perp a})\,\Delta\hat{f}^b_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})\cos(\varphi_1-\varphi_3+\phi_C^H)\,\hat{M}_1^0\,\hat{M}_3^0\,\Delta^N\hat{D}_{C/c^\uparrow}(z,k_{\perp C})$$

Sivers contributions

Transversity ⊗ Boer-Mulders

Transversity ⊗ Collins

Sivers ⊗ Boer-Mulders ⊗ Collins

$$[\Sigma(\uparrow,0)-\Sigma(\downarrow,0)]^{gq\to gq} =$$

$$\frac{1}{2}\,\Delta\hat{f}_{g/A^\uparrow}(x_g,\boldsymbol{k}_{\perp g})\,\hat{f}_{q/B}(x_q,k_{\perp q})\,\left[|\hat{M}_1^0|^2+|\hat{M}_2^0|^2\right]\,\hat{D}_{C/g}(z,k_{\perp C})$$

$$+\left[\Delta^-\hat{f}^g_{\mathcal{T}_1/\uparrow}(x_g,\boldsymbol{k}_{\perp g})\,\cos(\varphi_1-\varphi_2+2\phi_C^H)+\Delta\hat{f}^g_{\mathcal{T}_2/\uparrow}(x_g,\boldsymbol{k}_{\perp g})\,\sin(\varphi_1-\varphi_2+2\phi_C^H)\right]$$

$$\times\,\hat{f}_{q/B}(x_q,k_{\perp q})\,\hat{M}_1^0\,\hat{M}_2^0\,\Delta^N\hat{D}_{C/\mathcal{T}_1^g}(z,k_{\perp C})$$

$$[\Sigma(\uparrow,0)+\Sigma(\downarrow,0)]^{q_aq_b\to q_cq_d} =$$

“usual” collinear term

$$\hat{f}_{a/A}(x_a,k_{\perp a})\,\hat{f}_{b/B}(x_b,k_{\perp b})\,\left[|\hat{M}_1^0|^2+|\hat{M}_2^0|^2+|\hat{M}_3^0|^2\right]\,\hat{D}_{C/c}(z,k_{\perp C})$$

$$+\quad 2\,\Delta\hat{f}^a_{s_y/A}(x_a,\boldsymbol{k}_{\perp a})\,\Delta\hat{f}^b_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})\,\cos(\varphi_3-\varphi_2)\,\hat{M}_2^0\,\hat{M}_3^0\,\hat{D}_{C/c}(z,k_{\perp C})$$

BM ⊗ BM

$$+\quad\left[\hat{f}_{a/A}(x_a,k_{\perp a})\,\Delta\hat{f}^b_{s_y/B}(x_b,\boldsymbol{k}_{\perp b})\,\cos(\varphi_1-\varphi_3+\phi_C^H)\,\hat{M}_1^0\,\hat{M}_3^0\right.$$

$$\left.+\,\Delta\hat{f}^a_{s_y/A}(x_a,\boldsymbol{k}_{\perp a})\,\hat{f}_{b/B}(x_b,k_{\perp b})\,\cos(\varphi_1-\varphi_2+\phi_C^H)\,\hat{M}_1^0\hat{M}_2^0\right]\,\Delta^N\hat{D}_{C/c^\uparrow}(z,k_{\perp C})$$

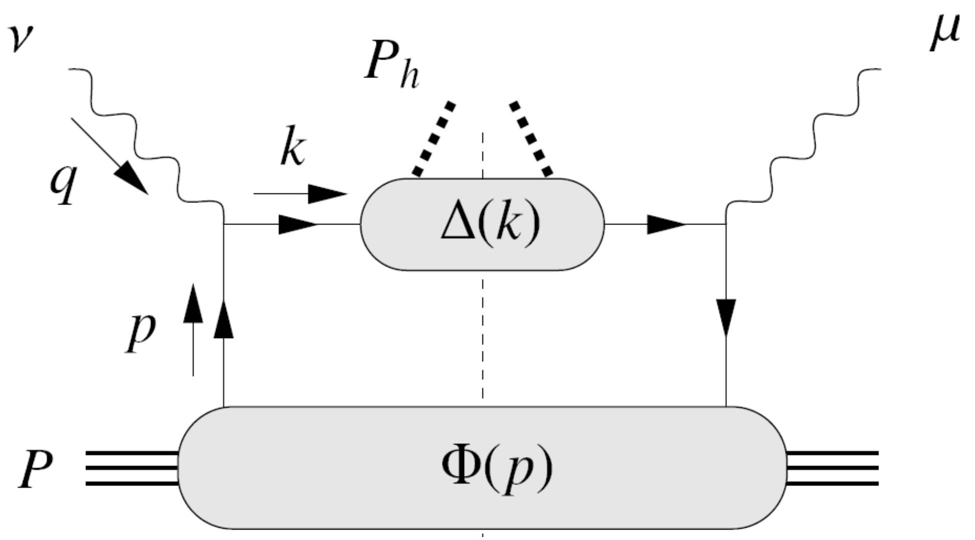
Boer-Mulders ⊗ Collins

$$[\Sigma(\uparrow,0)+\Sigma(\downarrow,0)]^{gq\to gq} =$$

$$\hat{f}_{g/A}(x_g,k_{\perp g})\,\hat{f}_{q/B}(x_q,k_{\perp q})\,\left[|\hat{M}_1^0|^2+|\hat{M}_2^0|^2\right]\,\hat{D}_{C/g}(z,k_{\perp C})$$

$$+\quad\Delta\hat{f}^g_{\mathcal{T}_1/A}(x_g,\boldsymbol{k}_{\perp g})\,\hat{f}_{q/B}(x_q,k_{\perp q})\,\cos(\varphi_1-\varphi_2+2\phi_C^H)\,\hat{M}_1^0\,\hat{M}_2^0\,\Delta^N\hat{D}_{C/\mathcal{T}_1^g}(z,k_{\perp C})$$

TMD Color gauge invariant approach (hadronic correlators)



The hadron tensor in semi-inclusive deep-inelastic scattering (SIDIS)

$$\Phi_{ij}(p;P,S) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{ip\cdot\xi} \langle P,S|\, \overline{\psi}_j(0)\, \psi_i(\xi)\, |P,S\rangle$$

$$\Delta_{ij}(k;P_h,S_h) = \sum\!\!\!\!\!\!\int_X \int \frac{d^4\xi}{(2\pi)^4}\, e^{-ik\cdot\xi} \langle 0|\, \psi_i(0)\, |P_h,S_h;X\rangle\langle P_h,S_h;X|\, \overline{\psi}_j(\xi)\, |0\rangle$$

Mulders, Boer, Pijlman, Bomhof

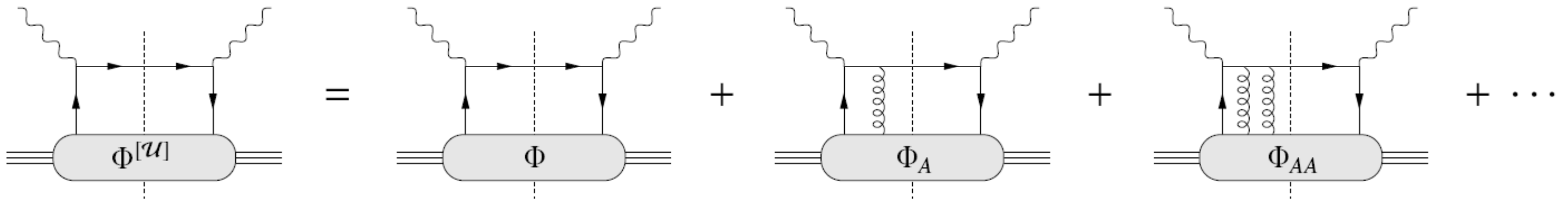
Hadronic correlators as given in previous slide are not invariant under local $SU_c(3)$ gauge transformations, since involve bilocal products of quark fields;
We need to connect the quark fields by means of a path-ordered gauge link (Wilson line)

$$\mathcal{U}^{C}_{[\zeta;\xi]} = \mathcal{P}\exp\left[-ig\int_C d\eta\cdot A^a(\eta)\,T^a\right]$$

C : integration path with endpoints ζ and ξ;
$\mathcal{P}$ denotes path-ordering: parametrize the integration path using
$\eta^\mu(s)$ with $s\in[0,1]$ and $\eta^\mu(0)=\zeta^\mu$ and $\eta^\mu(1)=\xi^\mu$

The properly invariant light-front ($\xi\cdot n$) definition of the correlator is therefore

$$\Phi^{[\mathcal{U}]}_{ij}(x,p_T) = \int\frac{d(\xi\cdot P)\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi}\,\langle P,S\,|\,\overline{\psi}_j(0)\,\mathcal{U}_{[0;\xi]}\,\psi_i(\xi)\,|P,S\,\rangle\,\rfloor_{\rm LF}$$



Parametrize the (color gauge invariant) hadronic correlator imposing general hermiticity, parity and time-reversal property. One ends up with the following parameterization (leading-twist only)

$$\begin{aligned}\Phi^{[\mathcal{U}]}(x,p_T;P,S) = \tfrac{1}{2}\Big\{ & f_1(x,p_T^2)\,\not{P} + \tfrac{1}{2}\,h_{1T}(x,p_T^2)\,\gamma_5[\not{S}_T,\not{P}] \\ & + S_L\,g_{1L}(x,p_T^2)\,\gamma_5\not{P} + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\,g_{1T}(x,p_T^2)\,\gamma_5\not{P} \\ & + S_L\,h_{1L}^{\perp}(x,p_T^2)\,\gamma_5\frac{[\not{p}_T,\not{P}]}{2M} + \frac{\boldsymbol{p}_T\cdot\boldsymbol{S}_T}{M}\,h_{1T}^{\perp}(x,p_T^2)\,\gamma_5\frac{[\not{p}_T,\not{P}]}{2M} \\ & + i h_1^{\perp}(x,p_T^2)\,\frac{[\not{p}_T,\not{P}]}{2M} - \frac{\epsilon_T^{p_T S_T}}{M}\,f_{1T}^{\perp}(x,p_T^2)\,\not{P}\Big\}.\end{aligned}$$

In the collinear (transverse momentum integrated) case this parameterization of the correlator reads

$$\Phi(x;P,S) = \tfrac{1}{2}\{\,f_1(x)\,\not{P} + S_L\,g_1(x)\,\gamma_5\not{P} + \tfrac{1}{2}\,h_1(x)\,\gamma_5[\not{S}_T,\not{P}]\,\}$$

Analogously, one can define a gauge invariant gluon correlator

$$\begin{aligned}\Gamma^{[\mathcal{U},\mathcal{U}']\mu\nu}(x,p_T) = \frac{n_\rho\,n_\sigma}{(p\cdot n)^2}\int\frac{d(\xi\cdot P)\,d^2\xi_T}{(2\pi)^3}\,e^{ip\cdot\xi} \\ \times\,\langle P,S|\,\mathrm{Tr}\,[\,F^{\mu\rho}(0)\,\mathcal{U}_{[0;\xi]}\,F^{\nu\sigma}(\xi)\,\mathcal{U}'_{[\xi;0]}\,]\,|P,S\rangle\,]_{\mathrm{LF}}\end{aligned}$$

In a pQCD scheme, without proper gauge links time-reversal invariance would imply the vanishing of the Sivers and Boer-Mulders (all T-odd) distribution functions

The same argument does not apply to the hadronic correlator for the fragmentation functions, due to the explicit appearance of hadronic final state interactions

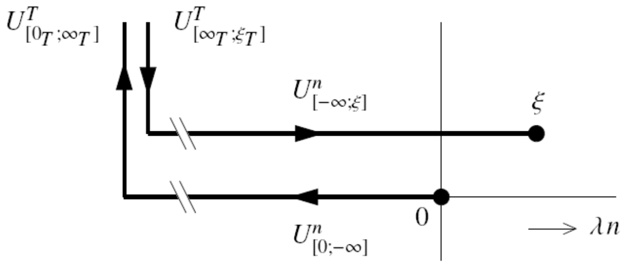
Historically, the Collins mechanism in the fragmentation process was introduced as an alternative way to explain large SSAs and to measure the transversity distribution in SIDIS

Gauge invariance requirement does not restrict the gauge -link path-ordered exponential.

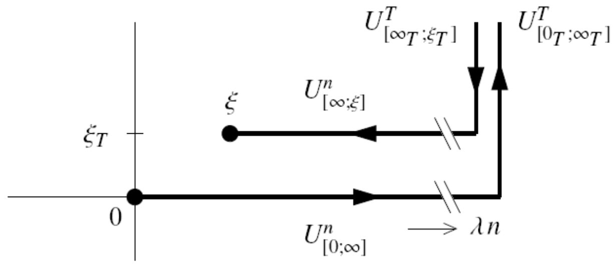
Wilson lines arise from resummation of gluonic initial and final state interactions between the hadronic remnants and the current (active) quarks. They may depend on the hard elementary scattering processes associated with the hadronic process considered

Gluonic initial and final state interactions lead to future pointing Wilson lines in SIDIS, while they give rise to past pointing Wilson lines in Drell-Yan processes

A pictorial representation



past pointing Wilson line $\mathcal{U}^{[-]}$



future pointing Wilson line $\mathcal{U}^{[+]}$

$$\mathcal{U}^{[\pm]}_{[0;\xi]} = U^n_{[0;\pm\infty]}\, U^T_{[0_T;\infty_T]}\, U^T_{[\infty_T;\xi_T]}\, U^n_{[\pm\infty;\xi]}$$

$$U^n_{[\xi;\xi+\eta]} = \mathcal{P}\exp\left[-ig\int_0^{\eta\cdot P/n\cdot P} d\lambda\; n\cdot A^a(\xi+\lambda n)\,T^a\right] \qquad U^T_{[\xi;\xi+\eta]} = \mathcal{P}\exp\left[-ig\int_{\xi_T}^{\xi_T+\eta_T} d\zeta_T\cdot A^a_T(\zeta)\,T^a\right]$$

It was a common belief that gauge fields should vanish at light-cone infinity and transverse Wilson lines could be ignored

In the axial gauge the Wilson lines U^n should reduce to simple unit operators

Gauge links could be effectively omitted ⇒ vanishing of naively T-odd distributions

In 2002 Brodsky, Hwang and Schmidt showed by an explicit calculation within a spectator diquark model of one-gluon FSI in semi-inclusive DIS that transverse single spin asymmetries can be nonvanishing at leading twist.

Soon after Collins proved that this result is related to the proper treatment of gauge links and that

time-reversal invariance does not imply the vanishing of the Sivers function

Instead, Sivers functions in SIDIS and Drell-Yan processes should have opposite signs

$$f_{1T}^{\perp}(x,p_T^2)\rfloor_{\text{Drell-Yan}} = -f_{1T}^{\perp}(x,p_T^2)\rfloor_{\text{SIDIS}}$$

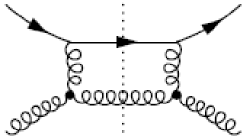
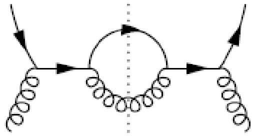
Wilson lines and the effects of gluonic initial and final state interactions are process dependent
In general the universality properties of the naively T-odd distributions can be spoiled.
For SIDIS and Drell-Yan processes a simple generalized form of universality seems to hold.

Collins and Metz have shown that inclusion of gauge links preserves universality
for the Collins fragmentation function.
Very important also from a phenomenological point of view

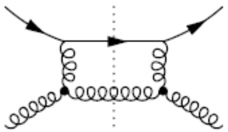
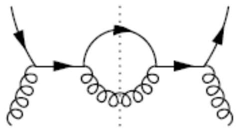
For basic hadronic processes, SIDIS, Drell-Yan, e+e- annihilation, the hard processes
are (at tree level) simple e.m. vertices
⇒ only future/past pointing Wilson lines occur

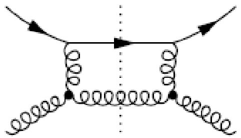
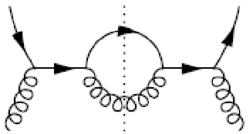
For processes involving hadrons both in the initial/final states the situation is much more involved:
even simple generalized universality properties can be lost
Different partonic processes can give rise to different gauge-link factors

Gluonic pole hard scattering cross sections (e.g. $qg \longrightarrow qg$)

$$\frac{d\hat{\sigma}_{qg\to qg}}{d\hat{t}} =$$

$$+$$

$$+ \cdots$$

$$= -\frac{4\pi\alpha_S^2 T_F^2}{\hat{s}^2}\,\frac{\hat{s}^2+\hat{u}^2}{2\,\hat{s}\hat{u}}\left\{\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} - \frac{1}{N^2}\right\}$$

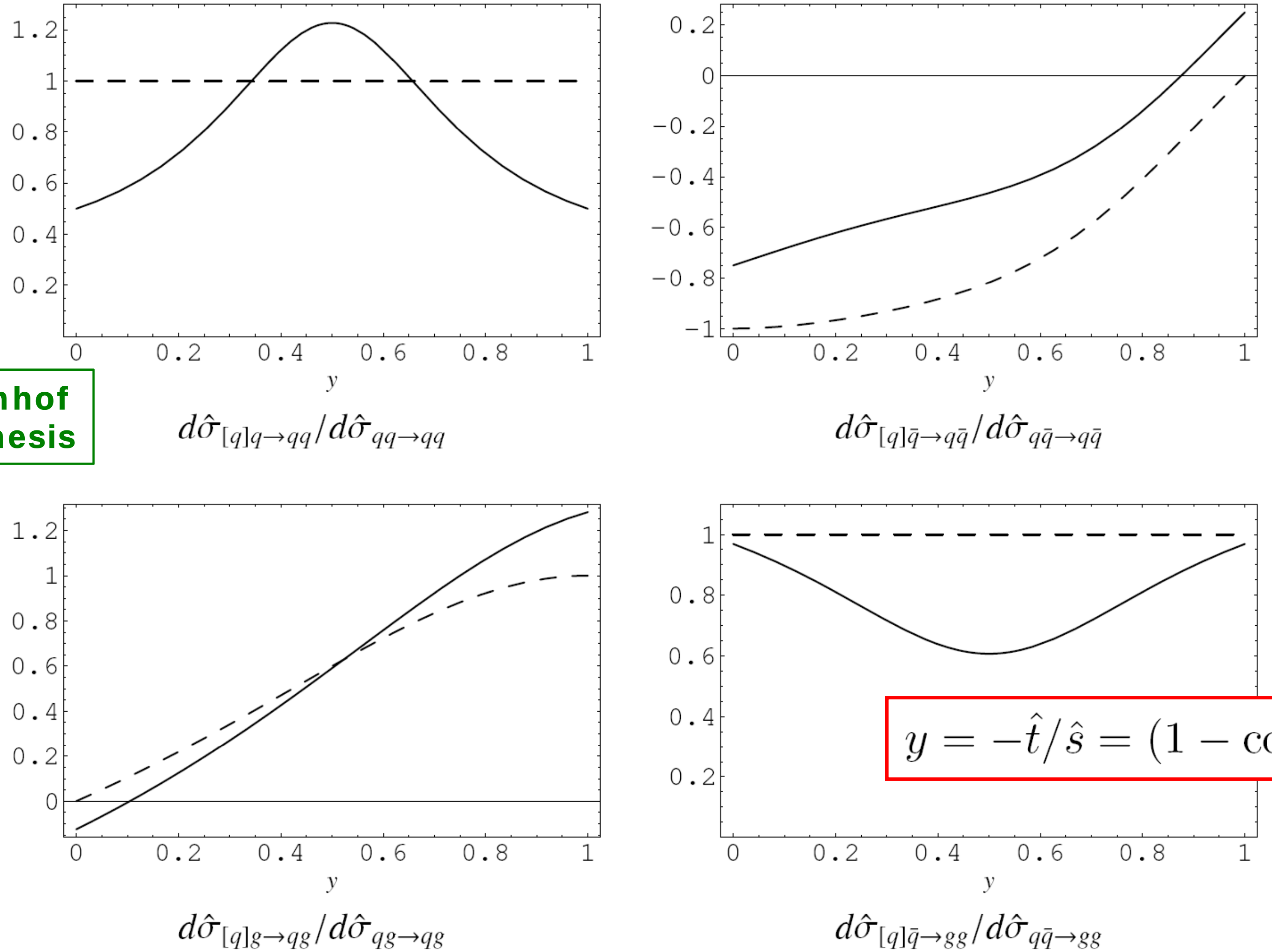
$$\frac{d\hat{\sigma}_{[q]g\to qg}}{d\hat{t}} = \Phi^{[D_1]}(x,p_T)\otimes$$

$$+\,\Phi^{[D_2]}(x,p_T)\otimes$$

$$+ \cdots$$

$$= C_G^{[D_1]}(q_i)$$

$$+\,C_G^{[D_2]}(q_i)$$

$$+ \cdots$$

gauge invariance preserved!

$$= -\frac{4\pi\alpha_S^2 T_F^2}{\hat{s}^2}\,\frac{\hat{s}^2+\hat{u}^2}{2\,\hat{s}\hat{u}}\left\{\frac{\hat{s}^2}{\hat{t}^2} - \frac{N^2+1}{N^2-1}\left\{\frac{\hat{u}^2}{\hat{t}^2} - \frac{1}{N^2}\right\}\right\},$$

C. Bomhof PhD Thesis



$$y = -\hat{t}/\hat{s} = (1 - \cos\hat{\theta})/2$$

Figure 4.3: Comparison of some of the unpolarized partonic and gluonic pole cross sections. We plot their ratios for N=3 (solid line) and $N\to\infty$ (dashed line) against the variable y defined in (2.8).

pQCD Collinear twist-three approach

(an almost pictorial discussion…)

consider the reaction

$$A(P,\vec{s}_T) + B(P') \longrightarrow h(\ell) + X$$

$$S = (P+P')^2 \simeq 2P\cdot P', \qquad T = (P-\ell)^2 \simeq -2P\cdot\ell$$
$$U = (P'-\ell)^2 \simeq -2P'\cdot\ell,$$

$$x_F = \frac{2\ell_z}{\sqrt{S}} = \frac{T-U}{S}$$

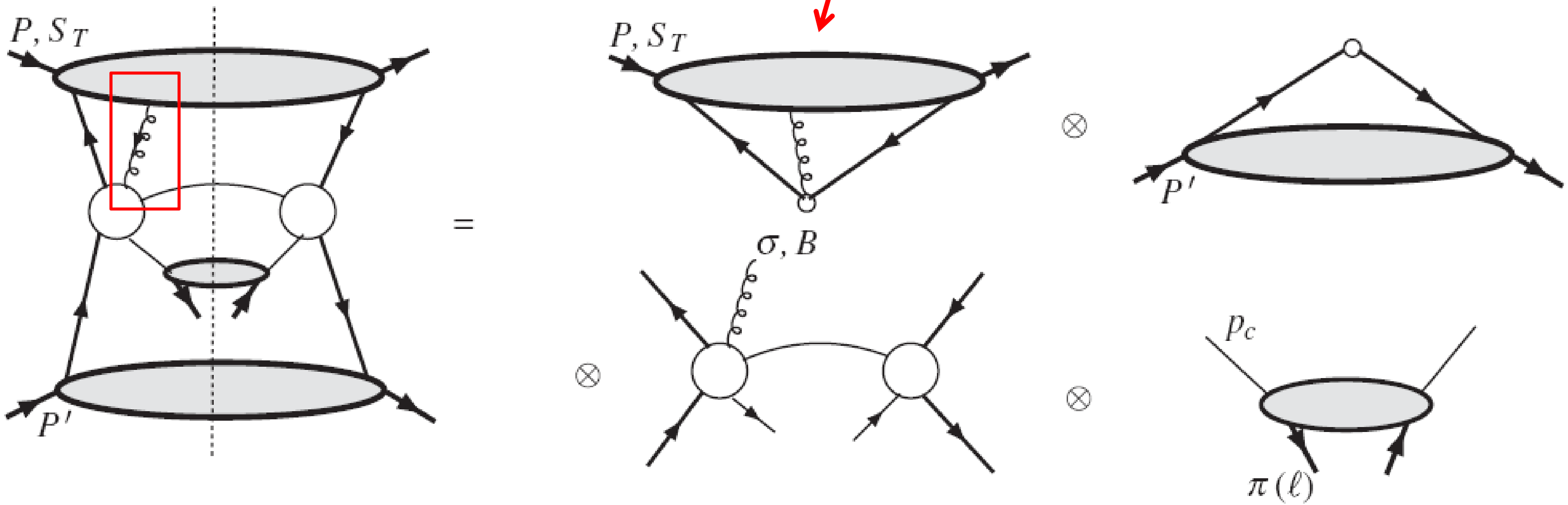
$$\begin{aligned}
d\Delta\sigma_{A^\uparrow B\to C+X} = & \sum_{abc} \phi^{(3)}_{a/A^\uparrow}(x_1,x_2) \otimes \phi_{b/B}(x') \otimes H_{a+b\to c} \otimes D_{c\to C}(z) \\
& + \sum_{abc} \delta q^{(2)}_{a/A}(x) \otimes \phi^{(3)}_{b/B}(x'_1,x'_2) \otimes H''_{a+b\to c} \otimes D_{c\to C}(z) \\
& + \sum_{abc} \delta q^{(2)}_{a/A}(x) \otimes \phi_{b/B}(x') \otimes H'_{a+b\to c} \otimes D^{(3)}_{c\to C}(z_1,z_2)
\end{aligned}$$

Qiu – Sterman(91-98), Qiu-Vogelsang-Yuan (2006), Koike and coworkers,…

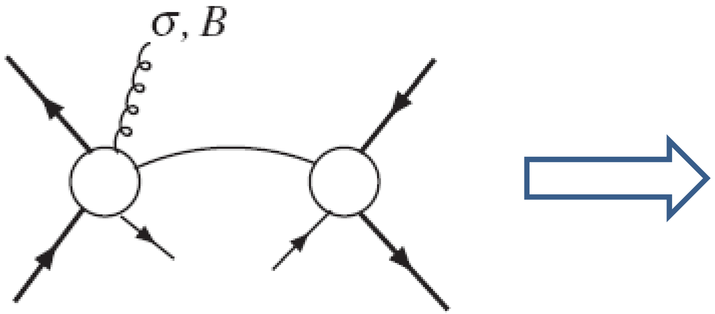
$$d\Delta\sigma \approx \sum_a \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} T_a(k_1,k_2) S_a(k_1,k_2)$$

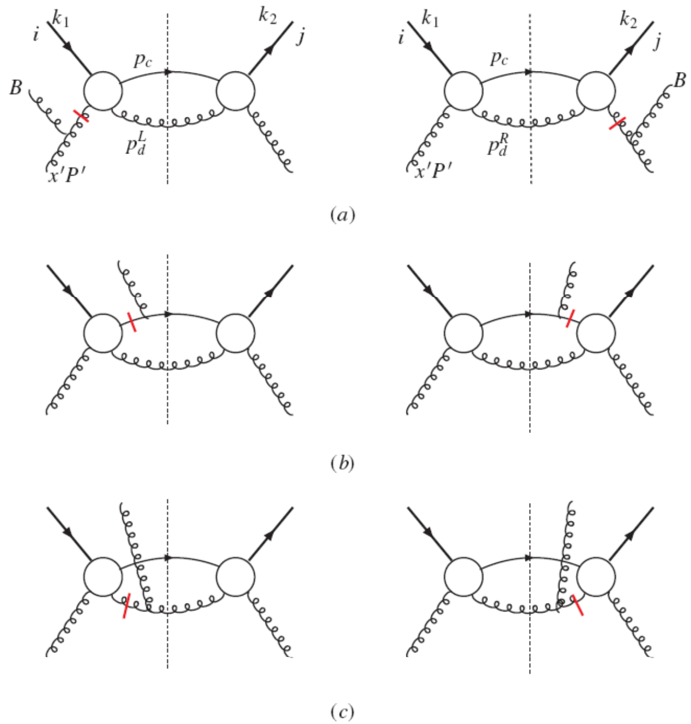
$$S_a(k_1,k_2) \approx \sum_{bc} \int \frac{dx'}{x'} \phi_{b/B}(x') \int dz\, H_{a+b\to c}(k_1,k_2,x',p_c) D_{c\to C}(z)$$

$$k_i^\mu = x_i P^\mu + k_{i,\perp}^\mu + k_{i,T}^2/(x_i S) P'^\mu$$



Kouvaris, Qiu, Vogelsang, Yuan, PRD 74 (2006)





Expanding $H_{ab\to c}$ in the partonic momenta, k_1 and k_2, around $k_1 = x_1 P$ and $k_2 = x_2 P$, respectively, we have

$$H_{ab\to c}(k_1, k_2) = H_{ab\to c}(x_1, x_2) + \frac{\partial H_{ab\to c}}{\partial k_1^\rho}(x_1, x_2)(k_1 - x_1 P)^\rho + \frac{\partial H_{ab\to c}}{\partial k_2^\rho}(x_1, x_2)(k_2 - x_2 P)^\rho + \dots \qquad (8)$$

This expansion enables us to integrate over three of the four components of the two loop momenta k_i, which reduce to convolutions over the parton light-cone momentum fractions x_i

$$T_a(k_1, k_2) \Longrightarrow T_{a,F}(x_1, x_2) = \int \frac{dy_1^- dy_2^-}{4\pi} \mathrm{e}^{ix_1P^+y_1^- + i(x_2-x_1)P^+y_2^-} \times \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ [\epsilon^{s_T\sigma n\bar{n}} F_\sigma^+(y_2^-)] \times \psi_a(y_1^-)|P, \vec{s}_T\rangle,$$

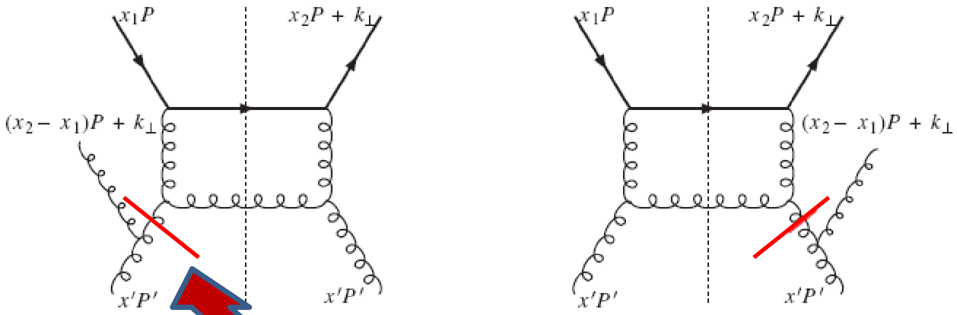
Additional path ordered exponentials of the gauge field rendering the matrix element gauge invariant have been suppressed; (valid in the light-cone gauge $A^+ = 0$)

This matrix element is a symmetric function of its argument s and is real

The phases required to generate SSAs have to arise in the hard scattering functions H
Imaginary parts in H can arise even at tree level, thanks to the pole structure of the hard scattering functions

$$\mathrm{d}\Delta\sigma = \frac{1}{2s}\sum_{abc}\int \mathrm{d}z\, D_{c\to C}(z) \int \frac{\mathrm{d}x'}{x'}\phi_{b/B}(x') \int \mathrm{d}x_1 \mathrm{d}x_2 T_{a,F}(x_1, x_2) \times \left[\mathrm{i}\epsilon^{\rho s_T n\bar{n}} \frac{\partial}{\partial k_2^\rho} H_{a+b\to c}(x_1, x_2, x', z)\right]_{k_2^\rho = 0},$$

Integrations over either x_1 or x_2 can be done by using the poles in H



$$\frac{1}{(x'P' + (x_2 - x_1)P + k_\perp)^2 + i\epsilon}$$
$$= \frac{1}{(x_2 - x_1)x'S + i\epsilon} + \mathcal{O}(k_T^2) \longrightarrow -\frac{i\pi}{x'S}\delta(x_2 - x_1)$$

Soft-gluon pole contributions

Restoring all factors

$$E_C \frac{d^3\Delta\sigma}{d^3 \boldsymbol{p}_C} = \frac{\alpha_s^2}{s} \sum_{a,b,c} \int_{z_{\min}}^1 \frac{dz}{z^2} D_{c\to C}(z) \int_{x'_{\min}}^1 \frac{dx'}{x'} \frac{1}{x's + t/z} \phi_{b/B}(x')$$
$$\times \sqrt{4\pi\alpha_s} \left(\frac{\epsilon^{p_C s_T n \bar{n}}}{z\hat{u}} \right) \frac{1}{x} \left[T_{a,F}(x,x) - x \left(\frac{d}{dx} T_{a,F}(x,x) \right) \right] H_{a+b\to c}(\hat{s},\hat{t},\hat{u})$$

$$H_{a+b\to c} = H^I_{a+b\to c}(\hat{s},\hat{t},\hat{u}) + H^F_{a+b\to c}(\hat{s},\hat{t},\hat{u}) \left(1 + \frac{\hat{u}}{\hat{t}} \right)$$

Initial state contributions

Final state contributions

SSAs in hadronic collisions are usually large in the forward region (large positive Feynman x), where on the average parton a inside the polarized hadron has a large (> x_F) light-cone momentum fraction x, while parton b inside the unpolarized hadron has a small light-cone momentum fraction x′.

all distributions vanish for large x as $(1-x)^\beta$, with $\beta > 0$, $(\partial/\partial x)T_{a,F}(x,x) \gg T_{a,F}(x,x)$ when $x \to 1$
Therefore, in the forward region, terms proportional to derivatives of the distributions $T_{a,F}$ dominate

$qg \to qg$ scattering:

$$H^U_{qg\to q}(\hat{s},\hat{t},\hat{u}) = \frac{C_F}{N_C}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right]\left[1-\frac{N_C}{C_F}\frac{\hat{s}\,\hat{u}}{\hat{t}^2}\right],$$

$$H^I_{qg\to q}(\hat{s},\hat{t},\hat{u}) = \frac{1}{2(N_C^2-1)}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right]\left[1-N_C^2\frac{\hat{u}^2}{\hat{t}^2}\right],$$

$$H^F_{qg\to q}(\hat{s},\hat{t},\hat{u}) = \frac{1}{2N_C^2(N_C^2-1)}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right]\left[1+2N_C^2\frac{\hat{s}\,\hat{u}}{\hat{t}^2}\right].$$

$qq \to qq$ scattering:

$$H^U_{qq\to q}(\hat{s},\hat{t},\hat{u}) = \frac{C_F}{N_C}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}-\frac{2}{N_C}\frac{\hat{s}^2}{\hat{t}\hat{u}}\right],$$

$$H^I_{qq\to q}(\hat{s},\hat{t},\hat{u}) = -\frac{1}{N_C^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}-\frac{N_C^2+1}{N_C}\frac{\hat{s}^2}{\hat{t}\hat{u}}\right],$$

$$H^F_{qq\to q}(\hat{s},\hat{t},\hat{u}) = -\frac{1}{2N_C^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]+\frac{N_C^2-2}{2N_C^2}\left[\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right] + \frac{1}{N_C^3}\frac{\hat{s}^2}{\hat{t}\hat{u}}.$$

Beside soft gluon pole contributions, there are hard-scattering diagrams possessing other poles, for which an initial quark becomes soft. These socalled “soft-fermion poles” are expected to play a less relevant role and are usually not considered in applications

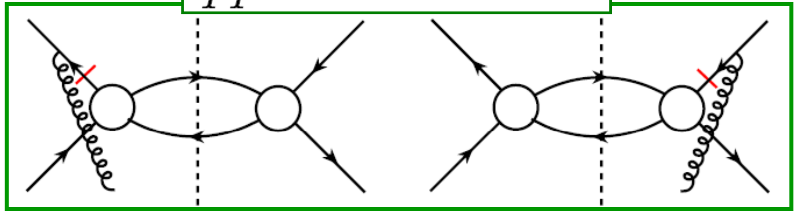
Quite recently, Koike has investigated the role of soft-fermion poles in SSAs for pion production at RHIC. If the corresponding twist-three quark-gluon correlation function is not suppressed in comparison with the one associated with soft-gluon poles,
its role could be not so negligible.

Additional contributions to the collinear expansion can arise from terms linear in the parton transverse momentum in other “hard”propagators or in the numerator of diagrams.
These terms do not lead to derivative contributions to the SSAs and
are also expected to be negligible

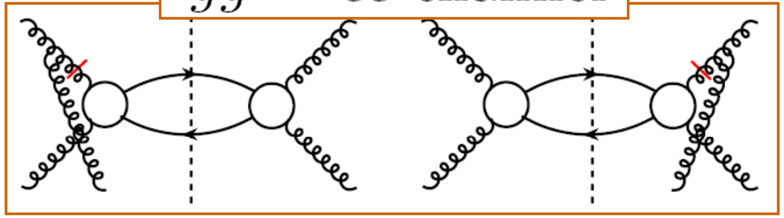
Contributions involving three-gluon twist-three correlation functions
were not included in the original formulation of the approach.
They could be relevant in the mid-rapidity region
Recently their role has been studied in SSAs in open charm production
at RHIC (gluon fusion channel dominant)

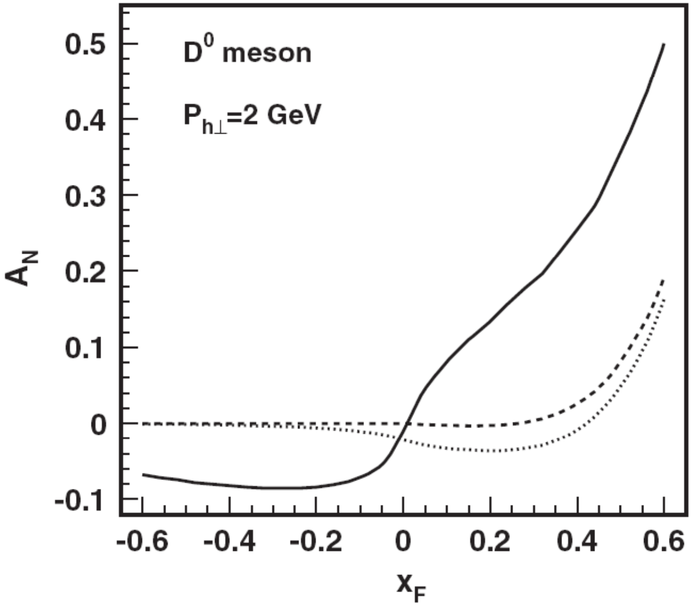
$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+ y_1^-} \frac{1}{xP^+} \langle P, s_T | F^+{}_\alpha(0) \times [\epsilon^{s_T \sigma n \bar{n}} F_\sigma{}^+(y_2^-)] F^{\alpha +}(y_1^-) | P, s_T \rangle,$$

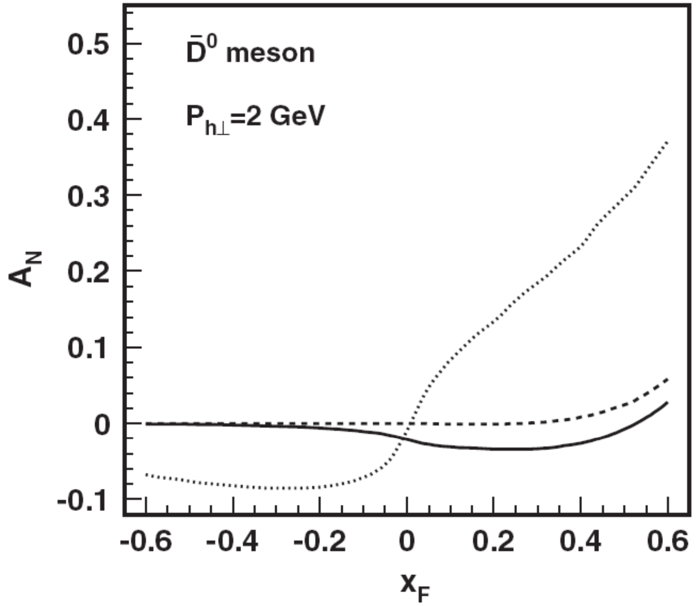
$q\bar{q} \to c\bar{c}$ channel



$gg \to c\bar{c}$ channel







Kang, Qiu, Vogelsang, Yuan PRD 78 (2008)

Relations between TMD and twist-three approaches

■ Factorization for TMD approaches has been proven for processes where two energy scales are present (a high perturbative scale (Q) and a low energy scale (q_T) associated with some trasverse momentum dependent observable [$\Lambda_{QCD} \sim q_T \ll Q$]), e.g. SIDIS, Drell-Yan process, two almost back-to-back particle production in pp collisions (still under some debate)
[Belitsky, Idilbi, Ji, Ma, Yuan; Boer, Bomhof, Mulders, Pijlman,…]
Factorization not yet proven for single particle production in hadronic collisions

■ Twist-three collinear approach follows usual factorization theorems of pQCD, proven also for single particle production in hadronic collisions.

■ It has been shown, for SIDIS and DY processes, that the TMD color gauge invariant approach and the twist-three formalism are equivalent and describe the same physics in the intermediate q_T region where they applicability domains overlap [$\Lambda_{QCD} \ll q_T \ll Q$]

■ This gives a link among the twist-three correlation functions T(x,x) and the k_T moments of the TMD distributions related to the same physical mechanism. As an example:

$$2M\, f_{1T}^{\perp(1)}(x) = -g T_F(x) \qquad\qquad f_{1T}^{\perp(1)}(x) = \int d^2 p_T\, \frac{\boldsymbol{p}_T^2}{2M^2}\, f_{1T}^{\perp}(x, p_T^2)\rfloor_{\rm SIDIS}$$

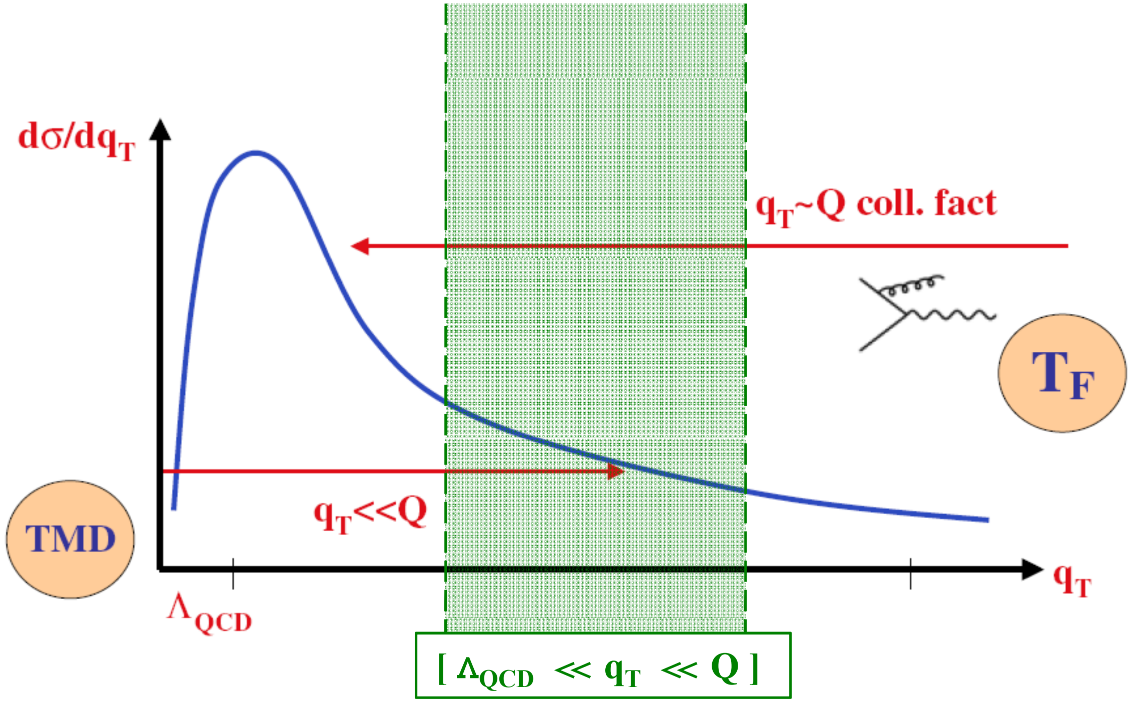


Figure 17. Cartoon of different kinematic regions $q_\perp \sim Q$ and $q_\perp \gg \Lambda_{\rm QCD}$ relevant for the single-spin asymmetry in the Drell–Yan process. In the region of overlap, $Q \gg q_\perp \gg \Lambda_{\rm QCD}$ both mechanisms describe the same physics [71].

Open points

- Factorization for single particle inclusive production in the TMD approach
- Evolution with scale of TMD distribution and fragmentation functions

  [Henneman, Boer, Mulders; evolution of first k_T moments of TMD PDFs and FFs]

  [Ceccopieri, Trentadue; Q^2-evolution of unpolarized TMD distr. in SIDIS]
- Soft factors from soft-gluon radiation [neglected in all phenomenological analyses]

  [Ji, Ma, Yuan; Collins, Metz; formal aspects of factorization in SIDIS and DY processes]
- Potential suppression of azimuthal asymmetries due to Sudakov factors [soft-gluon radiation]

  [Boer]
- Parton off-shellness , fully unintegrated parton correlation functions

  [Watt, Martin, Riskin; Linnyk, Leupold, Mosel; Collins, Rogers, Stasto]
- Experimental tests of generalized universality for Sivers Function [pQCD prediction]
- Improved parameterizations and phenomenological constraints for the TMD PDFs and FFs

To reach the simple configuration of the canonical amplitudes: start from the hadronic c.m. frame; perform a boost in the direction determined by $\mathbf{q} = \mathbf{p}_a + \mathbf{p}_b$ [so that the boosted three-vector $\mathbf{p}'_a + \mathbf{p}'_b$ is equal to zero]. This will provide us with a c.m.-like ref. frame S′ where partons a and b collide head-on. Here the parton a and the parton c [resulting from the hard interaction between a and b] will have directions identified by (θ'_a, ϕ'_a) and (θ'_c, ϕ'_c) respectively. In general, the parton momenta in S′ are related to the initial ones (before the boost) by [i=a,b,c,d]:

$$\boldsymbol{p}'_i = \boldsymbol{p}_i - \frac{\boldsymbol{q}}{q^0 + \sqrt{q^2}} \left(\frac{p_i \cdot q}{\sqrt{q^2}} + p_i^0 \right)$$

Perform now two subsequent rotations, one around the Z axis by an angle ϕ'_a, and one around the Y axis, by an angle θ'_a, such that the collision axis of the two colliding initial partons turns out to be aligned with the Z axis. We call this frame S″. Under these boost and rotations the helicity states and consequently the scattering amplitudes acquire phases, $\xi_{a,b,c,d}$ and $\tilde{\xi}_{a,b,c,d}$:

$$\begin{aligned}
\cos\xi_j &= \frac{\cos\theta_q \sin\theta_j - \sin\theta_q \cos\theta_j \cos(\phi_q - \phi_j)}{\sin\theta_{qp_j}} \\
\sin\xi_j &= \frac{\sin\theta_q \sin(\phi_q - \phi_j)}{\sin\theta_{qp_j}}
\end{aligned}$$

$$\tilde{\xi}_j = \eta'_j + \xi'_j$$

$$\cos\eta'_j = \frac{\cos\theta'_a - \cos\theta'_j \cos\theta_{p'_a p'_j}}{\sin\theta'_j \sin\theta_{p'_a p'_j}}$$

$$\sin\eta'_j = \frac{\sin\theta'_a \sin(\phi'_a - \phi'_j)}{\sin\theta_{p'_a p'_j}}$$

$$\cos\xi'_j = \frac{\cos\theta_q \sin\theta'_j - \sin\theta_q \cos\theta'_j \cos(\phi_q - \phi'_j)}{\sin\theta_{q p'_j}}$$

$$\sin\xi'_j = \frac{-\sin\theta_q \sin(\phi_q - \phi'_j)}{\sin\theta_{q p'_j}}$$

In the S'' frame the direction of the parton c is characterised by an azimuthal angle ϕ''_c given by

$$\tan\phi''_c = \frac{\sin\theta'_c \sin(\phi'_c - \phi'_a)}{\sin\theta'_c \cos(\phi'_c - \phi'_a)\cos\theta'_a - \cos\theta'_c \sin\theta'_a}$$

LO helicity amplitudes for the elementary process ab → cd

$q_a q_b \to q_c q_d$ $\quad$ $\bar{q}_a \bar{q}_b \to \bar{q}_c \bar{q}_d$

$$|\hat{M}_1^0|^2 = \frac{8}{9} g_s^4 \left[\frac{\hat{s}^2}{\hat{t}^2} + \delta_{ab}\left(\frac{\hat{s}^2}{\hat{u}^2} - \frac{2}{3}\frac{\hat{s}^2}{\hat{t}\hat{u}}\right)\right] \qquad |\hat{M}_2^0|^2 = \frac{8}{9} g_s^4 \frac{\hat{u}^2}{\hat{t}^2}$$

$$|\hat{M}_3^0|^2 = \delta_{ab}\frac{8}{9} g_s^4 \frac{\hat{t}^2}{\hat{u}^2} \qquad \hat{M}_1^0\,\hat{M}_2^0 = \frac{8}{9} g_s^4 \left(-\frac{\hat{s}\hat{u}}{\hat{t}^2} + \delta_{ab}\frac{1}{3}\frac{\hat{s}}{\hat{t}}\right)$$

$$\hat{M}_1^0\,\hat{M}_3^0 = \delta_{ab}\frac{8}{9} g_s^4 \left(\frac{\hat{s}\hat{t}}{\hat{u}^2} - \frac{1}{3}\frac{\hat{s}}{\hat{u}}\right) \qquad \hat{M}_2^0\,\hat{M}_3^0 = \delta_{ab}\frac{8}{27} g_s^4$$

$q_a \bar{q}_b \to q_c \bar{q}_d$

$$|\hat{M}_1^0|^2 = \delta_{ac}\frac{8}{9} g_s^4 \frac{\hat{s}^2}{\hat{t}^2} \qquad |\hat{M}_2^0|^2 = \frac{8}{9} g_s^4 \left(\delta_{ab}\frac{\hat{u}^2}{\hat{s}^2} + \delta_{ac}\frac{\hat{u}^2}{\hat{t}^2} - \delta_{ab}\,\delta_{ac}\frac{2}{3}\frac{\hat{u}^2}{\hat{s}\hat{t}}\right)$$

$$|\hat{M}_3^0|^2 = \delta_{ab}\frac{8}{9} g_s^4 \frac{\hat{t}^2}{\hat{s}^2} \qquad \hat{M}_1^0\,\hat{M}_2^0 = \frac{8}{9} g_s^4\,\delta_{ac}\left(-\frac{\hat{s}\hat{u}}{\hat{t}^2} + \delta_{ab}\frac{1}{3}\frac{\hat{u}}{\hat{t}}\right)$$

$$\hat{M}_1^0\,\hat{M}_3^0 = \delta_{ab}\,\delta_{ac}\frac{8}{27} g_s^4 \qquad \hat{M}_2^0\,\hat{M}_3^0 = \frac{8}{9} g_s^4\,\delta_{ab}\left(\frac{\hat{u}\hat{t}}{\hat{s}^2} - \delta_{ac}\frac{1}{3}\frac{\hat{u}}{\hat{s}}\right)$$

LO helicity amplitudes for the elementary process ab → cd (2)

$$qg \longrightarrow qg$$

$$|\hat{M}_1^0|^2 = \frac{8}{9}\, g_s^4 \left(-\frac{\hat{s}}{\hat{u}} + \frac{9}{4}\frac{\hat{s}^2}{\hat{t}^2}\right) \qquad |\hat{M}_2^0|^2 = \frac{8}{9}\, g_s^4 \left(-\frac{\hat{u}}{\hat{s}} + \frac{9}{4}\frac{\hat{u}^2}{\hat{t}^2}\right)$$

$$\hat{M}_1^0 \hat{M}_2^0 = -\frac{8}{9}\, g_s^4 \left(-1 + \frac{9}{4}\frac{\hat{u}\hat{s}}{\hat{t}^2}\right).$$

$$q\bar{q} \longrightarrow gg$$

$$|\hat{M}_2^0|^2 = \frac{64}{27}\, g_s^4 \left(\frac{\hat{u}}{\hat{t}} - \frac{9}{4}\frac{\hat{u}^2}{\hat{s}^2}\right) \qquad |\hat{M}_3^0|^2 = \frac{64}{27}\, g_s^4 \left(\frac{\hat{t}}{\hat{u}} - \frac{9}{4}\frac{\hat{t}^2}{\hat{s}^2}\right)$$

$$\hat{M}_2^0 \hat{M}_3^0 = \frac{64}{27}\, g_s^4 \left(1 - \frac{\hat{t}\hat{u}}{\hat{s}^2}\right)$$

$$gg \longrightarrow gg$$

$$|\hat{M}_1^0|^2 = \frac{9}{2}\, g_s^4\, \hat{s}^2 \left(\frac{1}{\hat{t}^2} + \frac{1}{\hat{u}^2} + \frac{1}{\hat{t}\hat{u}}\right) \qquad |\hat{M}_2^0|^2 = \frac{9}{2}\, g_s^4\, \frac{\hat{u}^2}{\hat{s}^2} \left(1 + \frac{\hat{u}}{\hat{t}} + \frac{\hat{u}^2}{\hat{t}^2}\right)$$

$$|\hat{M}_3^0|^2 = \frac{9}{2}\, g_s^4\, \frac{\hat{t}^2}{\hat{s}^2} \left(1 + \frac{\hat{t}}{\hat{u}} + \frac{\hat{t}^2}{\hat{u}^2}\right) \qquad \hat{M}_1^0 \hat{M}_2^0 = \frac{9}{2}\, g_s^4 \left(1 + \frac{\hat{u}}{\hat{t}} + \frac{\hat{u}^2}{\hat{t}^2}\right)$$

$$\hat{M}_1^0 \hat{M}_3^0 = \frac{9}{2}\, g_s^4 \left(1 + \frac{\hat{t}}{\hat{u}} + \frac{\hat{t}^2}{\hat{u}^2}\right) \qquad \hat{M}_2^0 \hat{M}_3^0 = \frac{9}{2}\, g_s^4\, \frac{1}{\hat{s}^2} \left(\hat{u}^2 + \hat{t}^2 + \hat{u}\hat{t}\right)$$