THE TRANSVERSE STRUCTURE OF THE NUCLEON

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The Nucleon Structure – 12th HANUC Lecture Week – Torino

Lecture II - Phenomenology

- Semi-inclusive DIS (Cahn, Boer-Mulders, Sivers, Collins effects)
- e^+e^- annihilation into nearly back-to-back hadrons (Collins effect)
- Drell-Yan process (Transversity, Boer-Mulders and Sivers effects)
- Single particle production in hadronic collisions

  Pion, kaon, Λ hyperon production

  Prompt photon production

  (GPM and twist-three collinear approaches)

- Double particle production in hadronic collisions

  Prompt photon + jet production (universality, TMD CGI approach)

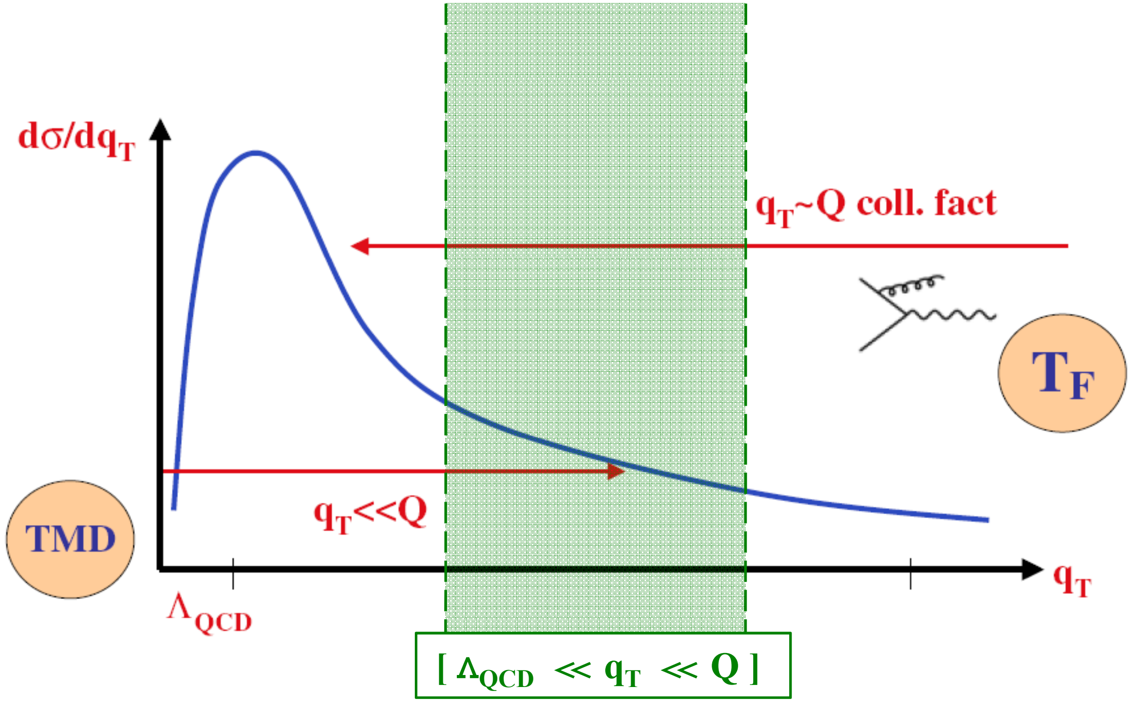


Figure 17. Cartoon of different kinematic regions $q_\perp \sim Q$ and $q_\perp \gg \Lambda_{\rm QCD}$ relevant for the single-spin asymmetry in the Drell–Yan process. In the region of overlap, $Q \gg q_\perp \gg \Lambda_{\rm QCD}$ both mechanisms describe the same physics [71].

Jaffe, Ji – Amsterdam group notation

f, g, h : unpolarized, longitudinally pol., transversely pol. quark

Subscript 1: leading twist distribution
Subscript L: longitudinally polarized hadron
Subscript T: transversely polarized hadron

Apex $\perp$: presence of transverse momenta with uncontracted Lorentz indices

$$h_{1L}^{\perp}$$

Transversely polarized quark

Uncontracted Lorentz indices

Leading twist distribution

Longitudinally polarized proton

Leading twist TMD PDFs and FFs

Sivers distribution function (chiral-even, naively T-odd)

Cagliari London Torino notation

$$\Delta^N f_{q/p^\uparrow}(x, |\boldsymbol{k}_\perp|) \; = \; -2\, \frac{|\boldsymbol{k}_\perp|}{M_p}\, f_{1T}^{\perp q}(x, |\boldsymbol{k}_\perp|)$$

Amsterdam notation

Boer-Mulders function (chiral odd, naively T-odd)

$$\Delta^N f_{q^\uparrow/p}(x, |\boldsymbol{k}_\perp|) \; = \; -\frac{|\boldsymbol{k}_\perp|}{M_p}\, h_1^{\perp q}(x, |\boldsymbol{k}_\perp|)$$

Collins fragmentation function (chiral-odd, naively T-odd)

$$\Delta^N D_{h/q^\uparrow}(z, |\boldsymbol{k}_{\perp h}|) \; = \; \frac{2|\boldsymbol{k}_{\perp h}|}{z M_h}\, H_1^{\perp q}(z, |\boldsymbol{k}_{\perp h}|)$$

“Polarizing” fragmentation function (chiral even, naively T-odd)

$$\Delta^N D_{\Lambda^\uparrow/q}(z, |\boldsymbol{k}_{\perp\Lambda}|) \; = \; \frac{|\boldsymbol{k}_{\perp\Lambda}|}{z M_\Lambda}\, D_{1T}^{\perp q}(z, |\boldsymbol{k}_{\perp\Lambda}|)$$

Theoretical groups who have performed phenomenological analyses of the processes of interest:

The Cagliari London Torino group

Anselmino, Boglione, D'Alesio, Kotzinian, Leader, Melis, FM, Prokudin, Türk

TMD approach, LT asymmetries in SIDIS, Drell Yan, e+e- collisions

HT SSAs in single particle production in pp collisions

The Bochum group

Goeke, Menzel, Metz, Schlegel, Schweitzer + Efremov and Collins

TMD approach, LT asymmetries in SIDIS, Drell Yan, e+e- collisions

Qiu, Sterman, Vogelsang, Yuan, Koike,…

Collinear twist-three approach for single and double particle production in pp collisions

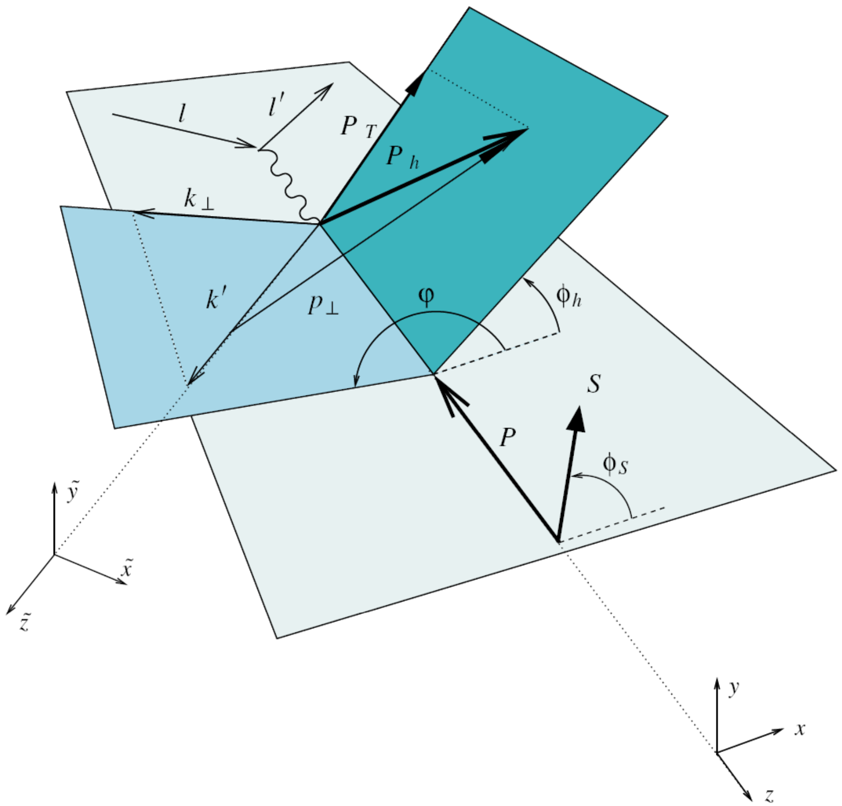
SIDIS and Drell-Yan in the intermediate -large q_T region

Amsterdam group

Boer, Mulders, Bacchetta, Pijlman, Bomhof

TMD CGI approach, double particle production with q_T imbalance in pp collisions

Phenomenological applications: SIDIS



Kinematical variables

$$q^2 = (\ell - \ell')^2 = -Q^2$$

$$x_B = Q^2/(2P \cdot q) \text{ Bjorken variable}$$

$$y = (P \cdot q)/(P \cdot \ell) \text{ inelasticity}$$

$$W^2 = (P + q)^2 \text{ c.m. energy of the } \gamma^* N \text{ system}$$

$$z_h = (P \cdot P_h)/(P \cdot q)$$

$$\hat{s} = xs - 2\ell \cdot \boldsymbol{k}_\perp - k_\perp^2 \frac{x_B}{x}\left(1 - \frac{x_B s}{Q^2}\right) \qquad \hat{t} = -Q^2$$

$$\hat{u} = -x\left(s - \frac{Q^2}{x_B}\right) + 2\ell \cdot \boldsymbol{k}_\perp - k_\perp^2 \frac{x_B^2 s}{x Q^2}.$$

$$x = \frac{1}{2} x_B \left(1 + \sqrt{1 + \frac{4k_\perp^2}{Q^2}}\right)$$

$$z = z_h + \frac{k_\perp P_{hT}}{Q^2} \frac{2x_B}{1 - x_B} \cos(\phi_h - \varphi) + \mathcal{O}\left(\frac{k_\perp^2}{Q^2}\right)$$

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$

$$+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$

$$+S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$

$$+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]$$

$$+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$

$$+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\},$$

$$\gamma=\frac{2Mx}{Q}$$

$$d\psi\approx d\phi_S$$

$$\varepsilon=\frac{1-y-\frac{1}{4}\gamma^2y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2y^2}$$

Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel JEHP 02 2007

Integration over the outgoing hadron transverse momentum gives

$$\frac{d\sigma}{dx\,dy\,d\psi\,dz} = \frac{2\alpha^2}{xyQ^2}\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon F_{UU,L} + S_\parallel \lambda_e \sqrt{1-\varepsilon^2}\,F_{LL} + |\boldsymbol{S}_\perp|\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + |\boldsymbol{S}_\perp|\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\Big\}$$

$$F_{UU,T}(x,z,Q^2) = \int d^2\boldsymbol{P}_{h\perp}\,F_{UU,T}(x,z,P_{h\perp}^2,Q^2)$$

Integrating over z and summing over all hadrons in the final state we recover the fully inclusive case

$$\frac{d\sigma}{dx\,dy\,d\psi} = \frac{2\alpha^2}{xyQ^2}\frac{y^2}{2\,(1-\varepsilon)}\Big\{F_T + \varepsilon F_L + S_\parallel\lambda_e\sqrt{1-\varepsilon^2}\,2x\,(g_1-\gamma^2 g_2) - |\boldsymbol{S}_\perp|\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,2x\gamma\,(g_1+g_2)\Big\}$$

$$\sum_h \int dz\, z\, F_{UU,T}(x,z,Q^2) = 2xF_1(x,Q^2) = F_T(x,Q^2)$$

$$\sum_h \int dz\, z\, F_{UU,L}(x,z,Q^2) = (1+\gamma^2)F_2(x,Q^2) - 2xF_1(x,Q^2) = F_L(x,Q^2)$$

$$\sum_h \int dz\, z\, F_{LL}(x,z,Q^2) = 2x\,\big(g_1(x,Q^2) - \gamma^2 g_2(x,Q^2)\big),$$

$$\sum_h \int dz\, z\, F_{LT}^{\cos\phi_S}(x,z,Q^2) = -2x\gamma\,\big(g_1(x,Q^2) + g_2(x,Q^2)\big)$$

$$\mathcal{C}\left[w\,f\,D\right] = x\sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right],$$

$$F_{UU,L} = 0,$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh\,H_1^\perp + \frac{M_h}{M}\,f_1\frac{\tilde{D}^\perp}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf^\perp D_1 + \frac{M_h}{M}\,h_1^\perp\frac{\tilde{H}}{z}\right)\right],$$

Cahn effect (LT)

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^\perp H_1^\perp\right],$$

Cahn (HT) and BM⊗Collins (LT) effects

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe\,H_1^\perp + \frac{M_h}{M}\,f_1\frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg^\perp D_1 + \frac{M_h}{M}\,h_1^\perp\frac{\tilde{E}}{z}\right)\right],$$

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh_L H_1^\perp + \frac{M_h}{M}\,g_{1L}\frac{\tilde{G}^\perp}{z}\right) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf_L^\perp D_1 - \frac{M_h}{M}\,h_{1L}^\perp\frac{\tilde{H}}{z}\right)\right],$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^\perp H_1^\perp\right],$$

$$F_{LL} = \mathcal{C}\left[g_{1L} D_1\right],$$

$$F_{LL}^{\cos\phi_h} = \frac{2M}{Q}\mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xe_L H_1^\perp - \frac{M_h}{M}\,g_{1L}\frac{\tilde{D}^\perp}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xg_L^\perp D_1 + \frac{M_h}{M}\,h_{1L}^\perp\frac{\tilde{E}}{z}\right)\right]$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

Sivers effect (LT)

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = 0,$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

Collins effect (LT)

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)\,(\boldsymbol{p}_T\cdot\boldsymbol{k}_T) + \boldsymbol{p}_T^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T) - 4\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{2M^2 M_h}\, h_{1T}^{\perp} H_1^{\perp}\right],$$

$$F_{UT}^{\sin\phi_S} = \frac{2M}{Q}\,\mathcal{C}\left\{\left(x f_T D_1 - \frac{M_h}{M}\, h_1 \frac{\tilde{H}}{z}\right) - \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\right) - \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\left\{\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{2M^2}\left(x f_T^{\perp} D_1 - \frac{M_h}{M}\, h_{1T}^{\perp}\frac{\tilde{H}}{z}\right) - \frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\right) + \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} g_{1T} D_1\right],$$

$$F_{LT}^{\cos\phi_S} = \frac{2M}{Q}\,\mathcal{C}\left\{-\left(x g_T D_1 + \frac{M_h}{M}\,h_1\frac{\tilde{E}}{z}\right) + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x e_T H_1^{\perp} - \frac{M_h}{M} g_{1T}\,\frac{\tilde{D}^{\perp}}{z}\right) + \left(x e_T^{\perp} H_1^{\perp} + \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{G}^{\perp}}{z}\right)\right]\right\},$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\left\{-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2}{2M^2}\left(x g_T^{\perp} D_1 + \frac{M_h}{M}\,h_{1T}^{\perp}\frac{\tilde{E}}{z}\right) + \frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x e_T H_1^{\perp} - \frac{M_h}{M} g_{1T}\,\frac{\tilde{D}^{\perp}}{z}\right) - \left(x e_T^{\perp} H_1^{\perp} + \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{G}^{\perp}}{z}\right)\right]\right\}.$$

Azimuthal moments in SIDIS

$$A_{S_B\,S_T}^{W(\phi_h,\phi_S)} = 2\langle\, W(\phi_h,\phi_S)\,\rangle = 2\,\frac{\int d\phi_h d\phi_S\, W(\phi_h,\phi_S)\,\left[\, d\sigma(\phi_h,\phi_S) - d\sigma(\phi_h,\phi_S+\pi)\,\right]}{\int d\phi_h d\phi_S\left[\, d\sigma(\phi_h,\phi_S) + d\sigma(\phi_h,\phi_S+\pi)\,\right]}$$

$B =$ lepton beam, $S_B = U,\, L$ $\qquad$ $T =$ nucleon target, $S_T = U,\, L,\, T$

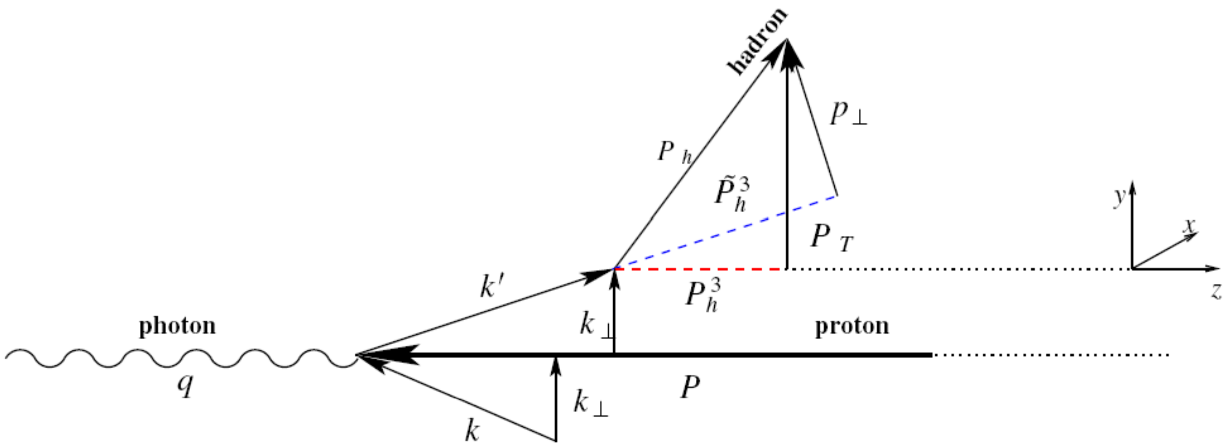
Note: electron and muon beams are naturally polarized in the transverse direction. However, we do not consider transversely polarized lepton beams here: to see the related effects one should be able to measure also the final lepton polarization.

A phenomenologically relevant example:
the azimuthal moments related to the Sivers and Collins effects

$$\langle\, \sin(\phi_h \pm \phi_S)\,\rangle = \frac{\int d\phi_h d\phi_S\, \sin(\phi_h \pm \phi_S)\,\left[\, d\sigma(\phi_h,\phi_S) - d\sigma(\phi_h,\phi_S+\pi)\,\right]}{\int d\phi_h d\phi_S\left[\, d\sigma(\phi_h,\phi_S) + d\sigma(\phi_h,\phi_S+\pi)\,\right]}$$

Unpolarized case (I): Cahn effect

$$F_{UU,T} \qquad F_{UU}^{\cos\phi_h} \qquad F_{UU}^{\cos 2\phi_h}$$



$$\frac{d^5\sigma^{\ell p\to\ell hX}}{dx_B dQ^2 dz_h d^2\boldsymbol{P}_T} = \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp f_q(x,k_\perp)\frac{2\pi\alpha^2}{x_B^2 s^2}\frac{\hat{s}^2+\hat{u}^2}{Q^4} D_q^h(z,p_\perp)\frac{z}{z_h}\frac{x_B}{x}\left(1+\frac{x_B^2}{x^2}\frac{k_\perp^2}{Q^2}\right)^{-1}$$

$$\propto A + B_{\rm Cahn}^{LT}\left(\frac{P_T}{Q}\right)\cos\phi_h + C_{\rm Cahn}^{HT}\left(\frac{P_T}{Q}\right)^2\cos 2\phi_h$$

Unpolarized case (I): Cahn effect (LO)

Assume a simple factorized, flavour-independent, Gaussian k_T shape

$$f_q(x, k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$D_q^h(z, p_\perp) = D_q^h(z)\,\frac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

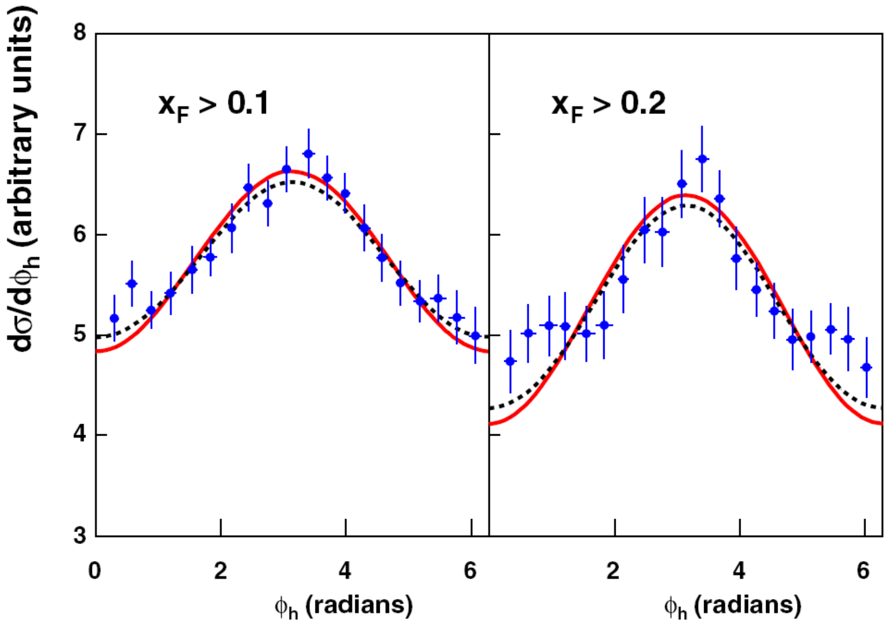
$$\frac{d^5\sigma^{\ell p\to\ell hX}}{dx_B\,dQ^2\,dz_h\,d^2\boldsymbol{P}_T} = \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp f_q(x, k_\perp)\,\frac{2\pi\alpha^2}{x_B^2 s^2}\,\frac{\hat{s}^2+\hat{u}^2}{Q^4}\,D_q^h(z, p_\perp)\,\frac{z}{z_h}\,\frac{x_B}{x}\left(1+\frac{x_B^2}{x^2}\frac{k_\perp^2}{Q^2}\right)^{-1}$$

$$\propto A + B_{\rm Cahn}^{LT}\left(\frac{P_T}{Q}\right)\cos\phi_h + C_{\rm Cahn}^{HT}\left(\frac{P_T}{Q}\right)^2\cos 2\phi_h$$

At leading twist in the (kT/Q) power expansion the k_T integration can be performed analitically:

$$\frac{d^5\sigma^{\ell p\to\ell hX}}{dx_B\,dQ^2\,dz_h\,d^2\boldsymbol{P}_T} \simeq \sum_q \frac{2\pi\alpha^2 e_q^2}{Q^4}\,f_q(x_B)\,D_q^h(z_h)\left[1+(1-y)^2 - 4\,\frac{(2-y)\sqrt{1-y}\,\langle k_\perp^2\rangle z_h P_T}{\langle P_T^2\rangle Q}\cos\phi_h\right]\frac{1}{\pi\langle P_T^2\rangle}\,e^{-P_T^2/\langle P_T^2\rangle}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$$



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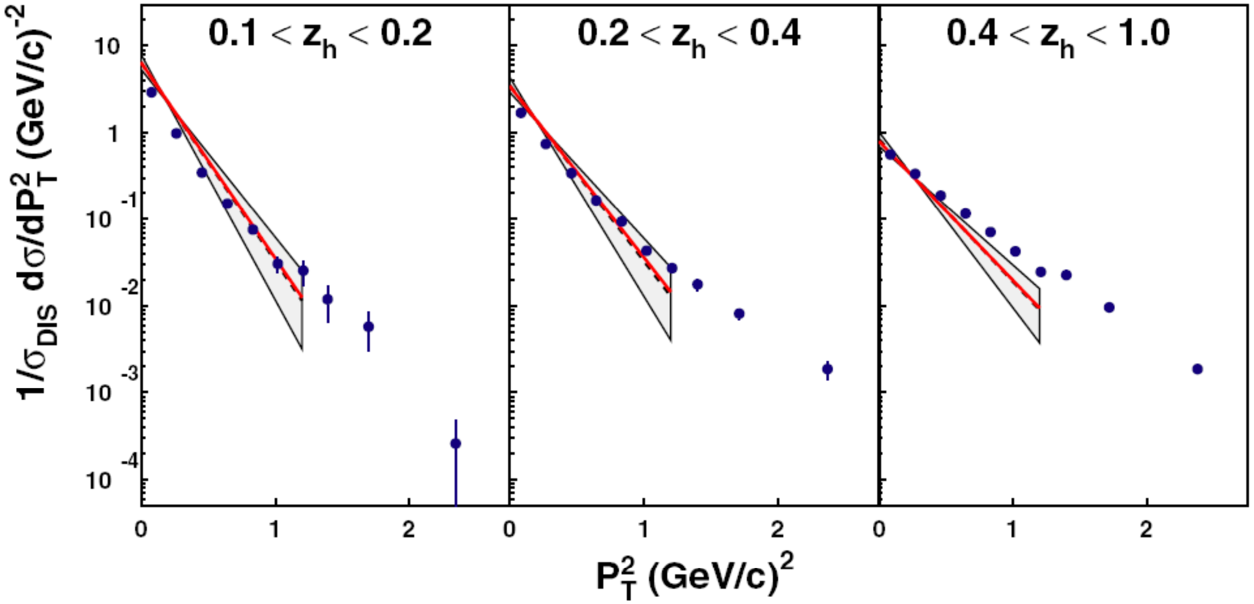
Kotzinian, FM, Prokudin
PRD 71 2005

$\langle k_\perp^2 \rangle = 0.25\,\mathrm{GeV}^2$ $\langle p_\perp^2 \rangle = 0.20\,\mathrm{GeV}^2$



Unpolarized case (I): Cahn effect (LO QCD)

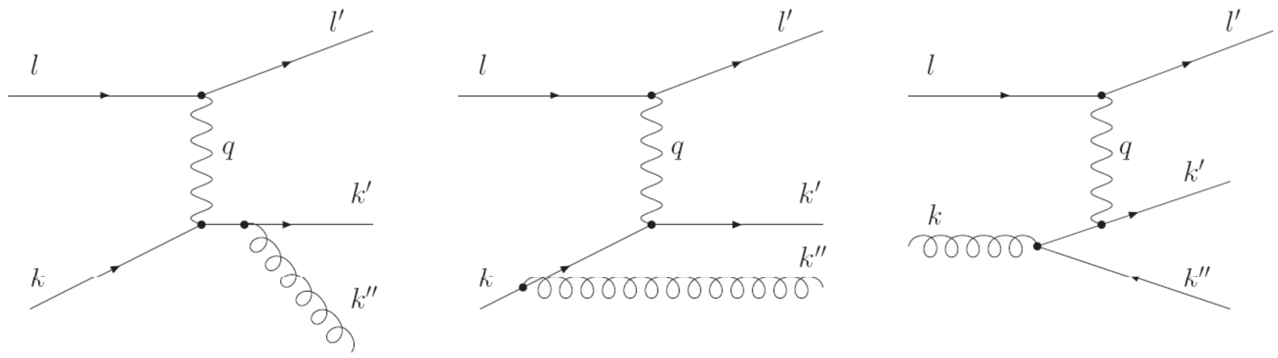


FIG. 2:
Feynman diagrams corresponding to ℓq and ℓg elementary scattering at first order in α_s.

$$\frac{d^5\sigma_1^{lp\to lhX}}{dx_{Bj}\,dy\,dz_h\,d^2\boldsymbol{P}_T} = \frac{\alpha^2\,e_q^2}{16\pi^2}\,\frac{y}{Q^4}\int_{x_{Bj}}^1 \frac{dx'}{x'P_T^2 + z_h^2(1-x')Q^2}\sum_{i,j} f_i\left(\frac{x_{Bj}}{x'},Q^2\right)\,L_{\mu\nu}\,M_{ij}^{\mu\nu}\,D_j^h\left(z_h + \frac{x'P_T^2}{z_h(1-x')Q^2},Q^2\right)$$

$$L_{\mu\nu}M_{ij}^{\mu\nu} \propto A_{ij}(x',z_h) + B_{ij}(x',z_h)\cos\phi_h + C_{ij}(x',z_h)\cos 2\phi_h$$

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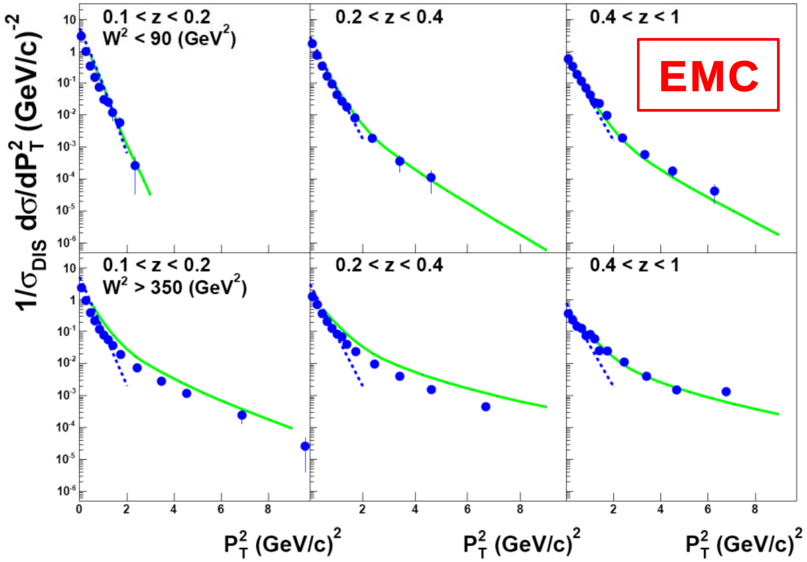
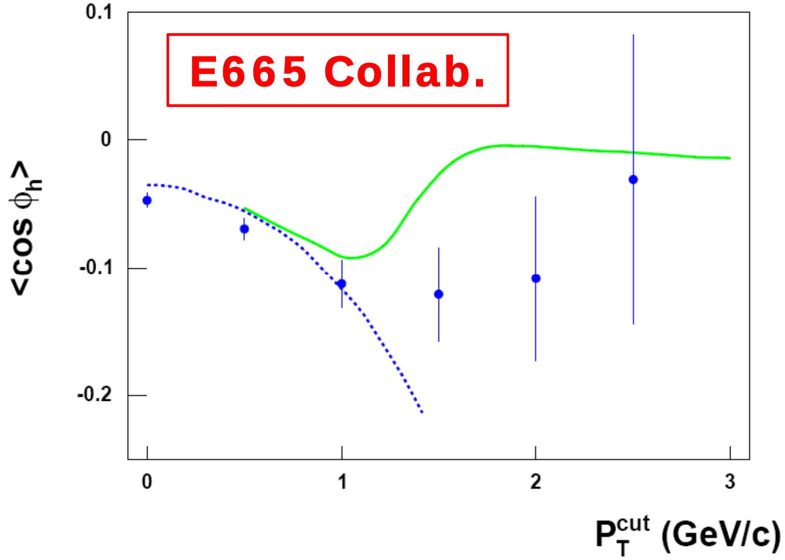
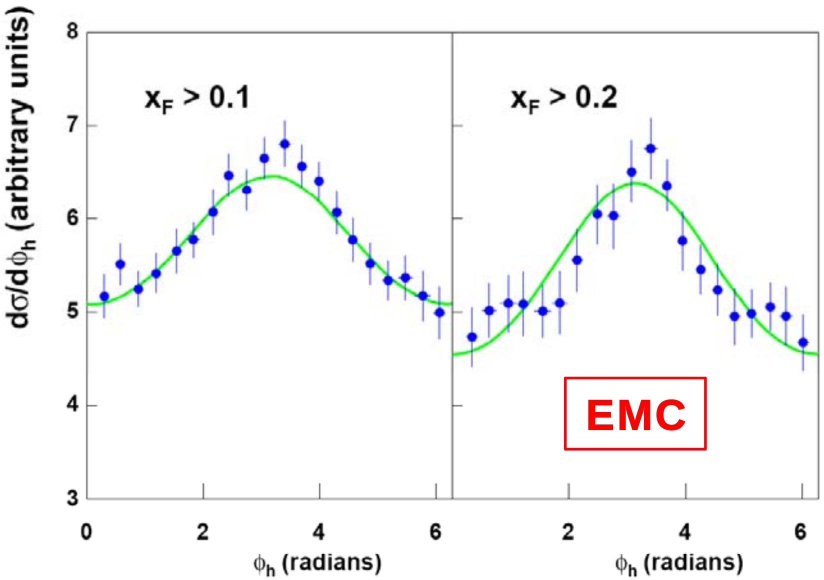


$$\langle k_\perp^2 \rangle = 0.28\,\mathrm{GeV}^2 \qquad \langle p_\perp^2 \rangle = 0.25\,\mathrm{GeV}^2$$





Unpolarized case (II): Boer-Mulders ⊗ Collins contribution

$$\langle \cos 2\phi \rangle = \frac{\int d\sigma^{(0)} \; \cos 2\phi \; + \; \int d\sigma^{(1)} \; \cos 2\phi}{\int d\sigma^{(0)} \; + \; \int d\sigma^{(1)}}$$

$$\left. \frac{d^5\sigma^{(0)}_{\rm BM}}{dx\, dy\, dz\, d^2\mathbf{P}_T} \right|_{\cos 2\phi} = \frac{4\pi\alpha^2_{\rm em} s}{Q^4} \sum_a e_a^2 \, x(1-y) \times \int d^2\mathbf{k}_T \int d^2\mathbf{p}_T \, \delta^2(\mathbf{P}_T - z\mathbf{k}_T - \mathbf{p}_T) \times \frac{2\,\mathbf{h}\cdot\mathbf{k}_T \, \mathbf{h}\cdot\mathbf{p}_T - \mathbf{k}_T\cdot\mathbf{p}_T}{zMM_h} h_1^{\perp a}(x, k_T^2)\, H_1^{\perp a}(z, p_T^2) \, \cos 2\phi$$

$$h_1^{\perp q} \sim -\kappa_T^q \qquad f_{1T}^{\perp q} \sim -\kappa^q$$

$$h_1^{\perp q}(x, k_T^2) = \frac{\kappa_T^q}{\kappa^q} f_{1T}^{\perp q}(x, k_T^2)$$

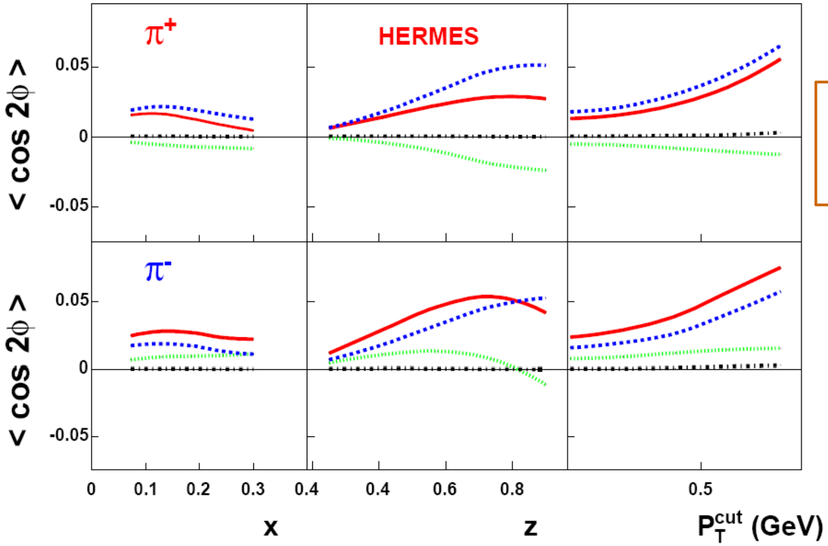
$$\kappa^u \simeq 1.67 \text{ and } \kappa^d \simeq -2.03$$

$$\kappa_T^u \simeq 3, \; \kappa_T^d \simeq 1.9.$$

$$h_1^{\perp u} \simeq 1.80 \, f_{1T}^{\perp u} \,, \quad h_1^{\perp d} = -0.94 \, f_{1T}^{\perp d}$$

Barone Prokudin Ma PRD 78 (2008)

FIG. 4: Our prediction for the cos 2ϕ asymmetry at HERMES. The dot–dashed line is the $\mathcal{O}(\alpha_s)$ QCD contribution, the dotted line is the Boer-Mulder contribution, the dashed line is the higher–twist Cahn contribution. The continuous line is the resulting asymmetry taking all contributions into account.



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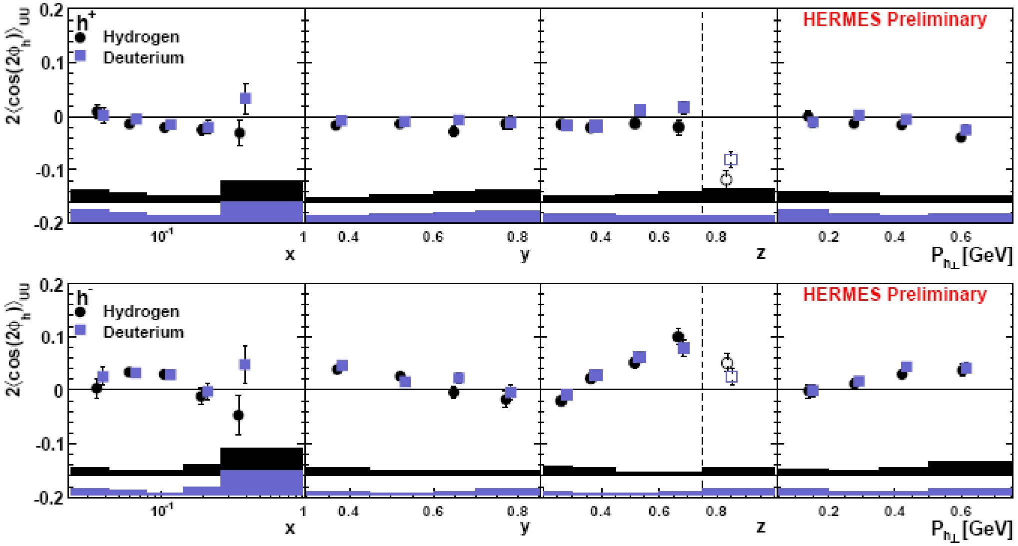
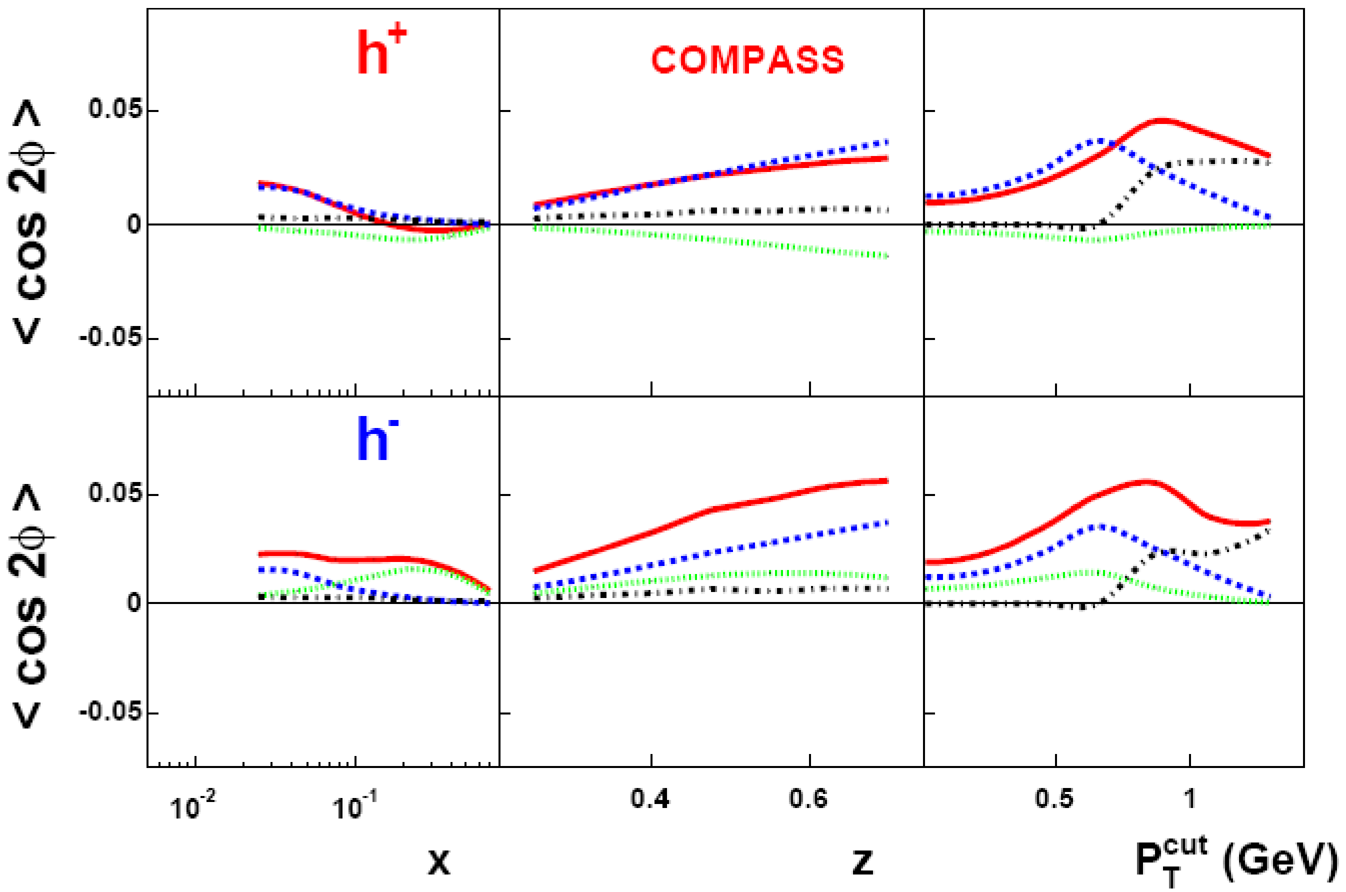


HERMES
arXiv:0901.2438
[hep-ex]



Deuteron target

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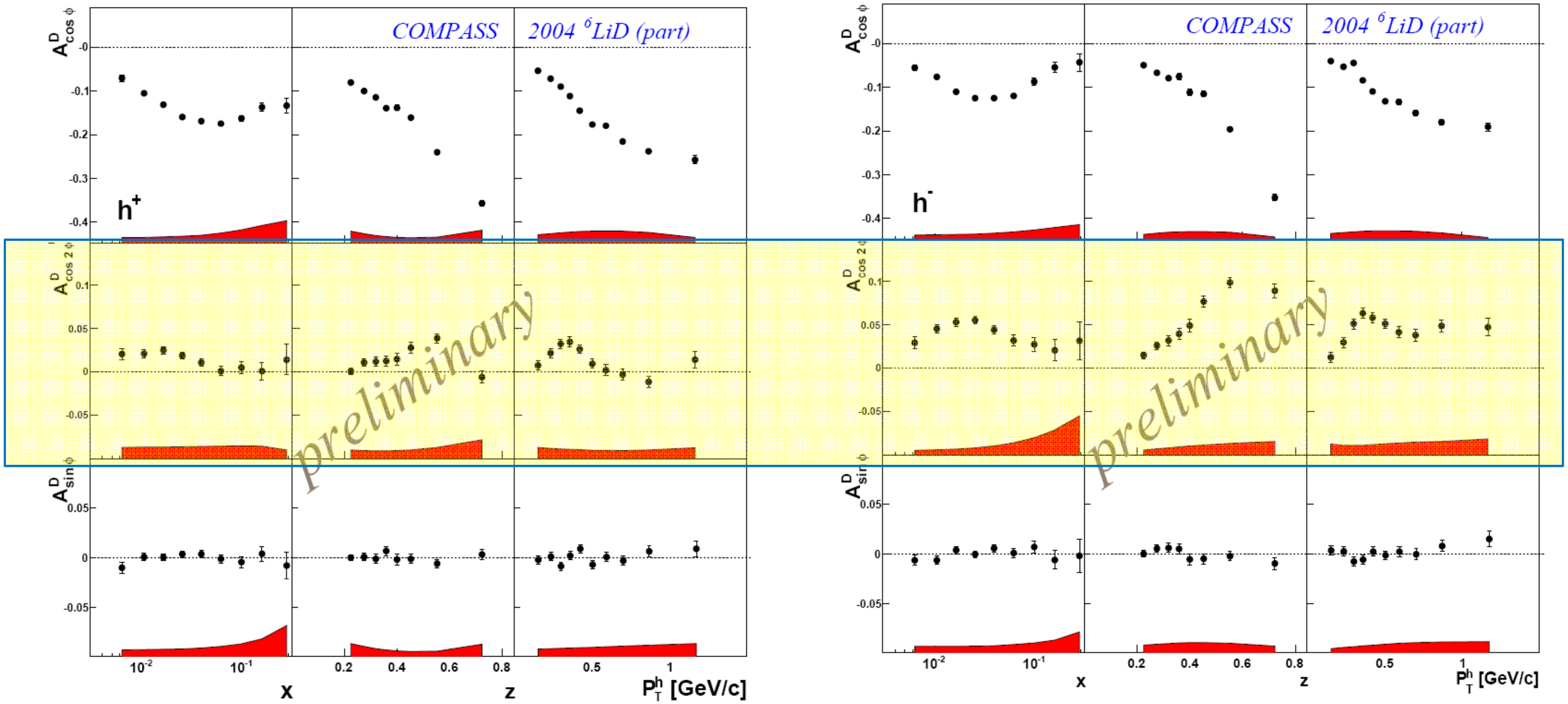


COMPASS arXiv:0808.0114 [hep-ex]

Transversely polarized hadron: Sivers and Collins effects

$$\frac{d^6\sigma^{\ell p^\uparrow \to \ell' h X}}{dx_B\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S} = A_0 + \tilde{A}_{UT}^{\rm Sivers}\sin(\phi_h-\phi_S) + \tilde{A}_{UT}^{\rm Collins}\sin(\phi_h+\phi_S)$$
$$+ \tilde{A}_{LT}^{[g_{1T}\otimes D_1]}\cos(\phi_h-\phi_S) + \tilde{A}_{UT}^{[h_{1T}^\perp\otimes H_1^\perp]}\sin(3\phi_h-\phi_S)$$
$$+ \tilde{A}_{LT}^{[g_{1T}\otimes D_1]}\cos\phi_S + \tilde{A}_{LT}^{[g_{1T}\otimes D_1]}\cos(2\phi_h-\phi_S)$$
$$+ \tilde{A}_{UT}^{[f_{1T}^\perp\otimes D_1+\ldots]}\sin\phi_S + \tilde{A}_{UT}^{[f_{1T}^\perp\otimes D_1+\ldots]}\sin(2\phi_h-\phi_S)$$

$$A_{UT}^{\sin(\phi_h-\phi_s)} \propto \frac{f_{1T}^{\perp q}\otimes D_{1q}^h}{f_1^q\otimes D_{1q}^h},\quad A_{UT}^{\sin(\phi_h+\phi_s)} \propto \frac{h_1^q\otimes H_{1q}^{\perp h}}{f_1^q\otimes D_{1q}^h},$$

$$A_{LT}^{\cos(\phi_h-\phi_s)} \propto \frac{g_{1T}^q\otimes D_{1q}^h}{f_1^q\otimes D_{1q}^h},\quad A_{UT}^{\sin(3\phi_h-\phi_s)} \propto \frac{h_{1T}^{\perp q}\otimes H_{1q}^{\perp h}}{f_1^q\otimes D_{1q}^h}$$

$$A_{LT}^{\cos(\phi_s)} \propto \frac{M}{Q}\frac{g_{1T}^q\otimes D_{1q}^h}{f_1^q\otimes D_{1q}^h},\quad A_{LT}^{\cos(2\phi_h-\phi_s)} \propto \frac{M}{Q}\frac{g_{1T}^q\otimes D_{1q}^h}{f_1^q\otimes D_{1q}^h},$$

$$A_{UT}^{\sin(\phi_s)} \propto \frac{M}{Q}\frac{h_1^q\otimes H_{1q}^{\perp h}+f_{1T}^{\perp q}\otimes D_{1q}^h}{f_1^q\otimes D_{1q}^h},$$

$$A_{UT}^{\sin(2\phi_h-\phi_s)} \propto \frac{M}{Q}\frac{h_{1T}^{\perp q}\otimes H_{1q}^{\perp h}+f_{1T}^{\perp q}\otimes D_{1q}^h}{f_1^q\otimes D_{1q}^h}.$$

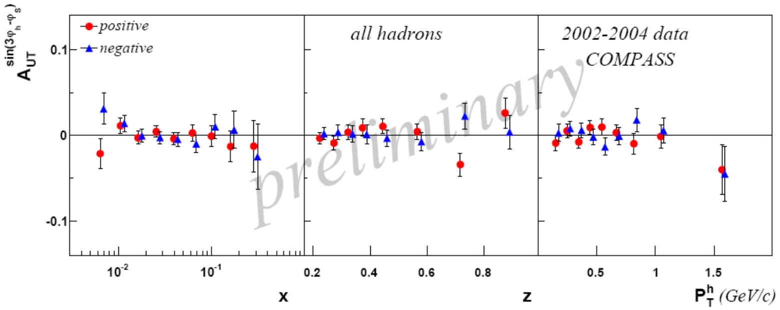


Fig. 1. $A_{UT}^{sin(3\phi_h-\phi_s)}$ asymmetry for positive (red circles) and negative (blue triangles) hadrons vs. x, z and p_t.

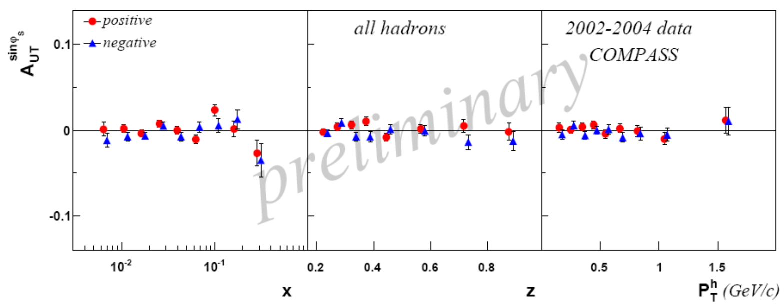


Fig. 2. $A_{UT}^{sin\phi_s}$ asymmetry for positive (red circles) and negative (blue triangles) hadrons vs. x, z and p_t.

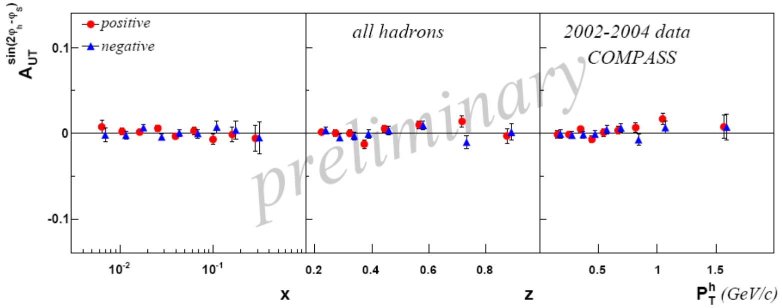


Fig. 3. $A_{UT}^{sin(2\phi_h-\phi_s)}$ asymmetry for positive (red circles) and negative (blue triangles) hadrons vs. x, z and p_t.

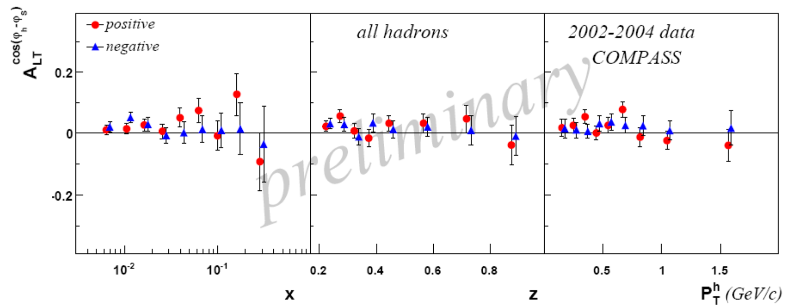


Fig. 4. $A_{LT}^{cos(\phi_h-\phi_s)}$ asymmetry for positive (red circles) and negative (blue triangles) hadrons vs. x, z and p_t.

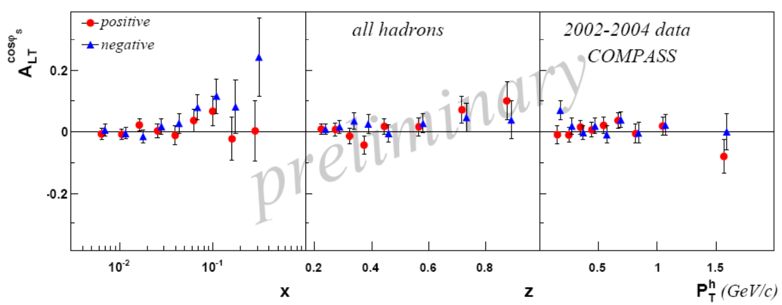


Fig. 5. $A_{LT}^{cos\phi_s}$ asymmetry for positive (red circles) and negative (blue triangles) hadrons vs. x, z and p_t.

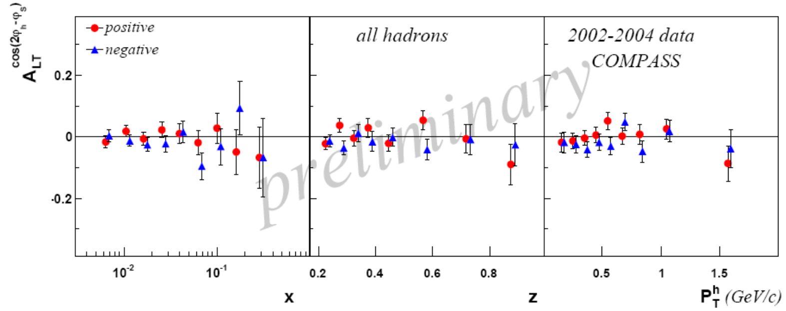


Fig. 6. $A_{LT}^{cos(2\phi_h-\phi_s)}$ asymmetry for positive (red circles) and negative (blue triangles) hadrons vs. x, z and p_t.

Transversely polarized hadron: Sivers and Collins asymmetries [O(k⊥/Q)]

$$A_{\text{Sivers}} \equiv \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp \Delta^N f_{q/p^\uparrow}(x, k_\perp) \sin(\varphi - \phi_S) \, \frac{d\hat{\sigma}}{dQ^2} D_{h/q}(z, p_\perp),$$

$$A_{\text{Collins}} \equiv \sum_q e_q^2 \int d^2\boldsymbol{k}_\perp \Delta_T q(x, k_\perp) \frac{d(\Delta\hat{\sigma})}{dQ^2} \Delta^N D_{h/q^\uparrow}(z, p_\perp) \sin(\phi_S + \varphi + \phi_q^h)$$

$$\frac{d\hat{\sigma}}{dy} = \frac{2\pi\alpha^2}{sxy^2}[1 + (1-y)^2]$$

$$\frac{d(\Delta\hat{\sigma})}{dQ^2} = \frac{d\hat{\sigma}^{\ell q^\uparrow \to \ell q^\uparrow}}{dQ^2} - \frac{d\hat{\sigma}^{\ell q^\uparrow \to \ell q^\downarrow}}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}(1-y)$$

Neglecting $\mathcal{O}(k_\perp^2/Q^2)$ terms, one finds

$$\cos\phi_q^h = \frac{P_T}{p_\perp}\cos(\phi_h - \varphi) - z\frac{k_\perp}{p_\perp}, \qquad \sin\phi_q^h = \frac{P_T}{p_\perp}\sin(\phi_h - \varphi)$$

$$\Delta_T q(x,k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\left[f_{q/p}(x) + \Delta_L q(x)\right]\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = 2\mathcal{N}_q^S(x)\, f_{q/p}(x)\sqrt{2e}\,\frac{k_\perp}{M'}\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}$$

$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\, D_{h/q}(z)\sqrt{2e}\,\frac{p_\perp}{M}\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_C}}{\pi\langle p_\perp^2\rangle},$$

$$\langle k_\perp^2\rangle_S = \frac{M'^2\langle k_\perp^2\rangle}{M'^2+\langle k_\perp^2\rangle}$$

$$\langle p_\perp^2\rangle_C = \frac{M^2\langle p_\perp^2\rangle}{M^2+\langle p_\perp^2\rangle}$$

$$|\mathcal{N}_q^{T,S,C}| \le 1$$

$$\langle P_T^2\rangle_S = \langle k_\perp^2\rangle_S + z^2\langle k_\perp^2\rangle$$

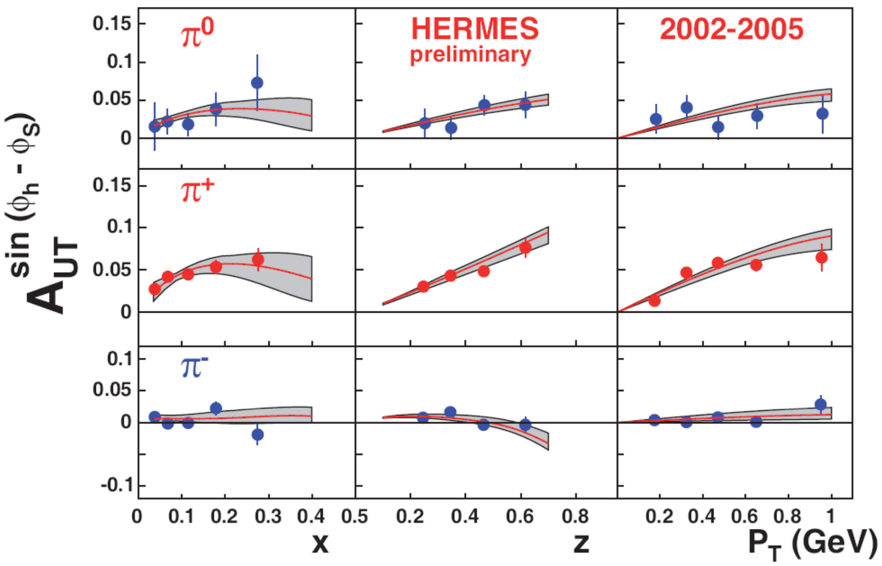
$$\langle P_T^2\rangle_C = \langle p_\perp^2\rangle_C + z^2\langle k_\perp^2\rangle$$

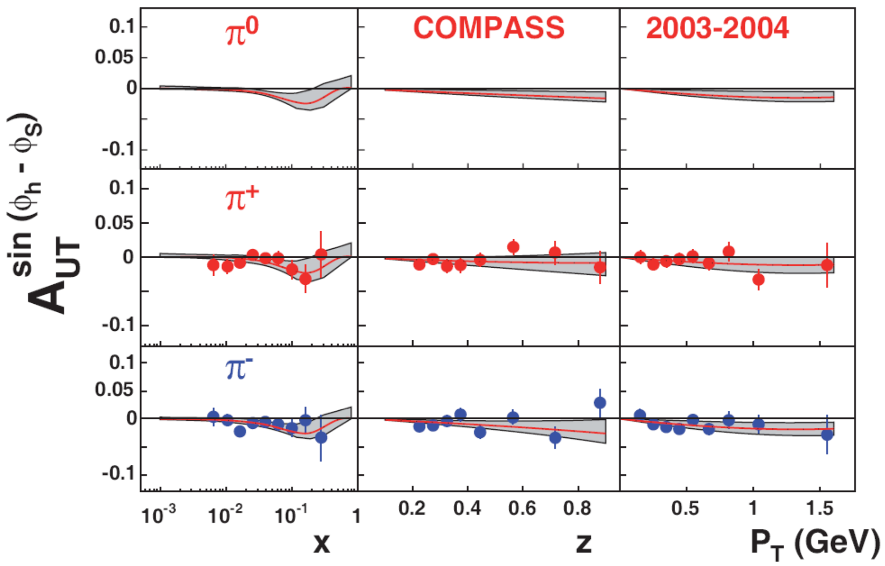
$$A_{UT}^{\sin(\phi_h-\phi_S)}(x,y,z,P_T) \simeq \frac{\Delta\sigma_{\rm Sivers}}{\sigma_0}, \qquad A_{UT}^{\sin(\phi_h+\phi_S)}(x,y,z,P_T) \simeq \frac{\Delta\sigma_{\rm Collins}}{\sigma_0}$$

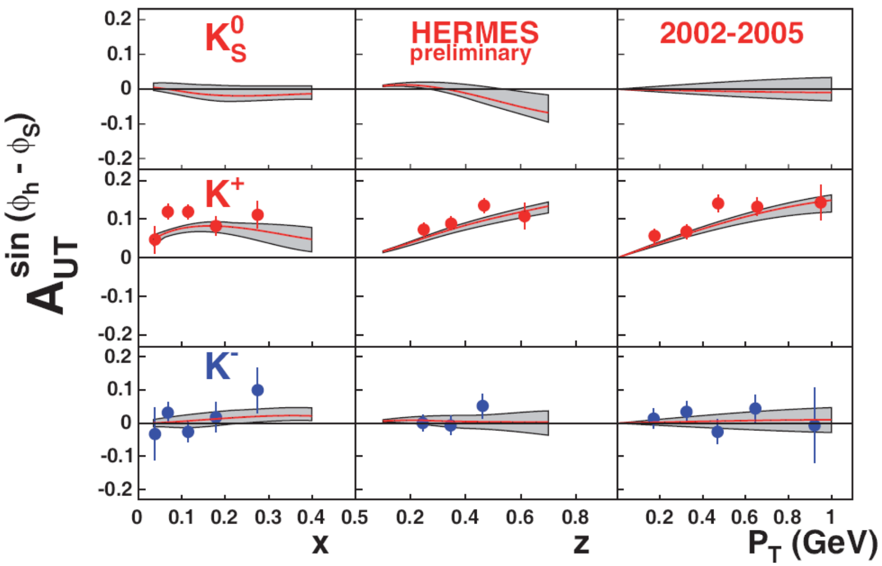
$$\Delta\sigma_{\rm Sivers} \propto \frac{z\,P_T}{M'}\,\frac{\sqrt{2e}\,\langle k_\perp^2\rangle_S^2}{\langle k_\perp^2\rangle}\,\frac{e^{-P_T^2/\langle P_T^2\rangle_S}}{\langle P_T^2\rangle_S^2}\left[1+(1-y)^2\right]\sum_q e_q^2\, 2\mathcal{N}_q^S(x)\, f_{q/p}(x)\, D_{h/q}(z)$$

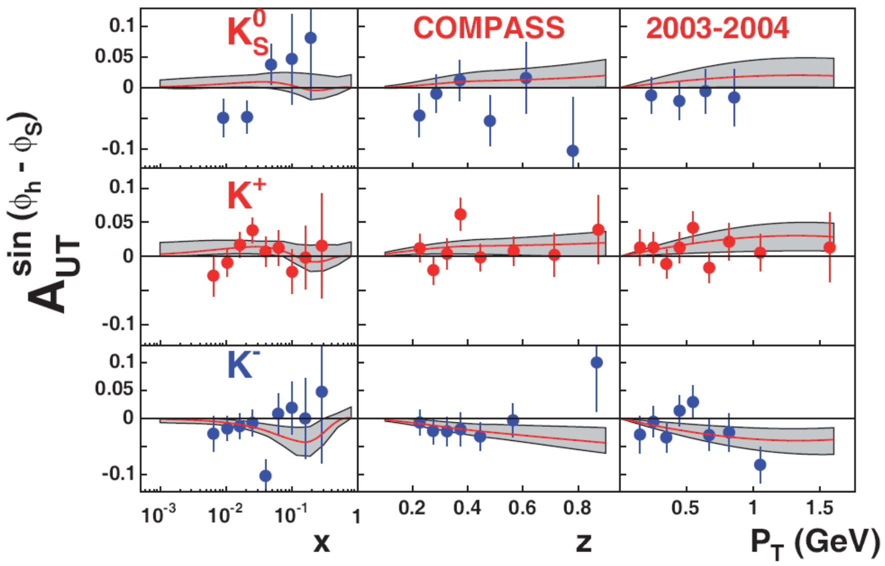
$$\Delta\sigma_{\rm Collins} \propto \frac{P_T}{M}\,\frac{\sqrt{2e}\,\langle p_\perp^2\rangle_C^2}{\langle p_\perp^2\rangle}\,\frac{e^{-P_T^2/\langle P_T^2\rangle_C}}{\langle P_T^2\rangle_C^2}\,(1-y)\sum_q e_q^2\,\mathcal{N}_q^T(x)\left[f_{q/p}(x)+\Delta_L q(x)\right]\mathcal{N}_q^C(z)\, D_{h/q}(z)$$

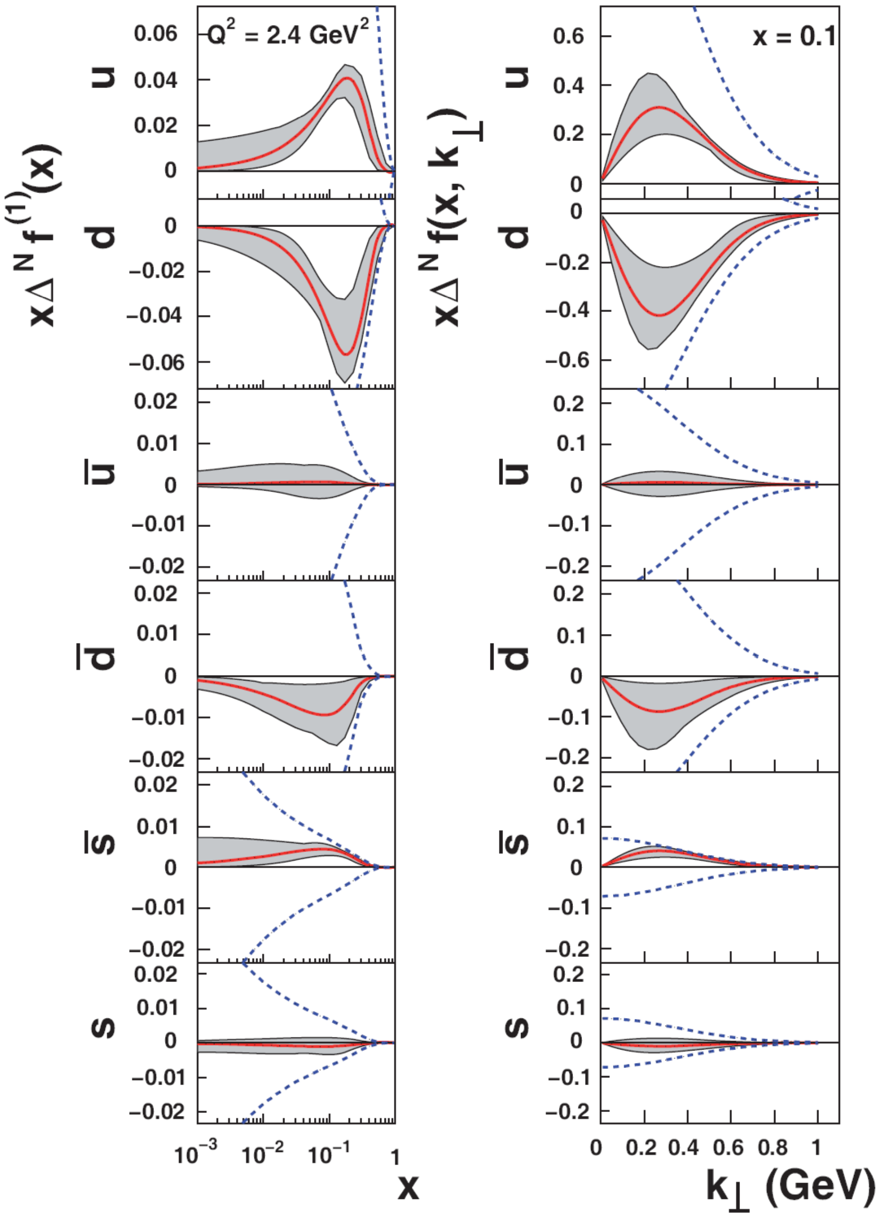
$$\sigma_0 \propto 2\pi\,\frac{1}{\pi\langle P_T^2\rangle}\, e^{-P_T^2/\langle P_T^2\rangle}\left[1+(1-y)^2\right]\sum_q e_q^2\, f_{q/p}(x)\, D_{h/q}(z),$$

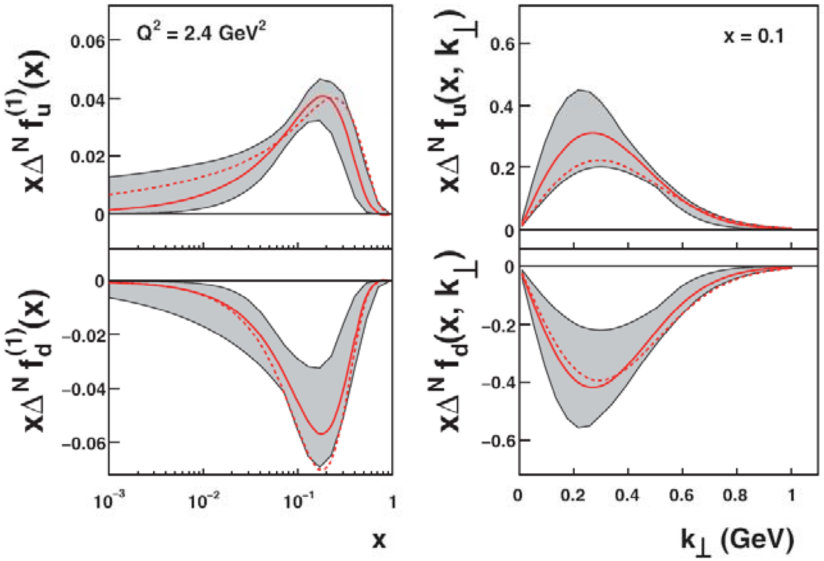

HERMES
preliminary
2002-2005
π^0
π^+
π^-
$A_{UT}^{\sin(\phi_h - \phi_S)}$
x
z
P_T (GeV)


COMPASS
2003-2004
π^0
π^+
π^-
$A_{UT}^{\sin(\phi_h - \phi_S)}$
x
z
P_T (GeV)


HERMES
preliminary
2002-2005
K_S^0
K^+
K^-
$A_{UT}^{\sin(\phi_h - \phi_S)}$
x
z
P_T (GeV)


COMPASS
2003-2004
K_S^0
K^+
K^-
$A_{UT}^{\sin(\phi_h - \phi_S)}$
x
z
P_T (GeV)





$$\sum_a \int \mathrm{d}x\, \mathrm{d}^2\boldsymbol{k}_\perp\, \boldsymbol{k}_\perp\, f_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) \equiv \sum_a \langle \boldsymbol{k}_\perp^a \rangle = 0$$

$$\langle k_\perp^u \rangle + \langle k_\perp^d \rangle = -17^{+37}_{-55}\,(\mathrm{MeV}/c),$$

$$\langle k_\perp^{\bar{u}} \rangle + \langle k_\perp^{\bar{d}} \rangle + \langle k_\perp^{s} \rangle + \langle k_\perp^{\bar{s}} \rangle = -14^{+43}_{-66}\,(\mathrm{MeV}/c)$$

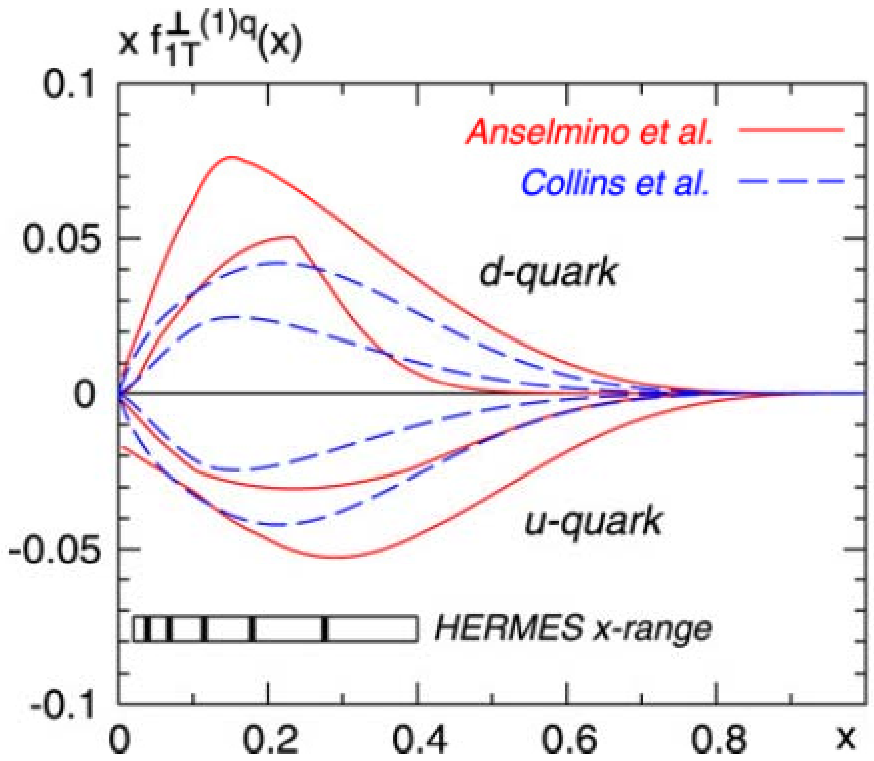
$$\langle k_\perp^u \rangle = 96^{+60}_{-28}\,(\mathrm{MeV}/c), \quad \langle k_\perp^d \rangle = -113^{+45}_{-51}\,(\mathrm{MeV}/c)$$

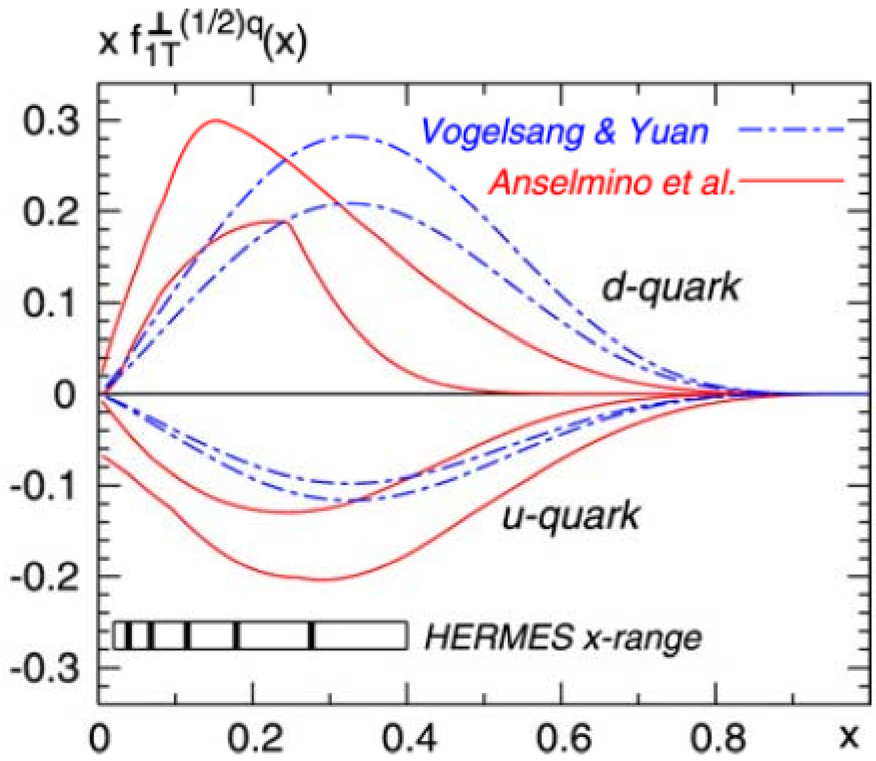
$$\langle k_\perp^{\bar{u}} \rangle = 2^{+24}_{-11}\,(\mathrm{MeV}/c), \quad \langle k_\perp^{\bar{d}} \rangle = -28^{+20}_{-60}\,(\mathrm{MeV}/c),$$

$$\langle k_\perp^{s} \rangle = -4^{+11}_{-15}\,(\mathrm{MeV}/c), \quad \langle k_\perp^{\bar{s}} \rangle = 17^{+30}_{-8}\,(\mathrm{MeV}/c),$$

$$-10 \leq \langle k_\perp^g \rangle \leq 48\,(\mathrm{MeV}/c)$$

Comparison of first and 1/2 transverse moments of the Sivers function as extracted by different theoretical groups [2005-2006]





Collins fragmentation function:
e+e- annihilation in two nearly back-to-back hadrons

Belle Collaboration
KEK-B

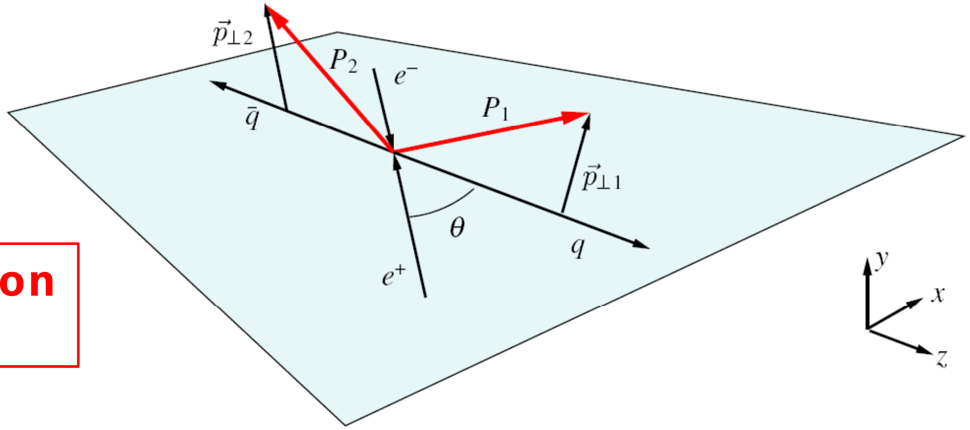


FIG. 2 (color online). Three-dimensional kinematics of the $e^+e^- \to h_1 h_2 X$ process, in the $q\bar{q}$ c.m. frame. In this configuration the reconstructed thrust axis identifies the $\hat{z}$ direction; the lepton-quark scattering plane defines the $\hat{x}\hat{z}$ plane.

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d^2\boldsymbol{p}_{\perp 1}\, d^2\boldsymbol{p}_{\perp 2}\, d\cos\theta} = \frac{3\pi\alpha^2}{2s} \sum_q e_q^2 \Big\{ (1+\cos^2\theta)\, D_{h_1/q}(z_1, p_{\perp 1})\, D_{h_2/\bar{q}}(z_2, p_{\perp 2}) + \frac{1}{4}\sin^2\theta\, \Delta^N D_{h_1/q^\uparrow}(z_1, p_{\perp 1})\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2, p_{\perp 2})\, \cos(\varphi_1 + \varphi_2) \Big\}$$

$$A(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle \mathrm{d}\sigma \rangle} \frac{\mathrm{d}\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, \mathrm{d}(\varphi_1 + \varphi_2)}$$

$$= 1 + \frac{1}{4} \frac{\sin^2\theta}{1+\cos^2\theta} \cos(\varphi_1 + \varphi_2) \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

$$\Delta^N D_{h/q^\uparrow}(z) = \int d^2\boldsymbol{p}_\perp \Delta^N D_{h/q^\uparrow}(z, p_\perp)$$
$$= \int d^2\boldsymbol{p}_\perp \frac{2p_\perp}{z m_h} H_1^{\perp q}(z, p_\perp) = 4\, H_1^{\perp(1/2)q}(z)\,.$$

$$\int d^2\boldsymbol{p}_\perp D_{h/q}(z, p_\perp) = D_{h/q}(z)$$

In order to eliminate false asymmetries the Belle Collab. consider the ratio R of the asymmetries for unlike-sign (U) pairs and like-sign (L) pairs [also all-charged (C) pion pairs have been considered]

$$R \equiv \frac{A_U}{A_L} = \frac{1 + \frac{1}{4} \cos(\varphi_1 + \varphi_2) \frac{\langle \sin^2\theta \rangle}{\langle 1+\cos^2\theta \rangle} P_U}{1 + \frac{1}{4} \cos(\varphi_1 + \varphi_2) \frac{\langle \sin^2\theta \rangle}{\langle 1+\cos^2\theta \rangle} P_L}$$

$$\simeq 1 + \frac{1}{4} \cos(\varphi_1 + \varphi_2) \frac{\langle \sin^2\theta \rangle}{\langle 1 + \cos^2\theta \rangle} (P_U - P_L)$$

$$\equiv 1 + \cos(\varphi_1 + \varphi_2)\, A_{12}(z_1, z_2)$$

$$P_U = \frac{\sum_q e_q^2 \, [\Delta^N D_{\pi^+/q^\uparrow}(z_1)\, \Delta^N D_{\pi^-/\bar{q}^\uparrow}(z_2) + \Delta^N D_{\pi^-/q^\uparrow}(z_1)\, \Delta^N D_{\pi^+/\bar{q}^\uparrow}(z_2)]}{\sum_q e_q^2 \, [D_{\pi^+/q}(z_1)\, D_{\pi^-/\bar{q}}(z_2) + D_{\pi^-/q}(z_1)\, D_{\pi^+/\bar{q}}(z_2)]}$$

$$P_L = \frac{\sum_q e_q^2 \, [\Delta^N D_{\pi^+/q^\uparrow}(z_1)\, \Delta^N D_{\pi^+/\bar{q}^\uparrow}(z_2) + \Delta^N D_{\pi^-/q^\uparrow}(z_1)\, \Delta^N D_{\pi^-/\bar{q}^\uparrow}(z_2)]}{\sum_q e_q^2 \, [D_{\pi^+/q}(z_1)\, D_{\pi^+/\bar{q}}(z_2) + D_{\pi^-/q}(z_1)\, D_{\pi^-/\bar{q}}(z_2)]}$$

$$A_{12}(z_1, z_2) = \frac{1}{4} \frac{\langle \sin^2\theta \rangle}{\langle 1 + \cos^2\theta \rangle} (P_U - P_L)\,.$$

$$D_{\pi^+/u,\bar{d}} = D_{\pi^-/d,\bar{u}} \equiv D_{\rm fav}\,,$$

$$D_{\pi^+/d,\bar{u}} = D_{\pi^-/u,\bar{d}} = D_{\pi^\pm/s,\bar{s}} \equiv D_{\rm unf}$$

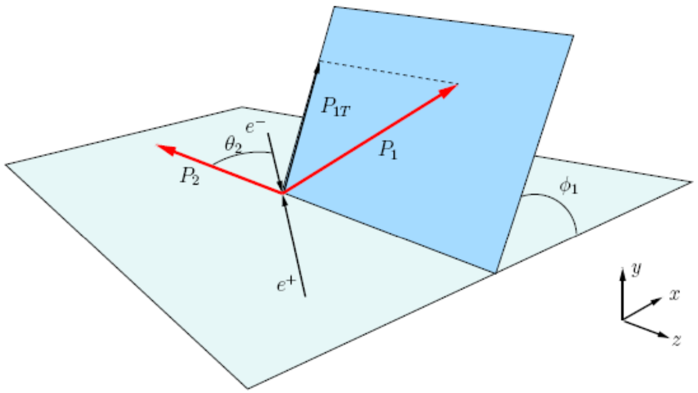


FIG. 3: Three dimensional kinematics of the $e^+e^- \to h_1 h_2 X$ process. In this configuration the $\hat{z}$ direction is identified by the momentum of the final hadron h_2, while h_1 is emitted at an azimuthal angle ϕ_1 with respect to the lepton-h_2 plane, defined as the $\hat{xz}$ plane.

$$A_0(z_1, z_2) = \frac{1}{\pi} \frac{z_1 z_2}{z_1^2 + z_2^2} \frac{\langle \sin^2\theta_2 \rangle}{\langle 1 + \cos^2\theta_2 \rangle} (P_U - P_L)$$

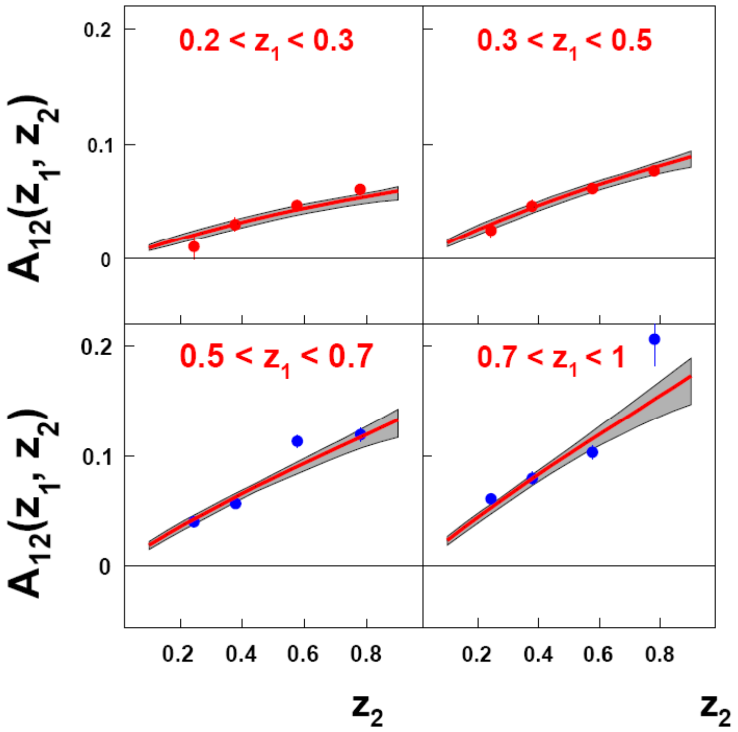


Figure 3. Fit of the Belle [23] data on the A_{12} asymmetry (the $\cos(\varphi_1 + \varphi_2)$ method).

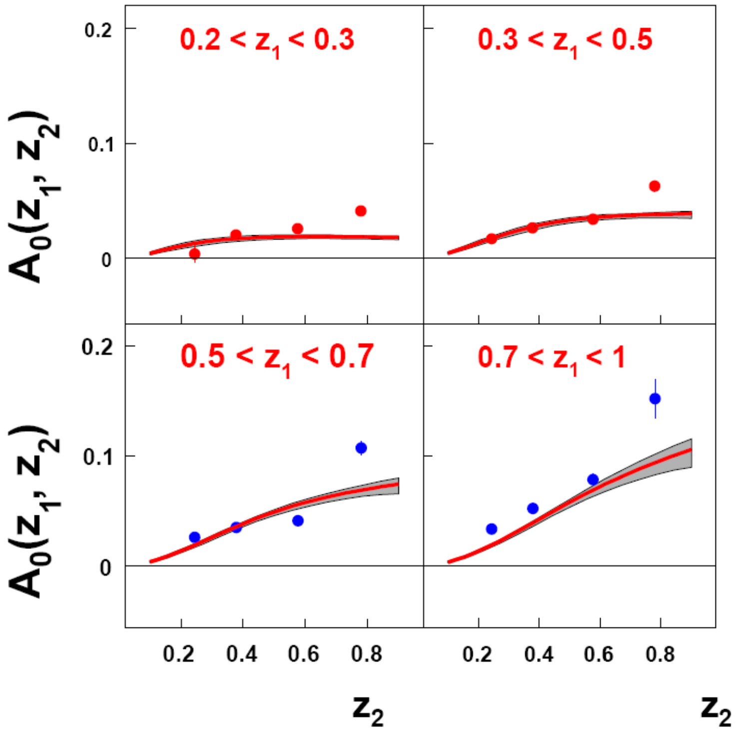
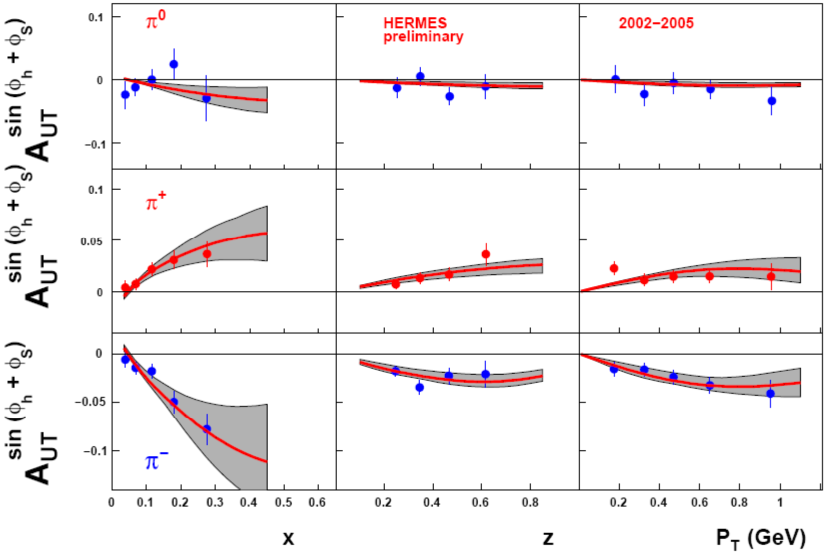
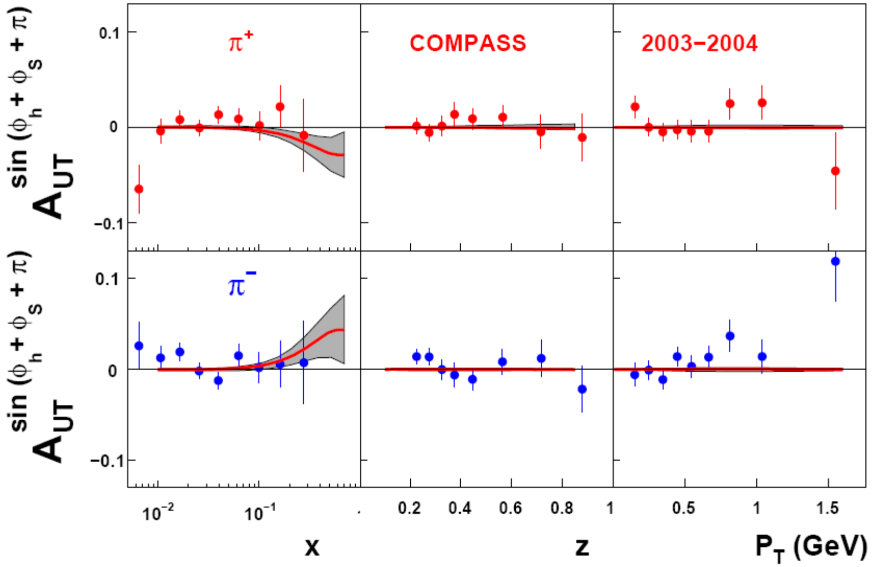


Figure 4. Comparison of our predictions with Belle [23] data for the A_0 Belle asymmetry (the $\cos(2\,\varphi_0)$ method).

Anselmino Boglione D'Alesio Kotzinian FM Prokudin Türk PRD 75 (2007)

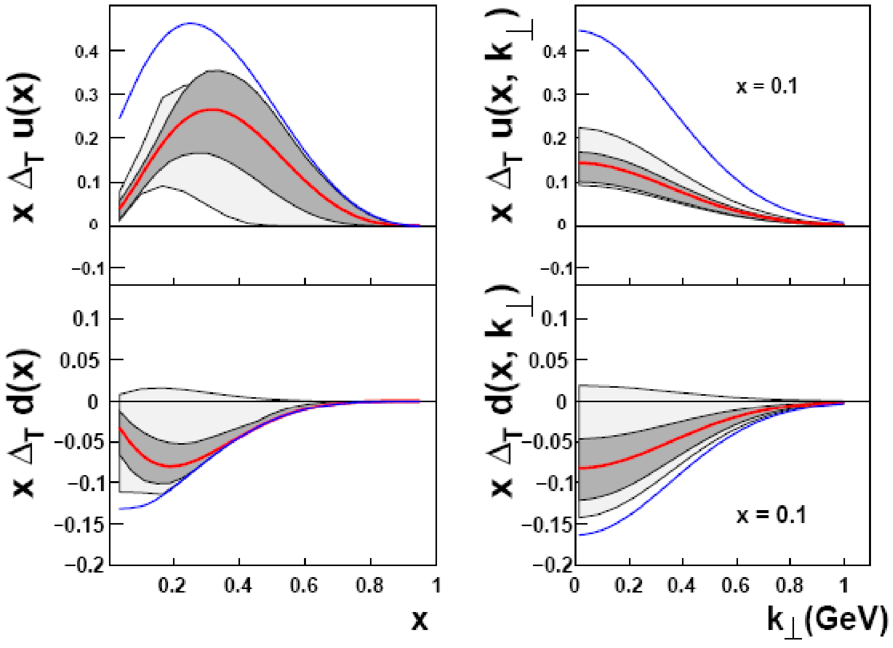


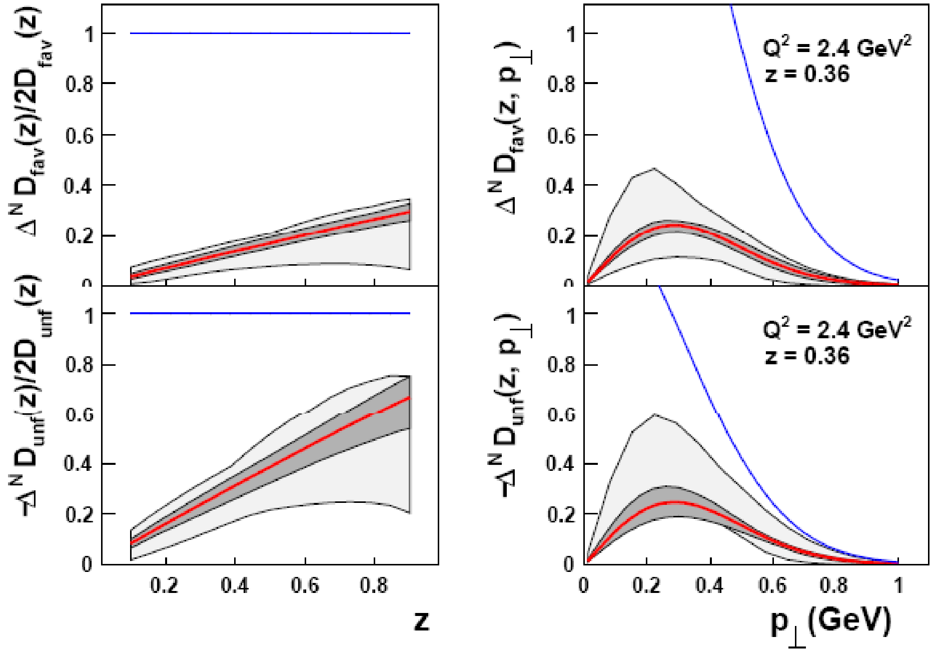
Deuteron target



u and d quark
transversity distribution

favored and unfavored
Collins functions





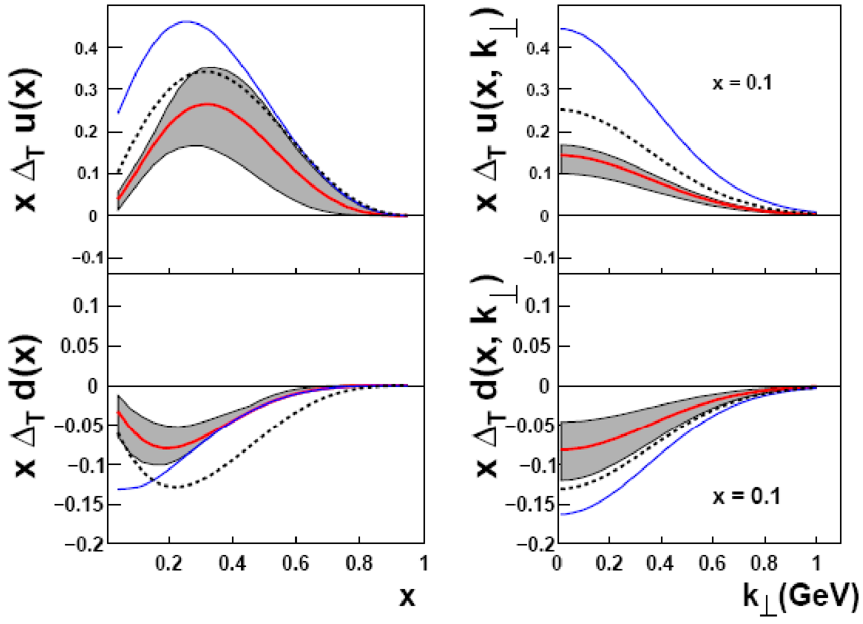


Figure 7. Comparison of the extracted transversity (solid line) with the helicity distribution (dashed line) at $Q^2 = 2.4\ \mathrm{GeV}^2$. The Soffer bound [46] (blue solid line) is also shown.

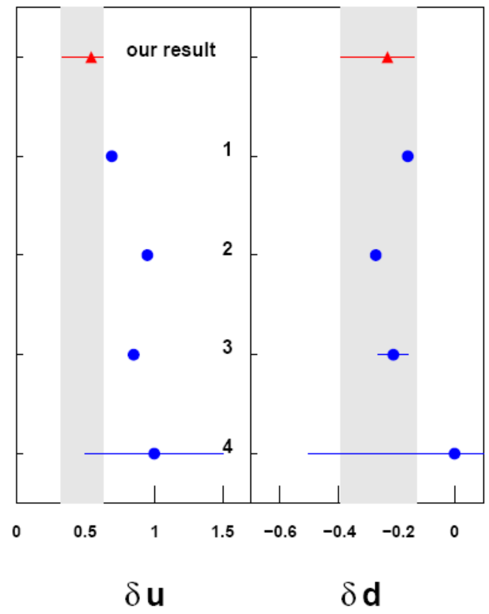


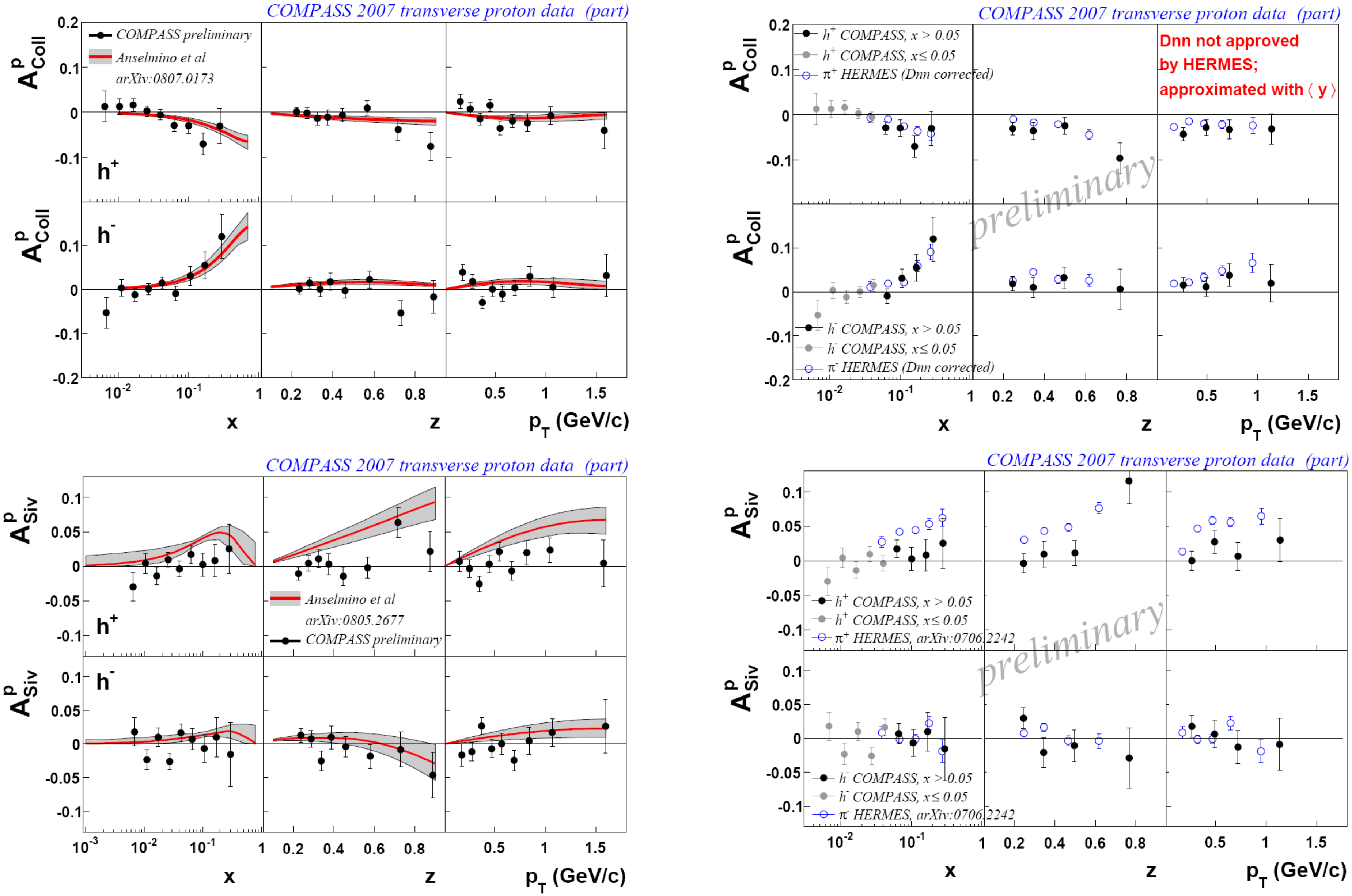
Figure 8. Tensor charge from different models compared to our result. 1: Quark-diquark model of Ref. [47], 2: Chiral quark soliton model of Ref. [48], 3: Lattice QCD [49], 4: QCD sum rules [50].

$$\delta q = \int_0^1 dx\,(\Delta_T q - \Delta_T \bar{q})$$

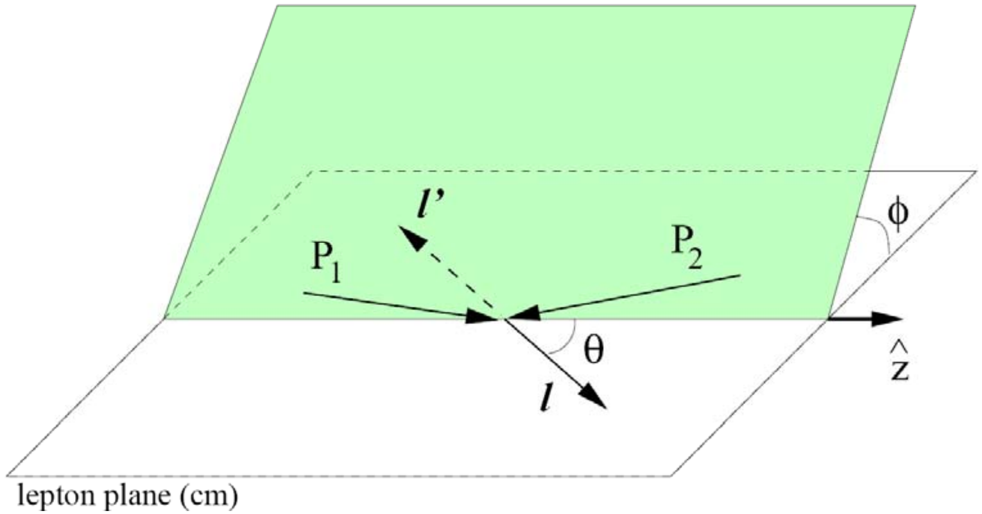
$$\delta u = 0.54^{+0.09}_{-0.22} \qquad \delta d = -0.23^{+0.09}_{-0.16}$$

$$Q^2 = 0.8\ \mathrm{GeV}^2$$

Compass – proton target - preliminary



Azimuthal asymmetries in unpolarized Drell-Yan processes



Kinematics of the Drell–Yan process in the lepton center of mass frame.

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos 2\phi\right)$$

In the naive parton model with massless quarks $\lambda = 1, \mu = \nu = 0$

Lam-Tung Relation [analogous to the Callan-Gross relation in DIS, valid in any cm frame]

$$\lambda = 1 - 2\nu$$

Exact at LO in collinear pQCD; small numerical violations at NLO

The complete structure for polarized case is much more complex…

[Arnold Metz Schlegel PRD 79 (2009)]

$$
\begin{aligned}
\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{\rm em}^2}{Fq^2}\{&((1+\cos^2\theta)F_{UU}^1 + (1-\cos^2\theta)F_{UU}^2 + \sin2\theta\cos\phi F_{UU}^{\cos\phi} + \sin^2\theta\cos2\phi F_{UU}^{\cos2\phi}) \\
&+ S_{aL}(\sin2\theta\sin\phi F_{LU}^{\sin\phi} + \sin^2\theta\sin2\phi F_{LU}^{\sin2\phi}) + S_{bL}(\sin2\theta\sin\phi F_{UL}^{\sin\phi} + \sin^2\theta\sin2\phi F_{UL}^{\sin2\phi}) \\
&+ |\vec{S}_{aT}|[\sin\phi_a((1+\cos^2\theta)F_{TU}^1 + (1-\cos^2\theta)F_{TU}^2 + \sin2\theta\cos\phi F_{TU}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TU}^{\cos2\phi}) \\
&+ \cos\phi_a(\sin2\theta\sin\phi F_{TU}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TU}^{\sin2\phi})] + |\vec{S}_{bT}|[\sin\phi_b((1+\cos^2\theta)F_{UT}^1 + (1-\cos^2\theta)F_{UT}^2 \\
&+ \sin2\theta\cos\phi F_{UT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{UT}^{\cos2\phi}) + \cos\phi_b(\sin2\theta\sin\phi F_{UT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{UT}^{\sin2\phi})] \\
&+ S_{aL}S_{bL}((1+\cos^2\theta)F_{LL}^1 + (1-\cos^2\theta)F_{LL}^2 + \sin2\theta\cos\phi F_{LL}^{\cos\phi} + \sin^2\theta\cos2\phi F_{LL}^{\cos2\phi}) \\
&+ S_{aL}|\vec{S}_{bT}|[\cos\phi_b((1+\cos^2\theta)F_{LT}^1 + (1-\cos^2\theta)F_{LT}^2 + \sin2\theta\cos\phi F_{LT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{LT}^{\cos2\phi}) \\
&+ \sin\phi_b(\sin2\theta\sin\phi F_{LT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{LT}^{\sin2\phi})] + |\vec{S}_{aT}|S_{bL}[\cos\phi_a((1+\cos^2\theta)F_{TL}^1 + (1-\cos^2\theta)F_{TL}^2 \\
&+ \sin2\theta\cos\phi F_{TL}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TL}^{\cos2\phi}) + \sin\phi_a(\sin2\theta\sin\phi F_{TL}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TL}^{\sin2\phi})] \\
&+ |\vec{S}_{aT}||\vec{S}_{bT}|[\cos(\phi_a+\phi_b)((1+\cos^2\theta)F_{TT}^1 + (1-\cos^2\theta)F_{TT}^2 + \sin2\theta\cos\phi F_{TT}^{\cos\phi} + \sin^2\theta\cos2\phi F_{TT}^{\cos2\phi}) \\
&+ \cos(\phi_a-\phi_b)((1+\cos^2\theta)\bar{F}_{TT}^1 + (1-\cos^2\theta)\bar{F}_{TT}^2 + \sin2\theta\cos\phi \bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos2\phi \bar{F}_{TT}^{\cos2\phi}) \\
&+ \sin(\phi_a+\phi_b)(\sin2\theta\sin\phi F_{TT}^{\sin\phi} + \sin^2\theta\sin2\phi F_{TT}^{\sin2\phi}) \\
&+ \sin(\phi_a-\phi_b)(\sin2\theta\sin\phi \bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin2\phi \bar{F}_{TT}^{\sin2\phi})]\}.
\end{aligned}
$$

Lam-Tung relation

$$\lambda = \frac{F_{UU}^1 - F_{UU}^2}{F_{UU}^1 + F_{UU}^2} \qquad \mu = \frac{F_{UU}^{\cos\phi}}{F_{UU}^1 + F_{UU}^2} \qquad \nu = \frac{2F_{UU}^{\cos2\phi}}{F_{UU}^1 + F_{UU}^2} \qquad F_{UU}^2 = 2F_{UU}^{\cos2\phi}$$

$$F_{UU}^{1} = \mathcal{C}[f_1 \bar{f}_1],$$

$$F_{UU}^{\cos 2\phi} = \mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b} h_1^\perp \bar{h}_1^\perp\Big],$$

$$F_{LU}^{\sin 2\phi} = \mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b} h_{1L}^\perp \bar{h}_1^\perp\Big],$$

$$F_{UL}^{\sin 2\phi} = -\mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b} h_1^\perp \bar{h}_{1L}^\perp\Big],$$

$$F_{TU}^{1} = -\mathcal{C}\Big[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a} f_{1T}^\perp \bar{f}_1\Big],$$

$$F_{TU}^{\sin(2\phi-\phi_a)} = \mathcal{C}\Big[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b} h_1 \bar{h}_1^\perp\Big],$$

$$F_{TU}^{\sin(2\phi+\phi_a)} = \mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{aT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}] - \vec{k}_{aT}^2(\vec{h}\cdot\vec{k}_{bT})}{2M_a^2 M_b} h_{1T}^\perp \bar{h}_1^\perp\Big],$$

$$F_{UT}^{1} = \mathcal{C}\Big[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b} f_1 \bar{f}_{1T}^\perp\Big],$$

$$F_{UT}^{\sin(2\phi-\phi_b)} = -\mathcal{C}\Big[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a} h_1^\perp \bar{h}_1\Big],$$

$$F_{UT}^{\sin(2\phi+\phi_b)} = -\mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{bT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}] - \vec{k}_{bT}^2(\vec{h}\cdot\vec{k}_{aT})}{2M_a M_b^2} h_1^\perp \bar{h}_{1T}^\perp\Big],$$

$$F_{LL}^{1} = -\mathcal{C}[g_{1L}\bar{g}_{1L}],$$

$$F_{LL}^{\cos 2\phi} = \mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b} h_{1L}^\perp \bar{h}_{1L}^\perp\Big],$$

$$F_{LT}^{1} = -\mathcal{C}\Big[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b} g_{1L}\bar{g}_{1T}\Big],$$

$$F_{LT}^{\cos(2\phi-\phi_b)} = \mathcal{C}\Big[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a} h_{1L}^\perp \bar{h}_1\Big],$$

$$F_{LT}^{\cos(2\phi+\phi_b)} = \mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{bT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}] - \vec{k}_{bT}^2(\vec{h}\cdot\vec{k}_{aT})}{2M_a M_b^2} h_{1L}^\perp \bar{h}_{1T}^\perp\Big],$$

$$F_{TL}^{1} = -\mathcal{C}\Big[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a} g_{1T}\bar{g}_{1L}\Big],$$

$$F_{TL}^{\cos(2\phi-\phi_a)} = \mathcal{C}\Big[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b} h_1 \bar{h}_{1L}^\perp\Big],$$

$$F_{TL}^{\cos(2\phi+\phi_a)} = \mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{aT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}] - \vec{k}_{aT}^2(\vec{h}\cdot\vec{k}_{bT})}{2M_a^2 M_b} h_{1T}^\perp \bar{h}_{1L}^\perp\Big],$$

$$F_{TT}^{1} = \mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{2M_a M_b}(f_{1T}^\perp \bar{f}_{1T}^\perp - g_{1T}\bar{g}_{1T})\Big],$$

$$\bar{F}_{TT}^{1} = -\mathcal{C}\Big[\frac{\vec{k}_{aT}\cdot\vec{k}_{bT}}{2M_a M_b}(f_{1T}^\perp \bar{f}_{1T}^\perp + g_{1T}\bar{g}_{1T})\Big],$$

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b)} = \mathcal{C}[h_1\bar{h}_1],$$

$$F_{TT}^{\cos(2\phi-\phi_a+\phi_b)} = \mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{bT})^2 - \vec{k}_{bT}^2}{2M_b^2} h_1 \bar{h}_{1T}^\perp\Big],$$

$$F_{TT}^{\cos(2\phi+\phi_a-\phi_b)} = \mathcal{C}\Big[\frac{2(\vec{h}\cdot\vec{k}_{aT})^2 - \vec{k}_{aT}^2}{2M_a^2} h_{1T}^\perp \bar{h}_1\Big],$$

$$F_{TT}^{\cos(2\phi+\phi_a+\phi_b)} = \mathcal{C}\Big[\Big(\frac{4(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT})[2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}]}{4M_a^2 M_b^2} + \frac{\vec{k}_{aT}^2\vec{k}_{bT}^2 - 2\vec{k}_{aT}^2(\vec{h}\cdot\vec{k}_{bT})^2 - 2\vec{k}_{bT}^2(\vec{h}\cdot\vec{k}_{aT})^2}{4M_a^2 M_b^2}\Big) h_{1T}^\perp \bar{h}_{1T}^\perp\Big].$$

Some of the more interesting azimuthal contributions

$$F_{UU}^{\cos 2\phi} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{M_a M_b} h_1^\perp \bar{h}_1^\perp\right]$$

Boer-Mulders effect

$$F_{TU}^{1} = -\mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{aT}}{M_a} f_{1T}^\perp \bar{f}_1\right]$$

A_N: Sivers effect
the only one surviving complete θ, ϕ integration

$$F_{TU}^{\sin(2\phi-\phi_a)} = \mathcal{C}\left[\frac{\vec{h}\cdot\vec{k}_{bT}}{M_b} h_1 \bar{h}_1^\perp\right]$$

A_N: BM⊗Transversity

$$F_{TT}^{\cos(2\phi-\phi_a-\phi_b)} = \mathcal{C}[h_1 \bar{h}_1]$$

A_{TT}: Transversity!
Survives in collinear case

$$F_{TT}^{1} = \mathcal{C}\left[\frac{2(\vec{h}\cdot\vec{k}_{aT})(\vec{h}\cdot\vec{k}_{bT}) - \vec{k}_{aT}\cdot\vec{k}_{bT}}{2M_a M_b}(f_{1T}^\perp \bar{f}_{1T}^\perp - g_{1T}\bar{g}_{1T})\right]$$

$$\bar{F}_{TT}^{1} = -\mathcal{C}\left[\frac{\vec{k}_{aT}\cdot\vec{k}_{bT}}{2M_a M_b}(f_{1T}^\perp \bar{f}_{1T}^\perp + g_{1T}\bar{g}_{1T})\right]$$

A_{TT}: Sivers and g_{1T} contributions

Unpol DY: Phenomenological results: pN

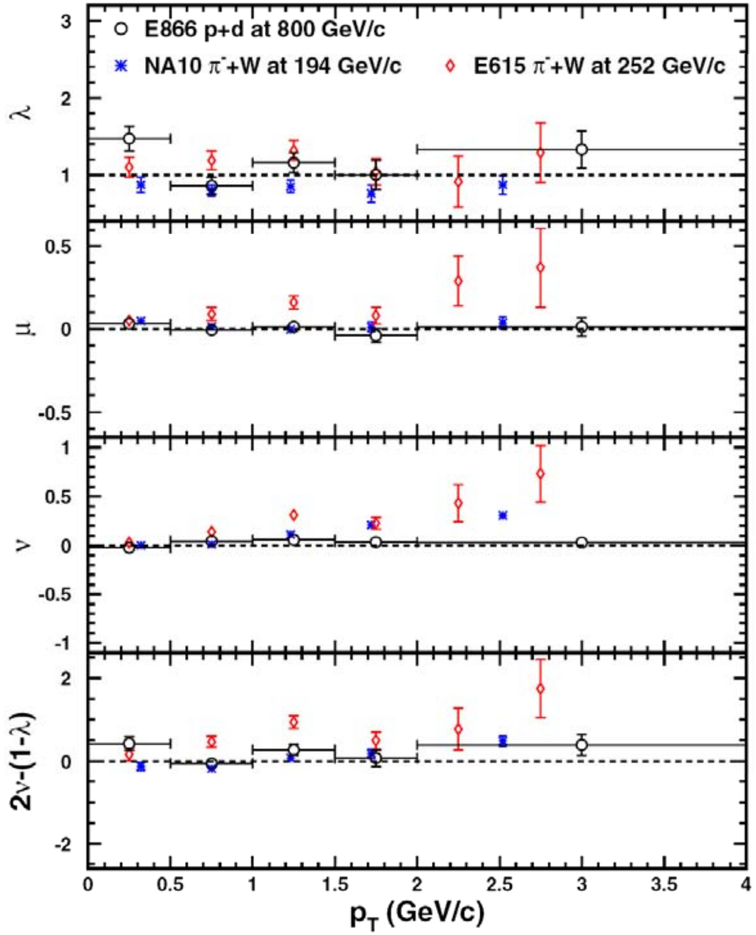


FIG. 1 (color online). Parameters λ, μ, ν, and $2\nu - (1 - \lambda)$ vs p_T in the Collins-Soper frame. Open circles are for E866 $p + d$ at 800 GeV/c, crosses are for NA10 $\pi^- + W$ at 194 GeV/c, and diamonds are E615 $\pi^- + W$ at 252 GeV/c. The error bars include the statistical uncertainties only.

E866/NuSea PRL 99 (2007)

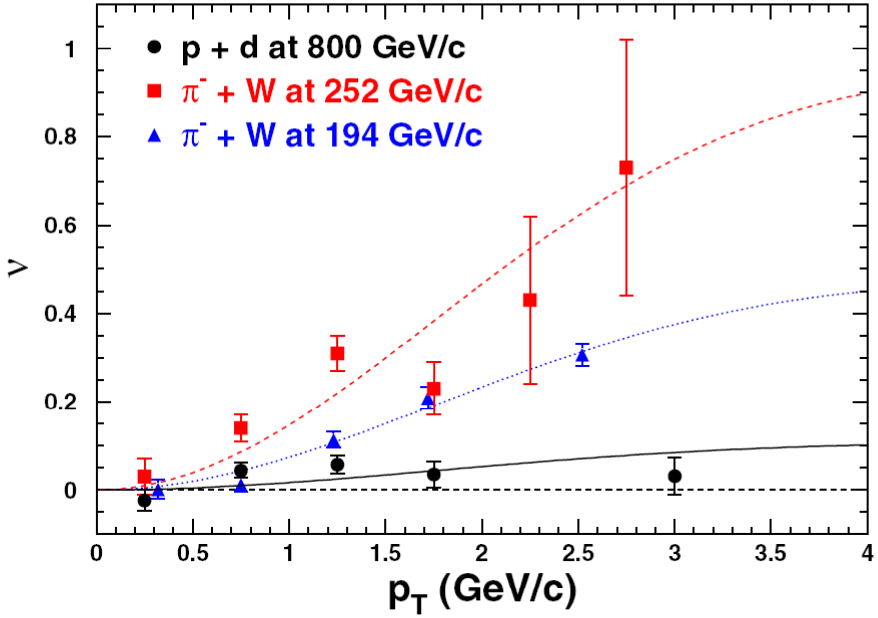


FIG. 2 (color online). Parameter ν vs p_T in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. (3) and $M_C = 2.4$ GeV/c^2 are also shown.

$$\nu = 16\kappa_1 \frac{p_T^2 M_C^2}{(p_T^2 + 4M_C^2)^2}$$

Simple model – Boer 98

Sizable BM functions for valence antiquarks in pion and valence quarks in the nucleon
Significantly smaller BM functions for antiquarks in the nucleon

Unpol. DY: Phenomenological results: pp

$$\nu = \frac{Q_\perp^2/Q^2}{1+\frac{3}{2}Q_\perp^2/Q^2}$$

E866/NuSea arXiv: 0811.4589 [nucl-ex]

	$p+p$ 800 GeV/c (E866)	$p+d$ 800 GeV/c (E866)	$\pi^- + W$ 194 GeV/c (NA10)
$\langle\lambda\rangle$	0.85 ± 0.10	1.07 ± 0.07	0.83 ± 0.04
$\langle\mu\rangle$	-0.026 ± 0.019	0.003 ± 0.013	0.008 ± 0.010
$\langle\nu\rangle$	0.040 ± 0.015	0.027 ± 0.010	0.091 ± 0.009
$\langle 2\nu - (1-\lambda)\rangle$	-0.07 ± 0.10	0.12 ± 0.07	0.01 ± 0.04

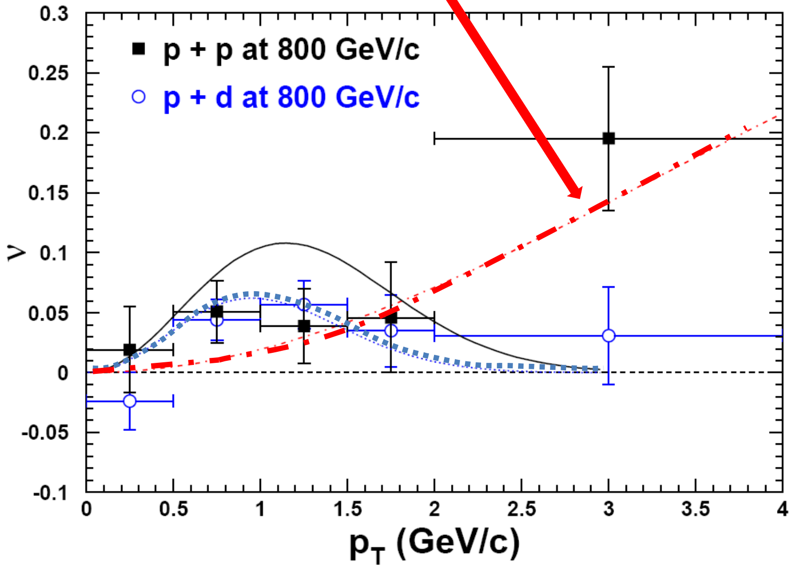


FIG. 2: (color online). Parameter ν vs. p_T in the Collins-Soper frame for the $p+p$ and $p+d$ Drell-Yan data. The solid and dotted curves are calculations [23] for $p+p$ and $p+d$, respectively, using parametrizations based on a fit to the $p+d$ data. The dot-dashed curve is the contribution from the QCD process (Eq. 2).

In summary, we report a measurement of the angular distributions of Drell-Yan dimuons for $p+p$ at 800 GeV/c. The pronounced $\cos 2\phi$ azimuthal angular dependence observed previously in pion-induced Drell-Yan is not observed in the $p+p$ reaction. The Lam-Tung relation remains valid for the $p+p$ Drell-Yan data. The overall magnitude of the $\cos 2\phi$ dependence for $p+p$ is consistent with, but slightly larger than that of $p+d$. The data suggest the presence of higher-order QCD correction at high p_T, and it is important to take this contribution into account before reliable extraction of the Boer-Mulders functions could be obtained.

Pol. DY: Phenomenological results: Sivers SSA

$$A_N = \frac{d\sigma^{A^\uparrow B\to \ell^+\ell^- X} - d\sigma^{A^\downarrow B\to \ell^+\ell^- X}}{d\sigma^{A^\uparrow B\to \ell^+\ell^- X} + d\sigma^{A^\downarrow B\to \ell^+\ell^- X}} \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

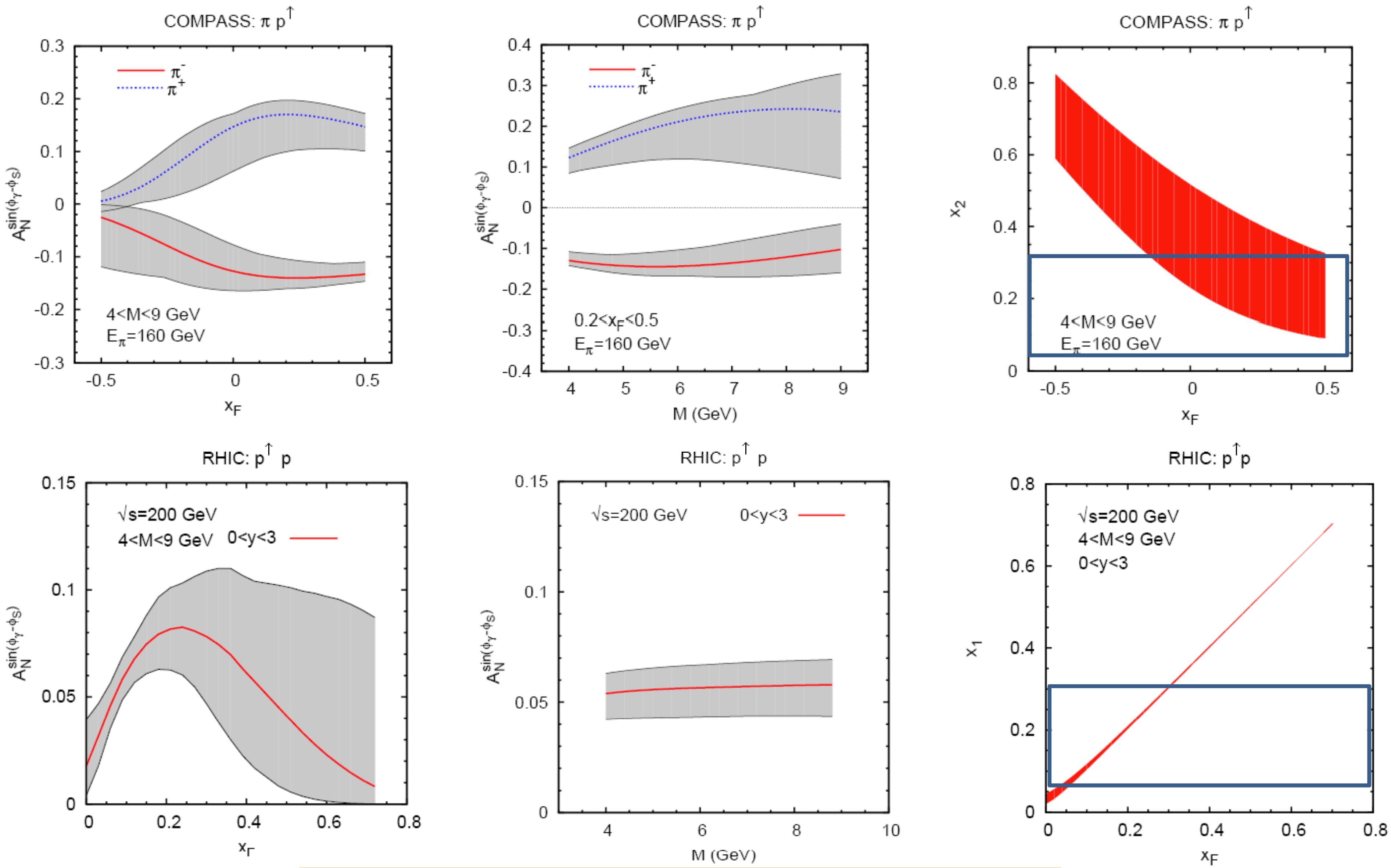
$$= \frac{\sum_q \int d^2\boldsymbol{k}_{\perp 1}\, d^2\boldsymbol{k}_{\perp 2}\, \delta^2(\boldsymbol{k}_{\perp 1} + \boldsymbol{k}_{\perp 2} - \boldsymbol{q}_T)\, \Delta^N f_{q/A^\uparrow}(x_1, \boldsymbol{k}_{\perp 1})\, f_{\bar{q}/B}(x_2, k_{\perp 2})\, \hat{\sigma}_0^{q\bar{q}}}{2\sum_q \int d^2\boldsymbol{k}_{\perp 1}\, d^2\boldsymbol{k}_{\perp 2}\, \delta^2(\boldsymbol{k}_{\perp 1} + \boldsymbol{k}_{\perp 2} - \boldsymbol{q}_T)\, f_{q/A}(x_1, k_{\perp 1})\, f_{\bar{q}/B}(x_2, k_{\perp 2})\, \hat{\sigma}_0^{q\bar{q}}}$$

$$\hat{\sigma}_0^{q\bar{q}} = e_q^2 \frac{4\pi\alpha^2}{9M^2} \qquad\qquad x_{1,2} = \frac{M}{\sqrt{s}}\, e^{\pm y} = \frac{\pm x_F + \sqrt{x_F^2 + 4\, M^2/s}}{2}$$

$$x_F = x_1 - x_2 \qquad\qquad |x_F| \le 1 - \frac{M^2}{s}$$

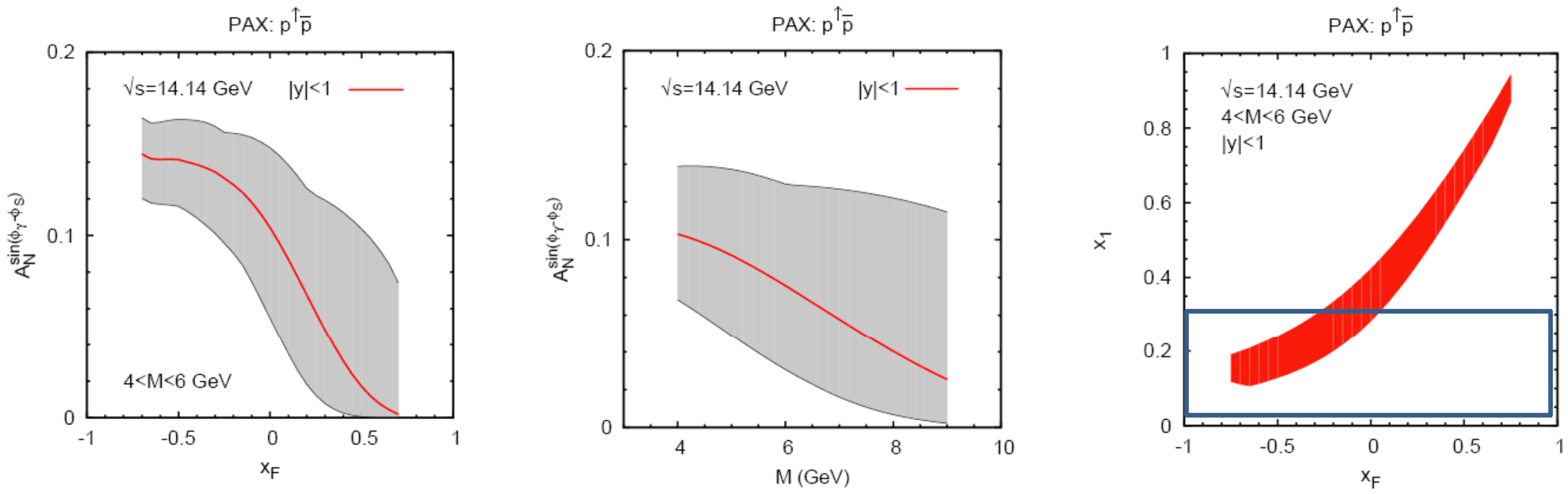
$$\frac{d^4\sigma}{dy\, dM^2\, d^2\boldsymbol{q}_T} = \frac{1}{s}\, \frac{d^4\sigma}{dx_1\, dx_2\, d^2\boldsymbol{q}_T} = (x_1 + x_2)\, \frac{d^4\sigma}{dx_F\, dM^2\, d^2\boldsymbol{q}_T} = \frac{1}{2}\, \frac{d^4\sigma}{d^4 q}$$

Pol. DY: Phenomenological results: Sivers SSA



Anselmino Boglione D'Alesio Melis FM Prokudin PRD 79 (2009)

Pol. DY: Phenomenological results: Sivers SSA



Anselmino Boglione D'Alesio Melis FM Prokudin PRD 79 (2009)

Unpol. cross sections and SSAs in hadronic collisions

$$\frac{E_C\, d\sigma^{(A,S_A)+(B,S_B)\to C+X}}{d^3\boldsymbol{p}_C} = \sum_{a,b,c,d,\{\lambda\}} \int \frac{dx_a\, dx_b\, dz}{16\pi^2 x_a x_b z^2 s}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, d^3\boldsymbol{k}_{\perp C}\, \delta(\boldsymbol{k}_{\perp C}\cdot\hat{\boldsymbol{p}}_c)\, J(\boldsymbol{k}_{\perp C})$$
$$\times\ \rho^{a/A,S_A}_{\lambda_a,\lambda'_a}\, \hat{f}_{a/A,S_A}(x_a,\boldsymbol{k}_{\perp a})\, \rho^{b/B,S_B}_{\lambda_b,\lambda'_b}\, \hat{f}_{b/B,S_B}(x_b,\boldsymbol{k}_{\perp b}) \qquad (1)$$
$$\times\ \hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}\, \hat{M}^*_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}\, \delta(\hat{s}+\hat{t}+\hat{u})\, \hat{D}^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,\boldsymbol{k}_{\perp C})\,,$$

There are different contributions involving all the allowed combinations of the TMD polarized distributions, fragmentation functions, and hard scattering cross sections

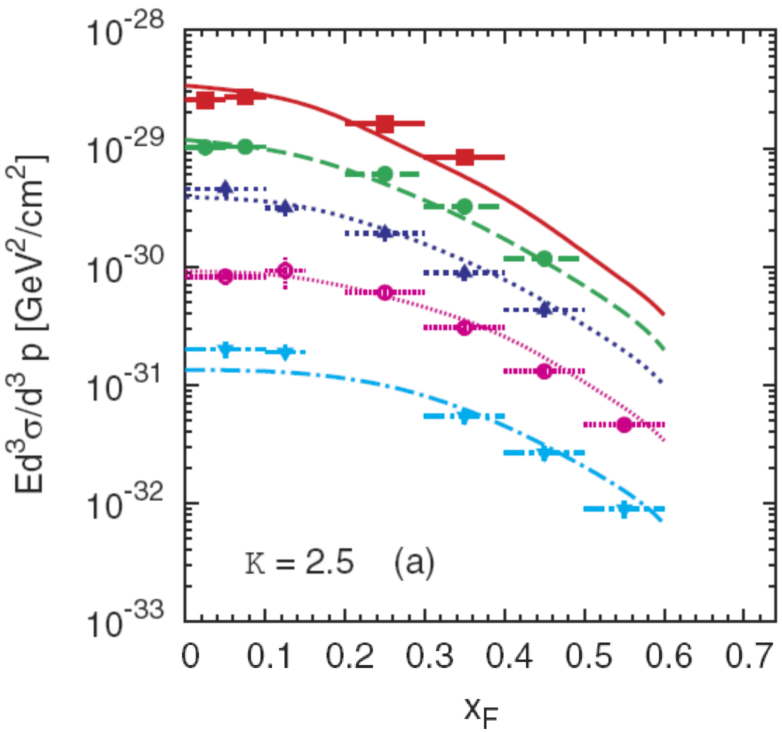
Unpolarized cross sections are dominated by the term already present in collinear configuration;

SSAs receive the potentially dominant contributions from the Sivers and Collins effects

Factorization for single particle production in hadronic collisions in the TMD approach has not been proven yet [see the twist-three approach]. Universality for TMD distributions might be invalidated. Hints from phenomenological tests are very important.

Use SIDIS and e+e- annihilation results; ASSUME UNIVERSALITY and look for SSAs in pp collisions

SSAs in hadronic collisions: fixed target – E704



$$\sqrt{s} \simeq 20 \text{ GeV}$$

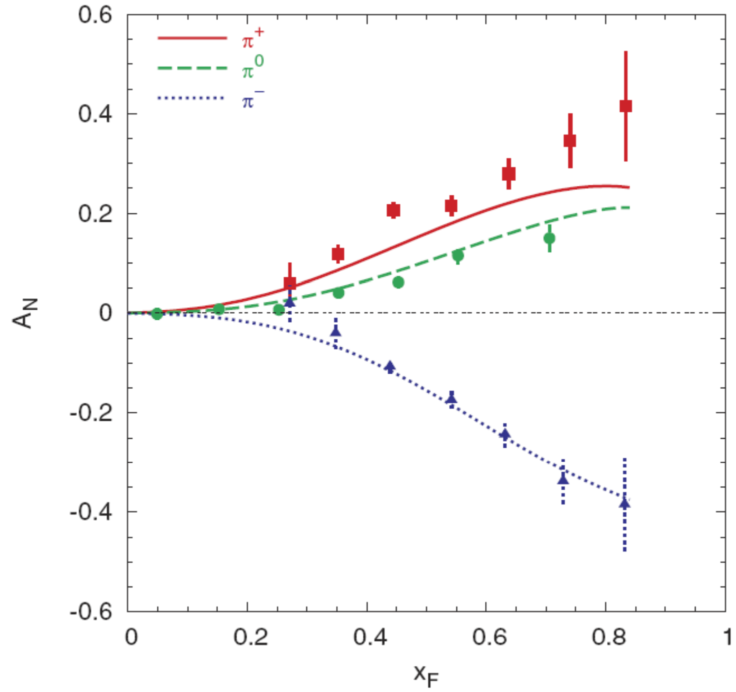
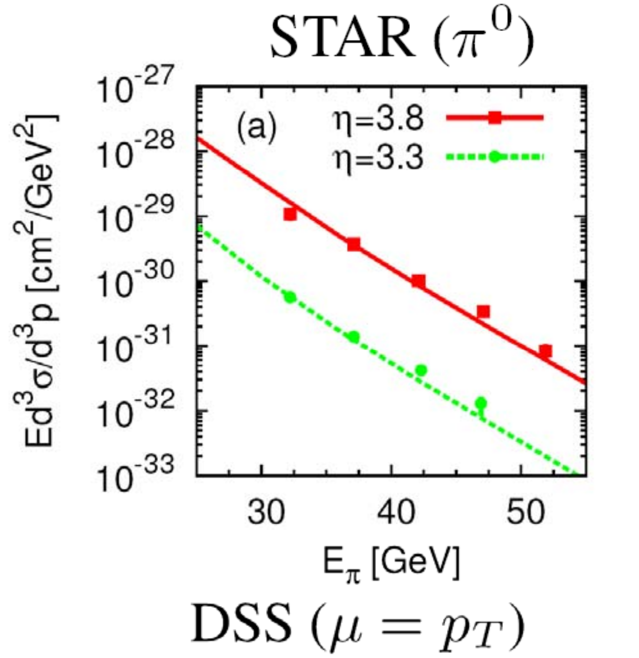
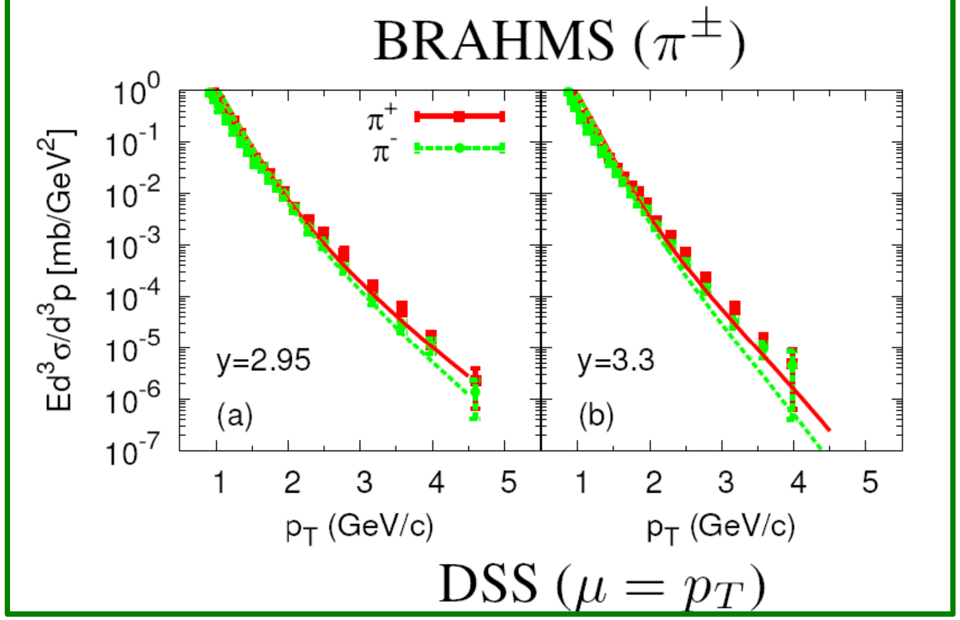


FIG. 16 (color online). A_N for inclusive pion production in pp collisions, at $\sqrt{s} = 19.4$ GeV and fixed $p_T = 1.5$ GeV$/c$, as a function of x_F. The parametrization MRST01 [25] for the unpolarized parton distributions is used; fragmentation function set is KKP-1 (see Section II C 2). For the Sivers function (see Eqs. (41) and (43)) parameters are given in Eq. (47), with $1/\beta = 0.8$ GeV$/c$ and $r = 0.7$. Data are from [45].

D'Alesio FM PRD 70 (2004)

Hadronic collisions at RHIC: unpol. cross sections

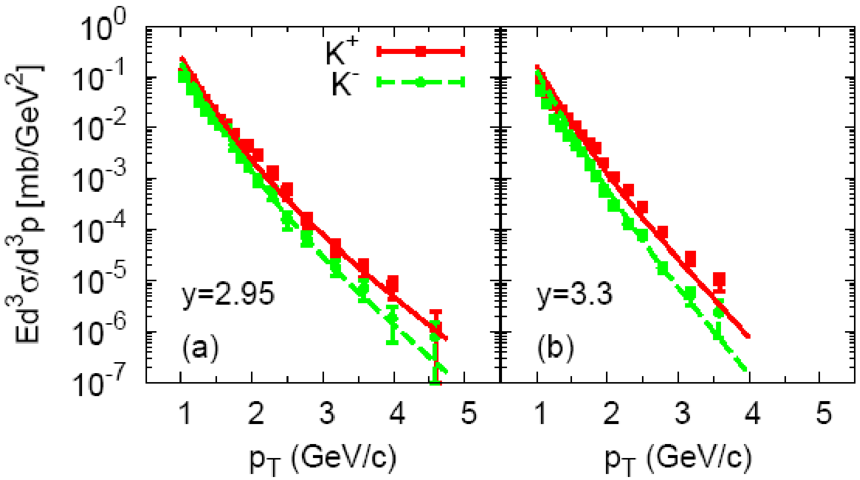




$\sqrt{s} = 200$ GeV

BRAHMS ($K^\pm$)

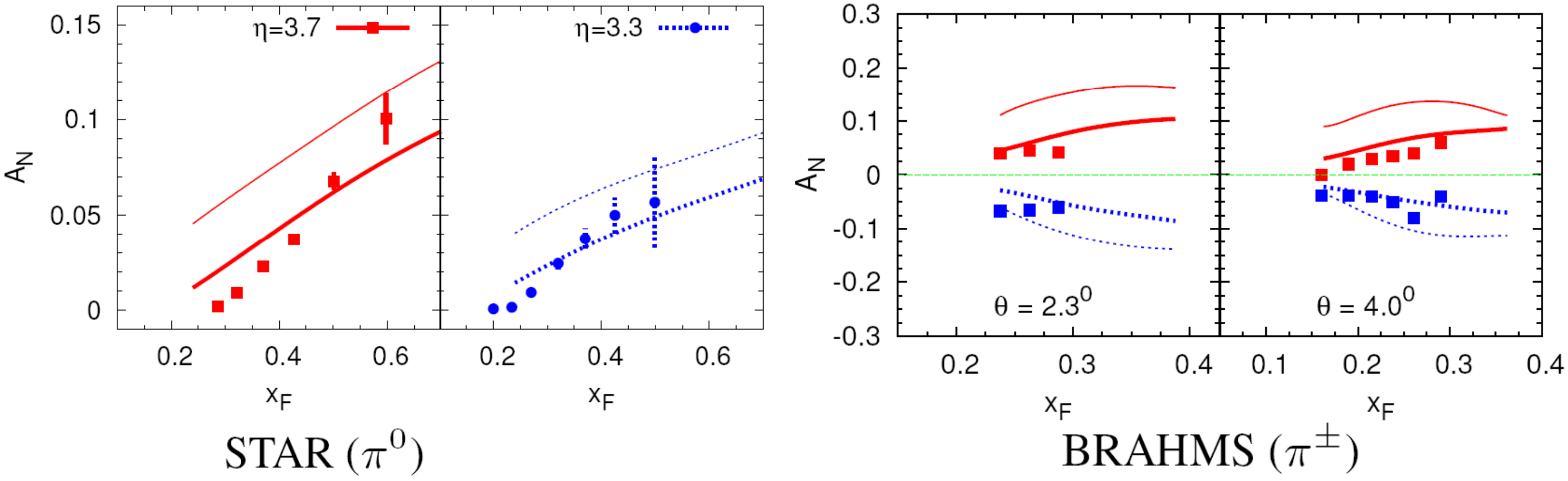
KRE ($\mu = \hat{p}_T/2$)



Boglione D'Alesio FM PRD 77 (2008)

Pion SSAs at RHIC:
maximized potenzial role of Sivers effect

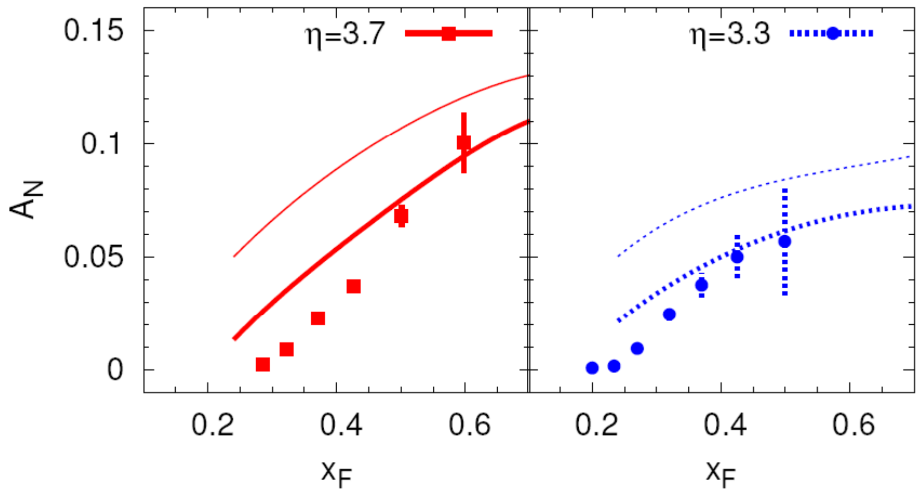
KRE (thin) vs. DSS (thick) lines



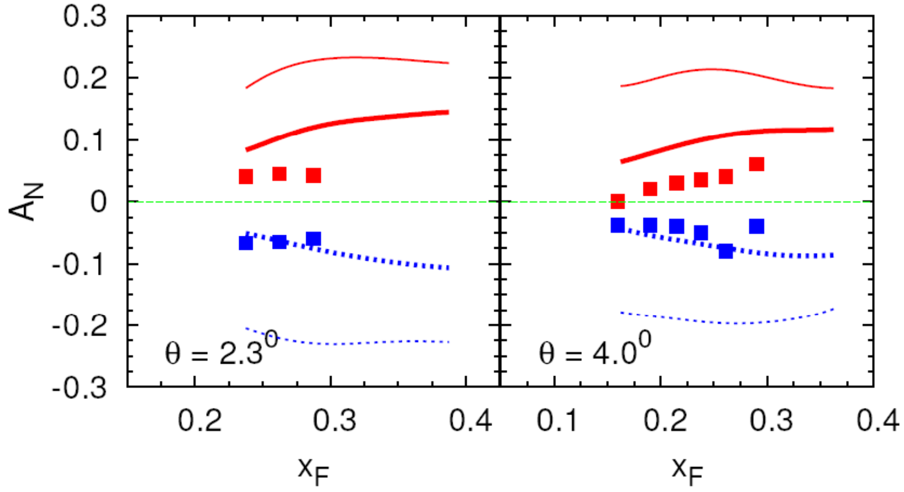
Anselmino Boglione D'Alesio Melis FM Prokudin - Preliminary

Pion SSAs at RHIC: maximized potenzial role of Collins (+ transversity) effect

KRE (thin) vs. DSS (thick) lines



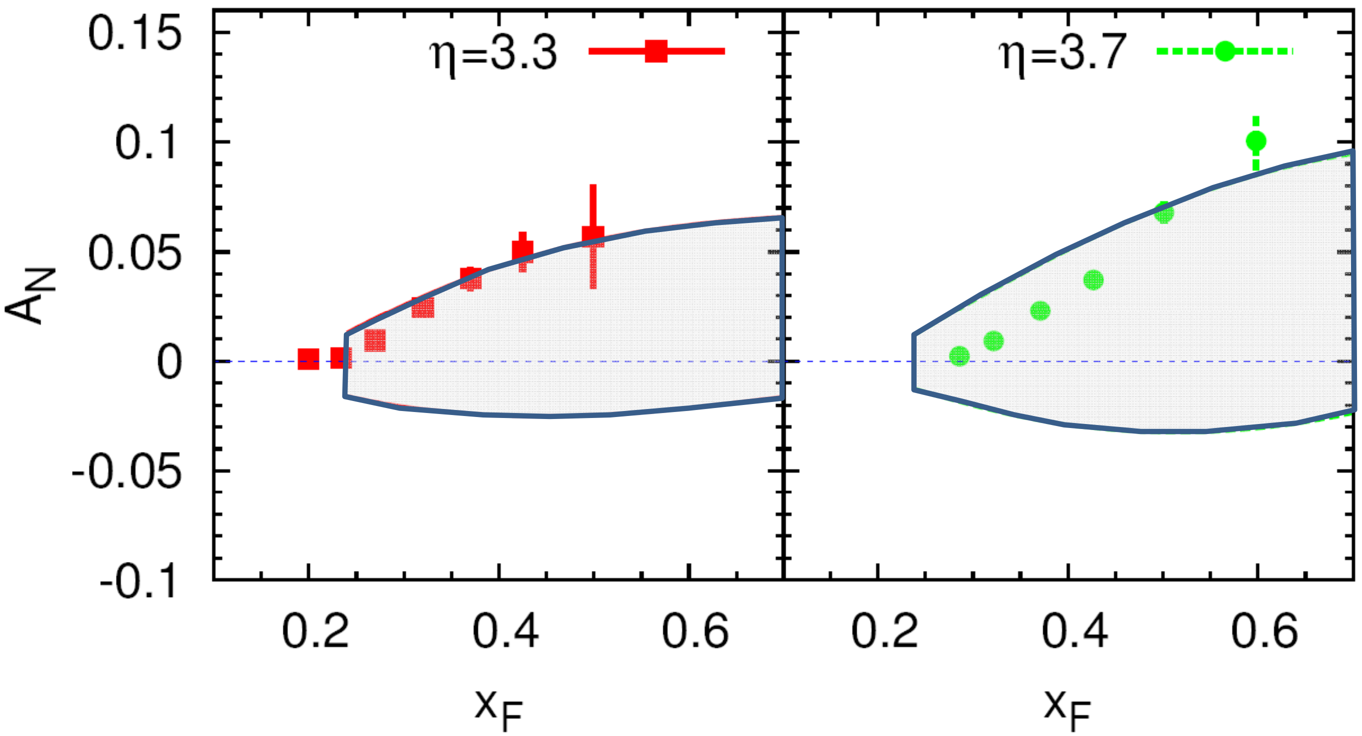
STAR (π^0)



BRAHMS ($\pi^{\pm}$)

Anselmino Boglione D'Alesio Melis FM Prokudin - Preliminary

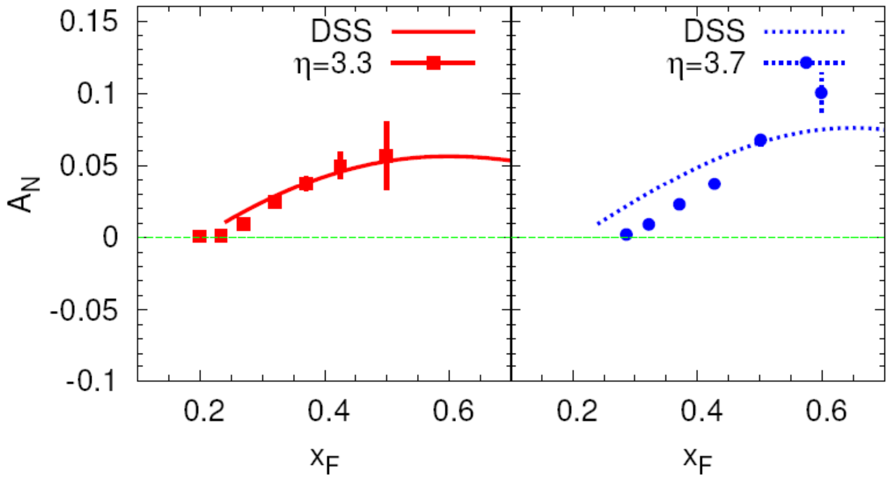
Pion SSAs at RHIC



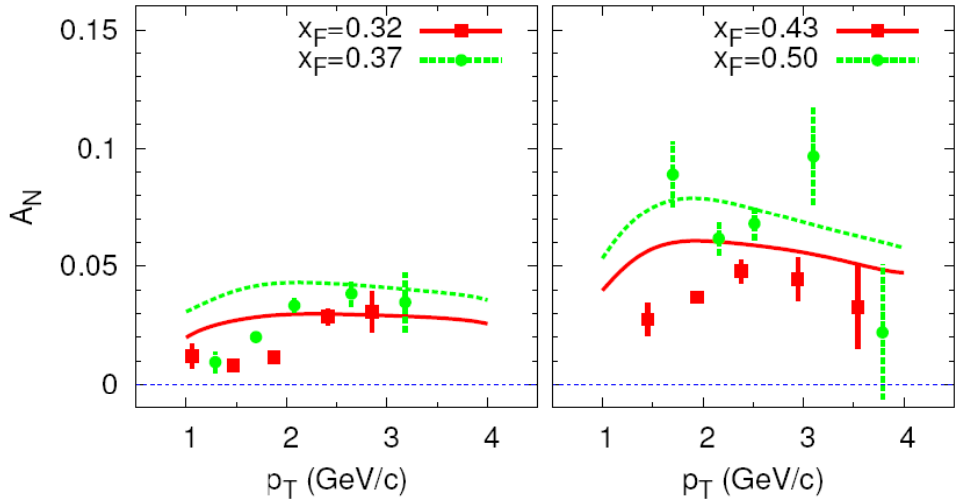
Fits to Sivers asymmetry in SIDIS constrain the Sivers distribution in a limited x range ($x < 0.3$). In particular, the parameter β_q governing its large-x behaviour [$\propto (1-x)^{\beta q}$] is almost uncostrained by these fits.

The shaded areas represent the covered region generated by a scan of the [β_u-β_d] space allowing for at most a 20% increase in the χ^2_{dof} of the SIDIS fit.

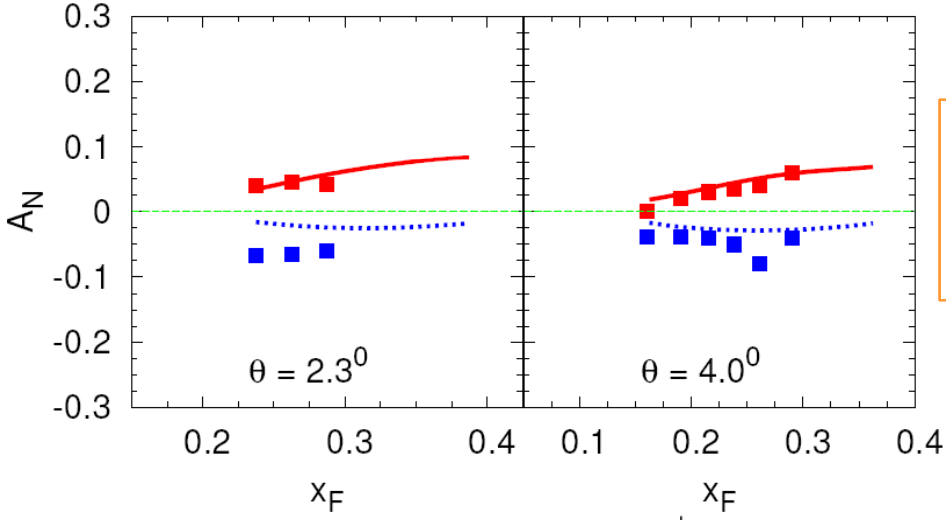
Sivers effect in pion SSAs at RHIC [fit of SIDIS data with $\chi^2_{dof} \simeq 1.2$]



STAR π^0



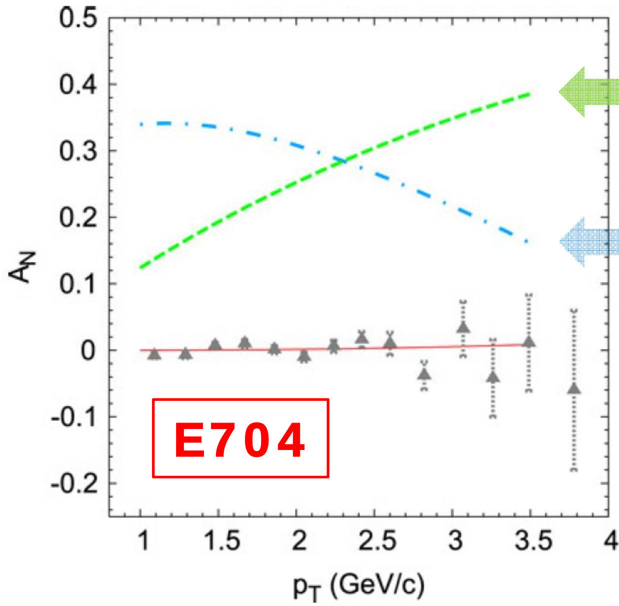
STAR π^0

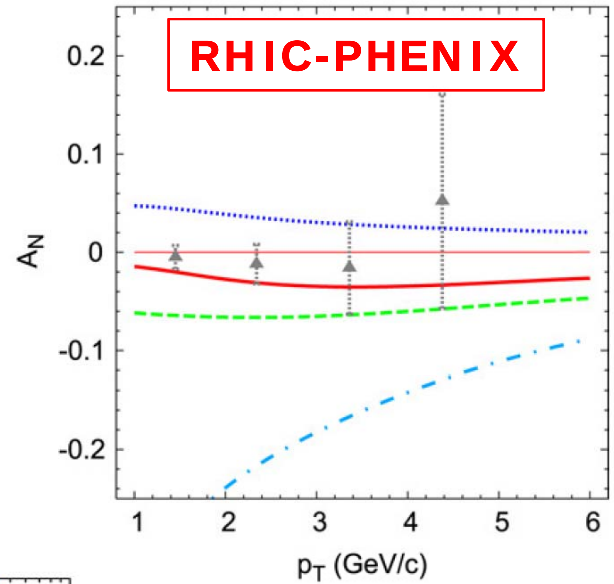


BRAHMS $\pi^{\pm}$

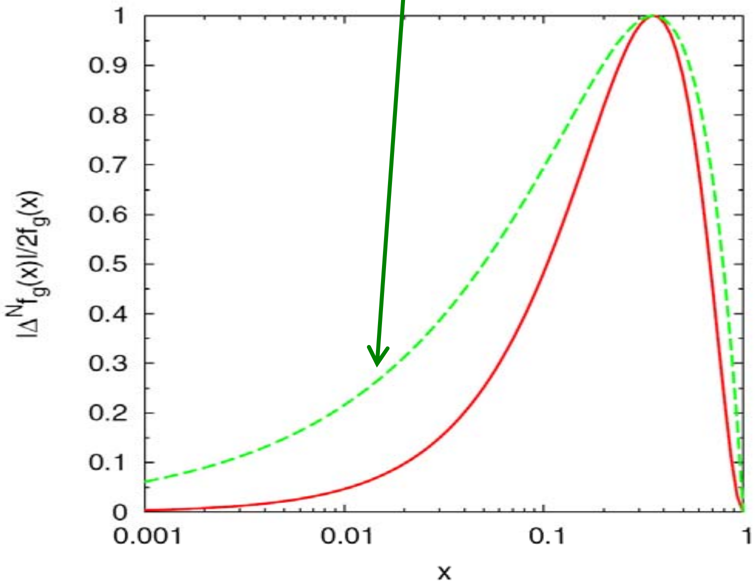
Anselmino Boglione D'Alesio Melis FM Prokudin Preliminary

Sivers effect in pion SSAs at RHIC: constraints on the gluon Sivers function





Bound on gluon Sivers distribution from PHENIX data



Similar conclusions have been reached by Brodsky and Gardner considering COMPASS data on the Sivers SSA with deuteron target

Anselmino D'Alesio Melis FM
PRD 74 (2006)

Twist-three approach and phenomenology

FNAL-E704

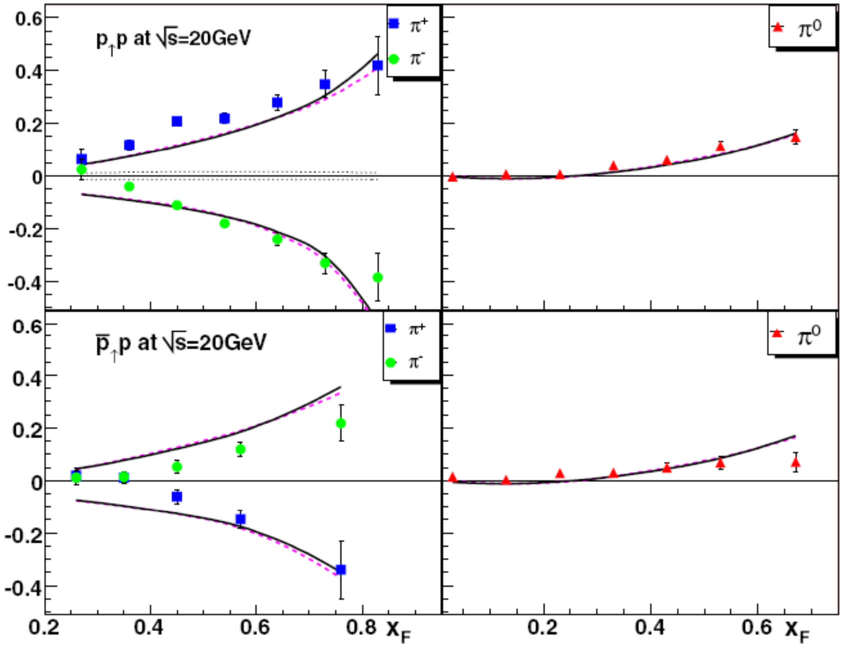
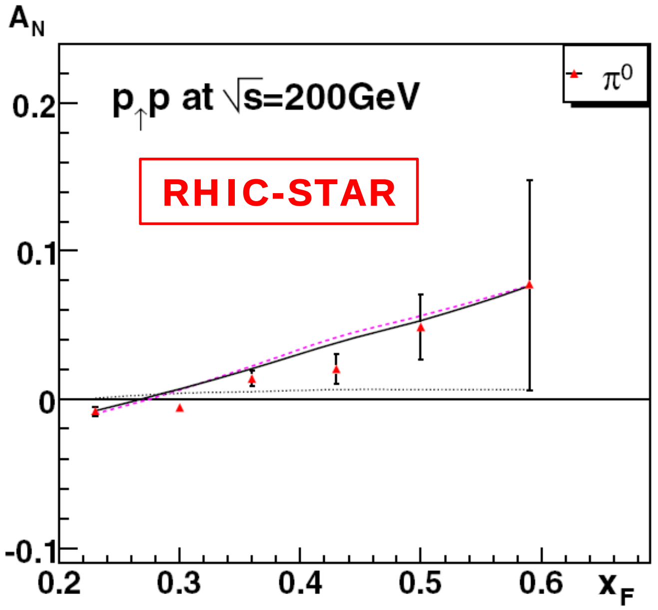
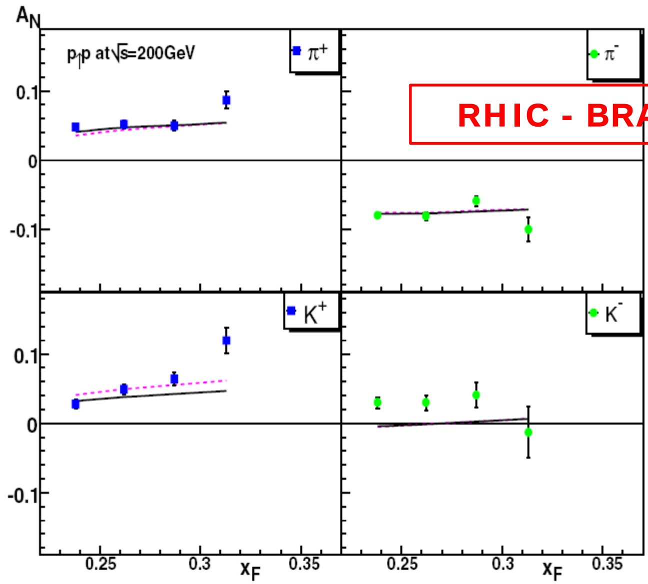


FIG. 5 (color online). Comparison of the single-spin asymmetries A_N using our fit results in Eqs. (30), (31), and (33) to the data from E704 [1]. The solid lines are for Fit I (Eq. (31)), and the dashed ones are for Fit II (Eq. (33)). The lower dotted lines in the upper left part of the figure show the contributions to A_N for $\pi^\pm$ production by the "nonderivative" terms alone, for Fit I. Note that the theory curves in the figure are normalized by $N_{E704} = 0.5$.

Kouvaris, Qiu, Vogelsang, Yuan, PRD 74 (2006)



RHIC-STAR



RHIC - BRAHMS

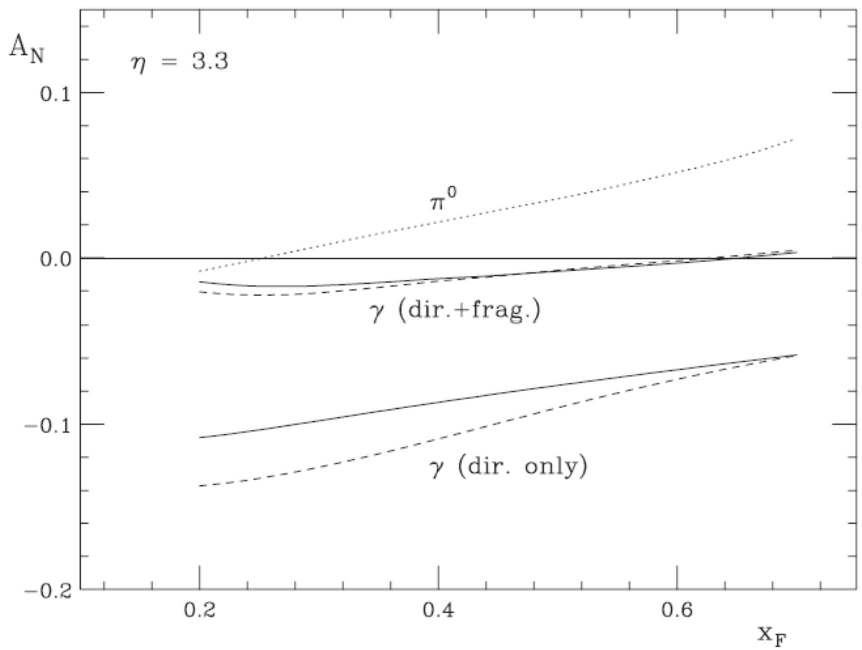


FIG. 12. Single-spin asymmetry for prompt-photon production at RHIC at $\sqrt{S} = 200$ GeV as a function of x_F for fixed pseudorapidity $\eta = 3.3$. We show the predictions for both Fit I (solid line) and Fit II (dashed line). We show separately the results for the cases when the fragmentation component is taken into account or neglected. The dotted curve shows the earlier result for π^0 production for Fit I.

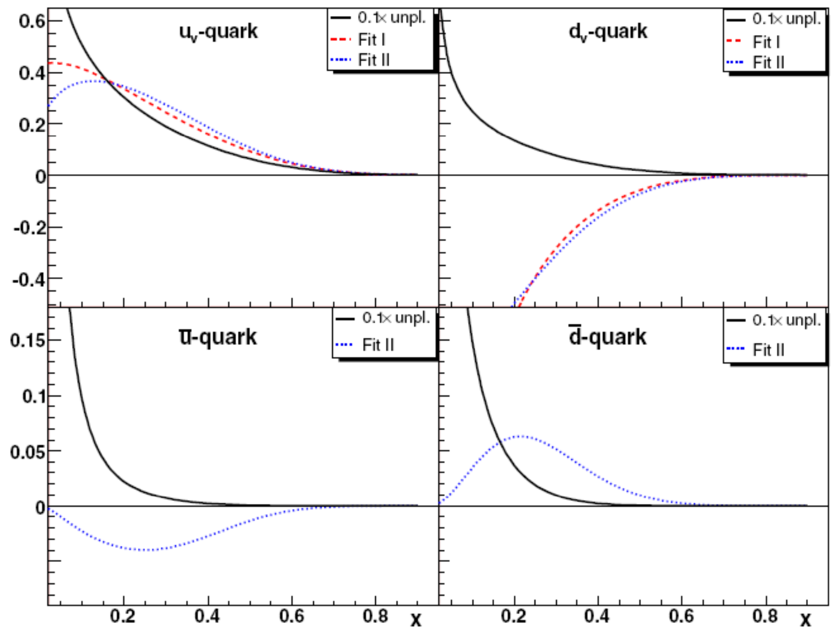


FIG. 7 (color online). $T_{a,F}$ distributions for $a = u_v$, d_v, $\bar{u}$, $\bar{d}$ resulting from our fits in Eqs. (31) and (33), at scale $\mu = 2$ GeV. We also show the corresponding unpolarized parton distribution functions [22], scaled by 1/10.

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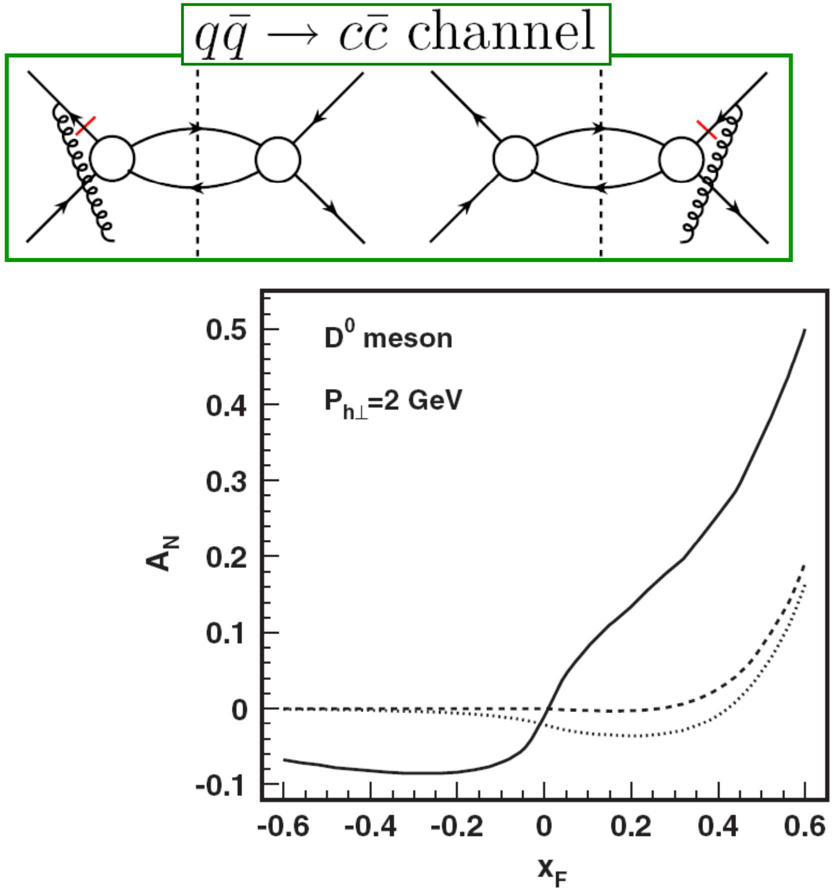
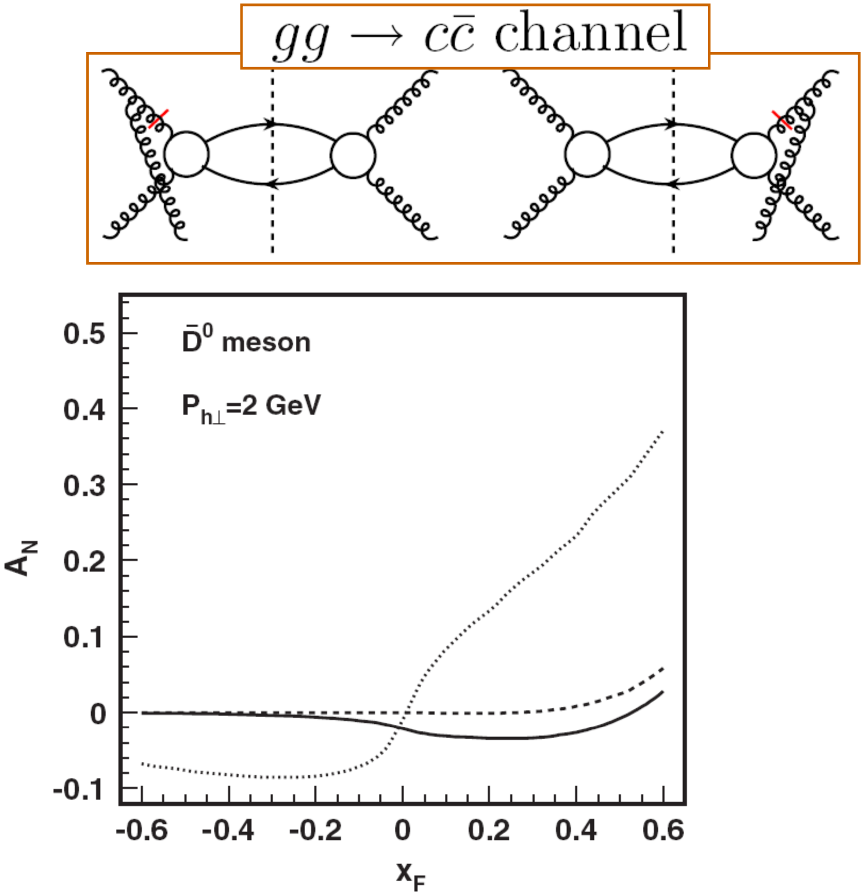
$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+ y_1^-} \frac{1}{xP^+} \langle P, s_T | F^+{}_\alpha(0) \times [\epsilon^{s_T \sigma n \bar{n}} F_\sigma{}^+(y_2^-)] F^{\alpha +}(y_1^-) | P, s_T \rangle,$$





Kang, Qiu, Vogelsang, Yuan PRD 78 (2008)

Prompt photon production in pp collisions at RHIC: a discriminating tool [GPM vs. Twist3]

Twist-three

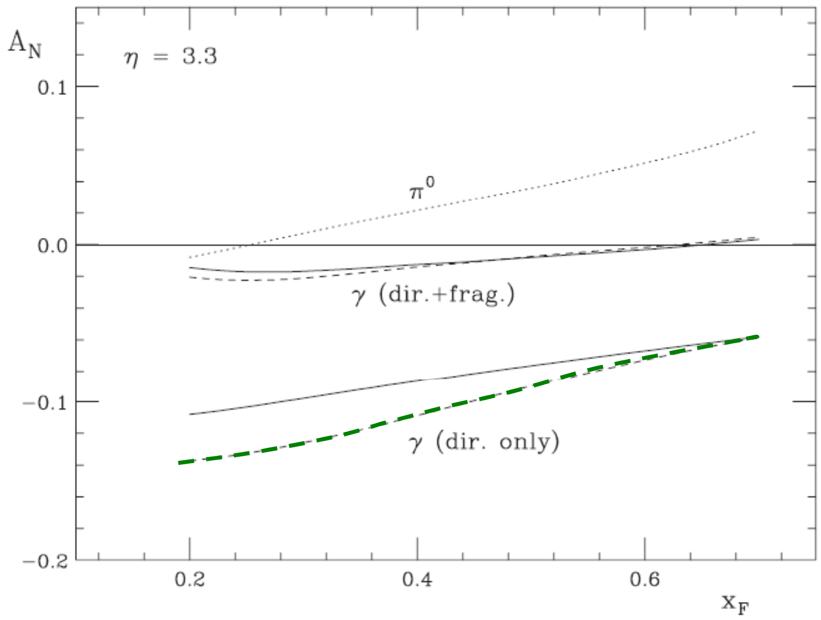


FIG. 12. Single-spin asymmetry for prompt-photon production at RHIC at $\sqrt{S} = 200$ GeV as a function of x_F for fixed pseudorapidity $\eta = 3.3$. We show the predictions for both Fit I (solid line) and Fit II (dashed line). We show separately the results for the cases when the fragmentation component is taken into account or neglected. The dotted curve shows the earlier result for π^0 production for Fit I.

Generalized Parton Model

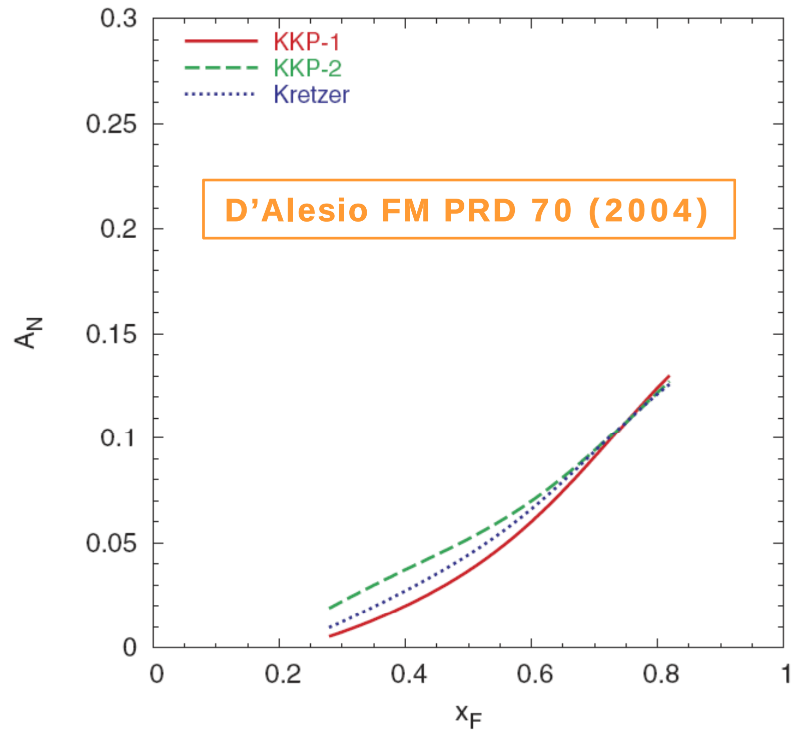


FIG. 20 (color online). A_N for inclusive photon production in pp collisions, at $\sqrt{s} = 200$ GeV and fixed rapidity $y = 3.8$, as a function of x_F. The parametrization MRST01 [25] for the unpolarized parton distributions is used. Curves correspond to different Sivers function parameterization sets (see text).

Prompt photon + jet production in pp collisions at RHIC: a discriminating tool for TMD scenarios [Generalized parton model and color gauge invariant]

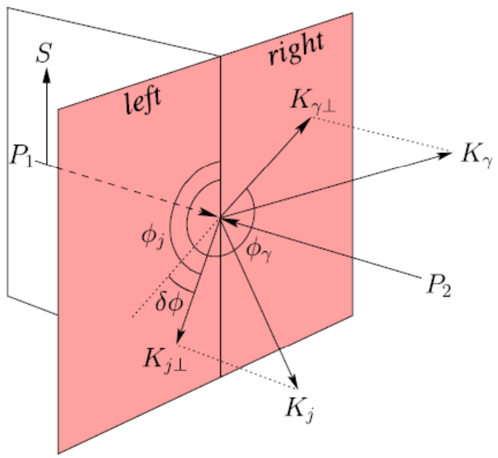


FIG. 1 (color online). Azimuthal angles involved in the process. The vectors $K_{\gamma\perp}$, $K_{j\perp}$ lie on the plane perpendicular to P_1.

$$d\hat{\sigma}_{[q]g\to\gamma q} = -\frac{N_c^2+1}{N_c^2-1} d\hat{\sigma}_{qg\to\gamma q}$$

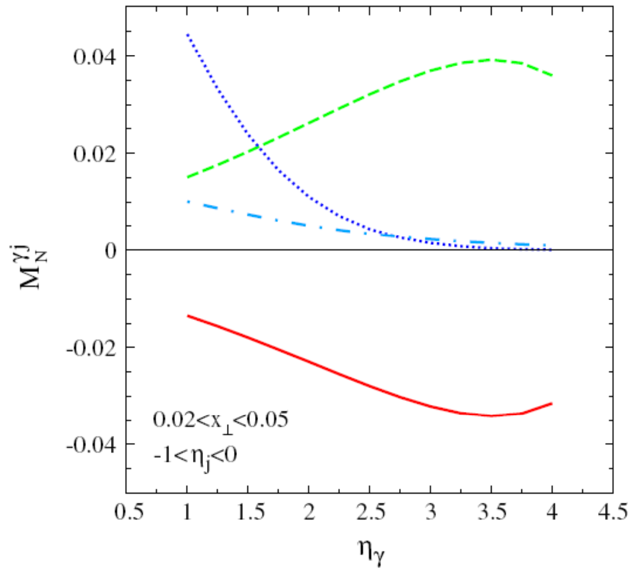


FIG. 5 (color online). Prediction for the azimuthal moment $M_N^{\gamma j}$ at $\sqrt{s} = 200$ GeV, as a function of η_γ, integrated over $-1 \le \eta_j \le 0$ and $0.02 \le x_\perp \le 0.05$. Solid line: using gluonic-pole cross sections. Dashed line: using standard partonic cross sections. Dotted line: maximum contribution from the gluon Sivers function (absolute value). Dot-dashed line: maximum contribution from the Boer-Mulders function (absolute value).

Bacchetta Bomhof D'Alesio Mulders FM PRL 99 (2007)

Useful references on spin physics
[reversed cronological order]

- U. D'Alesio, FM, Prog. Part. Nucl. Phys. 61, 394 (2008)
- V. Vogelsang, J. Phys. G 34, S149 (2007)
- S.D. Bass, Rev. Mod. Phys. 77 1257 (2005)
- V. Barone, P. Ratcliffe, Transverse Spin Physics, World Scientific (2003)
- V. Barone, A. Drago. P. Ratcliffe, Phys. Rep. 359, 1 (2002)
- E. Leader, Spin in Particle Physics, Cambridge UP (2001)
- B.W. Filippone, X.-D. Ji, Adv. Nucl. Phys. 26, 1 (2001)
- G. Bunce, N. Saito, J. Soffer, W. Vogelsang, Ann. Rev. Nucl. Part. Sci. 50, 525 (2000)
- B. Lampe, E. Reya, Phys. Rep. 332, 1 (2000)
- Z.-t. Liang, C. Boros, Int. J. Mod. Phys. A 15, 927 (2000)
- J. Felix, Mod. Phys. Lett. A 14, 827 (1999)
- S.B. Nurushev, Int. J. Mod. Phys. A 12, 3433 (1997)
- U. Stiegler, Phys. Rep. 277, 1 (1996)
- M. Anselmino, A. Efremov, E. Leader, Phys. Rep. 261, 1 (1995)
- S.M. Troshin, N.E. Tyurin, Spin Phenomena in Particle Interactions, World Scientific (1994)
- L.G. Pondrom, Phys. Rep. 122, 57 (1985)
- C. Bourrely, J. Soffer, E. Leader, Phys. Rep. 59, 95 (1980)

LO helicity amplitudes for the elementary process ab → cd

$q_a q_b \to q_c q_d$ $\qquad$ $\bar{q}_a \bar{q}_b \to \bar{q}_c \bar{q}_d$

$$|\hat{M}_1^0|^2 = \frac{8}{9}\, g_s^4 \left[\frac{\hat{s}^2}{\hat{t}^2} + \delta_{ab}\left(\frac{\hat{s}^2}{\hat{u}^2} - \frac{2}{3}\,\frac{\hat{s}^2}{\hat{t}\hat{u}}\right)\right] \qquad |\hat{M}_2^0|^2 = \frac{8}{9}\, g_s^4\, \frac{\hat{u}^2}{\hat{t}^2}$$

$$|\hat{M}_3^0|^2 = \delta_{ab}\,\frac{8}{9}\, g_s^4\, \frac{\hat{t}^2}{\hat{u}^2} \qquad \hat{M}_1^0\,\hat{M}_2^0 = \frac{8}{9}\, g_s^4 \left(-\frac{\hat{s}\hat{u}}{\hat{t}^2} + \delta_{ab}\,\frac{1}{3}\,\frac{\hat{s}}{\hat{t}}\right)$$

$$\hat{M}_1^0\,\hat{M}_3^0 = \delta_{ab}\,\frac{8}{9}\, g_s^4 \left(\frac{\hat{s}\hat{t}}{\hat{u}^2} - \frac{1}{3}\,\frac{\hat{s}}{\hat{u}}\right) \qquad \hat{M}_2^0\,\hat{M}_3^0 = \delta_{ab}\,\frac{8}{27}\, g_s^4$$

$q_a \bar{q}_b \to q_c \bar{q}_d$

$$|\hat{M}_1^0|^2 = \delta_{ac}\,\frac{8}{9}\, g_s^4\, \frac{\hat{s}^2}{\hat{t}^2} \qquad |\hat{M}_2^0|^2 = \frac{8}{9}\, g_s^4 \left(\delta_{ab}\,\frac{\hat{u}^2}{\hat{s}^2} + \delta_{ac}\,\frac{\hat{u}^2}{\hat{t}^2} - \delta_{ab}\,\delta_{ac}\,\frac{2}{3}\,\frac{\hat{u}^2}{\hat{s}\hat{t}}\right)$$

$$|\hat{M}_3^0|^2 = \delta_{ab}\,\frac{8}{9}\, g_s^4\, \frac{\hat{t}^2}{\hat{s}^2} \qquad \hat{M}_1^0\,\hat{M}_2^0 = \frac{8}{9}\, g_s^4\,\delta_{ac} \left(-\frac{\hat{s}\hat{u}}{\hat{t}^2} + \delta_{ab}\,\frac{1}{3}\,\frac{\hat{u}}{\hat{t}}\right)$$

$$\hat{M}_1^0\,\hat{M}_3^0 = \delta_{ab}\,\delta_{ac}\,\frac{8}{27}\, g_s^4 \qquad \hat{M}_2^0\,\hat{M}_3^0 = \frac{8}{9}\, g_s^4\,\delta_{ab} \left(\frac{\hat{u}\hat{t}}{\hat{s}^2} - \delta_{ac}\,\frac{1}{3}\,\frac{\hat{u}}{\hat{s}}\right)$$

LO helicity amplitudes for the elementary process ab $\to$ cd (2)

$$qg \longrightarrow qg$$

$$|\hat{M}_1^0|^2 = \frac{8}{9} g_s^4 \left(-\frac{\hat{s}}{\hat{u}} + \frac{9}{4} \frac{\hat{s}^2}{\hat{t}^2} \right) \qquad |\hat{M}_2^0|^2 = \frac{8}{9} g_s^4 \left(-\frac{\hat{u}}{\hat{s}} + \frac{9}{4} \frac{\hat{u}^2}{\hat{t}^2} \right)$$

$$\hat{M}_1^0 \hat{M}_2^0 = -\frac{8}{9} g_s^4 \left(-1 + \frac{9}{4} \frac{\hat{u}\hat{s}}{\hat{t}^2} \right).$$

$$q\bar{q} \longrightarrow gg$$

$$|\hat{M}_2^0|^2 = \frac{64}{27} g_s^4 \left(\frac{\hat{u}}{\hat{t}} - \frac{9}{4} \frac{\hat{u}^2}{\hat{s}^2} \right) \qquad |\hat{M}_3^0|^2 = \frac{64}{27} g_s^4 \left(\frac{\hat{t}}{\hat{u}} - \frac{9}{4} \frac{\hat{t}^2}{\hat{s}^2} \right)$$

$$\hat{M}_2^0 \hat{M}_3^0 = \frac{64}{27} g_s^4 \left(1 - \frac{\hat{t}\hat{u}}{\hat{s}^2} \right)$$

$$gg \longrightarrow gg$$

$$|\hat{M}_1^0|^2 = \frac{9}{2} g_s^4 \hat{s}^2 \left(\frac{1}{\hat{t}^2} + \frac{1}{\hat{u}^2} + \frac{1}{\hat{t}\hat{u}} \right) \qquad |\hat{M}_2^0|^2 = \frac{9}{2} g_s^4 \frac{\hat{u}^2}{\hat{s}^2} \left(1 + \frac{\hat{u}}{\hat{t}} + \frac{\hat{u}^2}{\hat{t}^2} \right)$$

$$|\hat{M}_3^0|^2 = \frac{9}{2} g_s^4 \frac{\hat{t}^2}{\hat{s}^2} \left(1 + \frac{\hat{t}}{\hat{u}} + \frac{\hat{t}^2}{\hat{u}^2} \right) \qquad \hat{M}_1^0 \hat{M}_2^0 = \frac{9}{2} g_s^4 \left(1 + \frac{\hat{u}}{\hat{t}} + \frac{\hat{u}^2}{\hat{t}^2} \right)$$

$$\hat{M}_1^0 \hat{M}_3^0 = \frac{9}{2} g_s^4 \left(1 + \frac{\hat{t}}{\hat{u}} + \frac{\hat{t}^2}{\hat{u}^2} \right) \qquad \hat{M}_2^0 \hat{M}_3^0 = \frac{9}{2} g_s^4 \frac{1}{\hat{s}^2} \left(\hat{u}^2 + \hat{t}^2 + \hat{u}\hat{t} \right)$$